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STRUCTURE FUNCTIONS AND LOW X WORKING GROUP SUMMARY THEORY

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This is a summary of the activities of the working group on *Structure functions and low x* during the DIS 2004 workshop held in Strbskie Pleso, Slovakia.

1 Introduction

Main topics discussed within the theoretical part of this working group were: higher order calculations (splitting functions and impact factor), NLLx BFKL evolution and resummation at small x, lattice calculations, dipole picture, parton saturation including nonlinear evolution and unintegrated parton distributions.

2 Lattice calculations

In lattice simulations a large effort has been devoted to the study of the systematics. The major source of the uncertainties are: non-perturbative renormalisation, quenching, approach to continuum limit, finite volume effects and chiral extrapolation. Shindler has shown that in particular finite volume effects can affect the matrix elements of the pion in a significant way even though in the calculation of the mass it does not seem to be the case, see Fig. 1. It is however expected that for the nucleon one can expect very large finite volume effects which might effect the evaluation of its mass.

3 Higher order calculations, NLL BFKL and resummation

One of the highlights of the DIS 2004 workshop was the full calculation of the splitting functions at NNLO presented by Vogt. The method of calculation employs the optical theorem and operator product expansion to calculate Mellin moments of the deep-inelastic structure functions. The final results have been checked to agree with all known partial results in the large and small x limits. Also the exact results are well within the bounds of the previous approximate estimates of the NNLO splitting functions. Only in the non-singlet sector a new term with a leading small x behaviour $\sim \ln^4 x$ has been found which was not predicted by the previous small

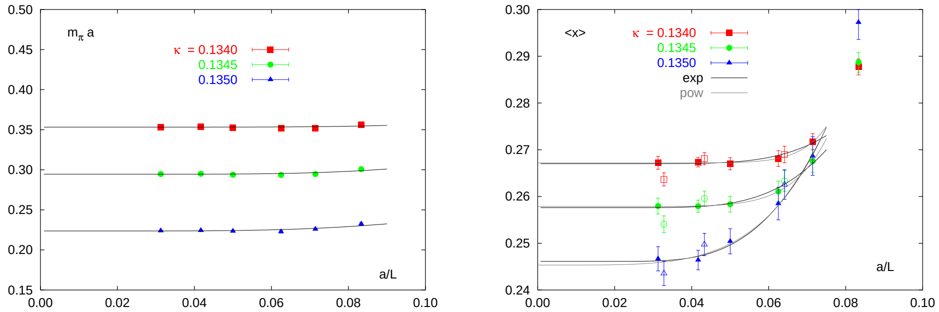

Figure 1. Left: finite size dependence of the pion mass for three values of the quark masses. Right: finite size dependence of the twist-2 operator matrix element for three values of the quark masses (talk by Shindler).

x resummations. The numerical difference between the NLO and NNLO results is not very large for moderate values of $x \geq 10^{-3}$. Of course the importance of the small x terms is greater with decreasing x. The convolutions $P \otimes f$ which enter the evolution are found to be quite sensitive to all terms, even ones which are suppressed at small x.

Apart from fixed order calculations considerable effort has been devoted to resummation of the small x terms in the gluon-gluon splitting function. Since the calculation of the BFKL equation at NLLx accuracy there were several attempts which aim to stabilise the series at low x. The common feature of these approaches is the fact that the resulting

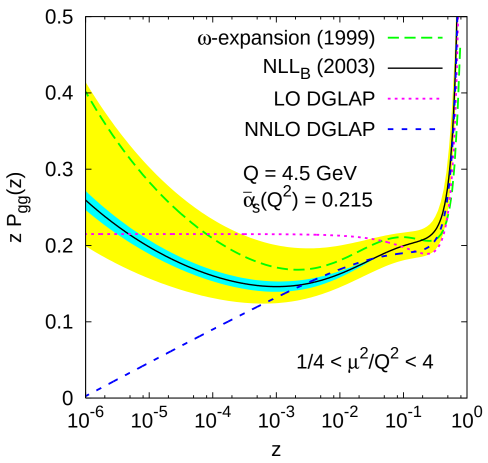

Figure 2. P_{gg} resummed splitting function determined from the collinearly improved small equation. Bands show the renormalisation scale dependence, whereas the dashed line shows the recently computed splitting function at NNLO (talk by Salam).

P_{gg} splitting function has a dip at moderately small values of x only later followed by the power increase at decreasing x, see Fig. 2. Salam has shown that within the framework of the collinearly improved small x equation this dip can be understood as a competition between a negative NLLx term $\sim \alpha_s^3 \ln 1/x$ and the positive LLx one $\sim \alpha_s^4 \ln^3 1/x$. It is interesting that the recently calculated splitting function at NNLO order is compatible with the resummed one down to the values of x characterising the dip. This can have some interesting implications for HERA.

Also some more phenomenological tests of NLLx and resummation schemes have been performed by Royon and collaborators. The idea is to try to compare the effective anomalous dimension measured from the HERA data together with the anomalous dimension resulting from evaluation of the NLLx or resummed kernel. First results show significant differences between various resummation schemes, though definite answer can be given only after a complete fit to HERA data has been performed.

The study of the BFKL equation at NLLx level using an iterative solution to the BFKL equation has been presented by Sabio-Vera. The solution includes running coupling in the MS-bar scheme and also allows to study the angular correlations. The results show the expected strong suppression of the NLLx intercept with respect to the LLx. Also the significant reduction of the scale dependence in the NLLx case is observed as nicely illustrated in Fig. 3. The iterative method has been also applied to the solution of the BFKL equation in the $N = 4$ supersymmetric Yang-Mills theory.

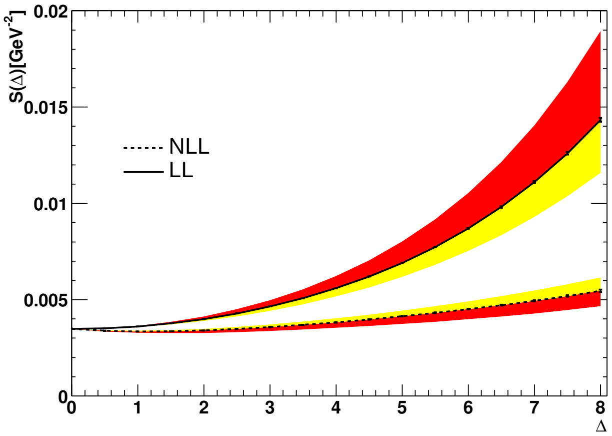

Figure 3. The gluon Green's function obtained from the iterative solution of the BFKL equation in the LLx and NLLx approximation. Bands show the renormalisation scale dependence in each of the approximations (talk by Sabio-Vera).

4 Other problems in low x physics

An analysis of the parton distributions down to very low values of Q^2 has been proposed by Kotikov by taking into account higher twist contributions. The initial condition for the leading twist part is taken to be flat in x at some small value of Q_0^2 and is subsequently evolved according to the standard DGLAP equation. The additional higher twist contribution is put in, which modifies both initial condition and the evolution itself. In this approach it is the higher twist contribution which is responsible for the slow rise of the structure function F_2 at lowest values of $Q^2 < 1\,\mathrm{GeV}^2$. The fit based on this model describes the structure function F_2 data very well down to lowest values of $Q^2 = 0.35\,\mathrm{GeV}^2$.

One of the interesting problems presented in this workshop was the study of BFKL Pomeron at finite temperature T performed by Lipatov. Since the advent of RHIC collider there have been intense studies of possible quark-gluon plasma formation. The current theoretical view is that after the quark-gluon plasma formation, the subsequent parton-parton scattering leads to the thermalisation of the plasma which after that cools down and hadronizes. One of the predictions of this theory is the suppression of the confining quark-antiquark potential. An interesting question arises whether the Pomeron, which can be viewed as a composite object of the reggeized gluons is affected by the presence of the medium. It turns out that the the resulting equation possesses still the conformal symmetry despite the presence of additional scale, temperature T. Also the intercept of the Pomeron is the same as in the $T = 0$ case. Further studies can be performed like for example non-linear Balitsky-Kovchegov equation in the non-zero temperature.

The calculation of the photon-gluon impact factor at NLLx is a very important issue in the field of small x physics mainly because of the two aspects. First, since the calculation of the NLLx BFKL it is known that the subleading corrections in the $\log 1/x$ hierarchy are quite large, and so they can affect the photon gluon impact factor as well. Secondly, the widely used dipole picture is formulated within the LLx accuracy, and so it is vital to know whether it can be extended to the NLLx level. A full calculation of the real and virtual corrections to the photon-gluon impact factor has been already performed [1], though the results are technically very complex and the integration over the remaining phase space has to be done. Kyrieleis presented first numerical results for the real corrections to the longitudinal virtual photon-gluon impact factor Φ_L. In this analysis, the real part of Φ_L has been found negative and depends on the scale choice. The photon-gluon impact factor is also a necessary ingredient in the calculation of the $\gamma^*\gamma^*$ cross section.

The problem of testing purely perturbative BFKL Pomeron is a very non-trivial one. The two scale processes in which two hard probes have large virtualities and are separated by significant value of rapidity have been proposed as a best kinematical setup for testing the BFKL dynamics. In particular the process of $\gamma^*\gamma^*$ scattering has been regarded as a very clean test of the BFKL Pomeron exchange. However, as shown by phenomenological analyses [2], at the energies of LEP the contribution from the reggeon exchanges can be still non-negligible. Lublinsky has shown that this contribution can be calculated within the perturbative QCD by summation of the ladder diagrams in the leading logarithmic approximation. The intercept calculated within this approach agrees well with the phenomenology and even though this contribution is formally subleading with respect to the Pomeron exchange it is numerically sizeable and has to be taken into account when evaluating the $\gamma^*\gamma^*$ total cross section.

In the DIS process only one large scale is involved and one might worry about potential soft contribution. Bondarenko presented a framework in which one takes soft Pomeron and the perturbative component in the common evolution equation. The soft component is estimated by summation of the ladder diagrams in which the gluon in the t-channel induces instanton transition between different vacua with subsequent multi-gluon production in each of the transition. The hard Pomeron is given by the BFKL kernel with running coupling. The effective Pomeron intercept which governs the energy dependence of the cross section has contributions which originate both from hard and soft processes. What is interesting in this model is that even at highest energies the soft contribution plays an important role.

5 Saturation

The study of parton saturation is a rapidly developing area of the small x physics and naturally it has attracted a lot of attention during the Workshop. The basic idea is that at very small x the density of gluons is so large that they start to recombine. This leads to the slow down of the growth of gluon density with energy, and new evolution equations emerge which contain nonlinear terms. The characteristic scale which divides the dilute and dense regimes is called saturation scale $Q_s(Y)$ and depends on energy (or rapidity). The perturbative saturation is believed to help in the restoration of the unitarity bound, otherwise violated by the solutions to the linear evolution equations. The Balitsky-Kovchegov equation [3] is the nonlinear equation for the dipole-nucleus scattering amplitude and it has recently attracted a lot of attention because it can be relatively easily studied at least numerically and yet contains the very nontrivial physics of parton saturation. Peschanski has shown that the Balitsky-Kovchegov equation can be well approximated by the nonlinear diffusion equation known as FKPP (Fisher-Kolmogorov-Petrovsky-Piscounov). From the solutions of this equation it is known that it has travelling wave solution at large values of rapidity Y. This property is equivalent to the geometrical scaling property for the amplitude for the dipole-nucleus scattering. The mathematical knowledge of the FKPP equation enabled to calculate universal terms in the expansion in rapidity of the saturation scale $Q_s(Y)$ as well as to get the analytical expressions for the solution around the transition region to the saturation. Similar analysis has been also performed in the case of running coupling. Analytical results from FKKP equation for the subleading terms in the expansion of the saturation scale have been confirmed by the numerical analysis of the BK equation presented by Golec. Naftali presented results on the numerical solution of the BK equation with impact parameter dependence. Using the modification of the kernel by restricting the allowed dipole sizes to be smaller than a given scale R (of the order of the target size) one can get the profile which is an exponential function in impact parameter. The agreement with the Froissart bound in that modified solution of the BK equation has also been demonstrated.

Iancu discussed the relation between the color dipole approach and the color glass condensate formalism in the description of high energy scattering. He showed that the scattering of two non-saturated color glasses in the center-of-mass frame gives the same result as obtained from onium-onium scattering in the color dipole picture thus proving that these two approaches are equivalent within certain kinematical regime. This regime is specified by the requirement that only multiple, incoherent scatterings are taken into account between two systems, however without the actual saturation of the onium wave function itself. He also discussed the role of fluctuations in the evaluation of the S-matrix in the high energy limit. This is recently very much discussed issue, see for example [4]. This phenomenon was first observed in the Monte-Carlo simulations of onium-onium scattering [5]. There the eikonal approximation, that is the assumption that the one-Pomeron amplitude is the same for each event, was found to work rather badly as compared with the exact result. As shown by Iancu typical configurations would lead to the scattering matrix S_Y which is much smaller than the one evaluated by taking into account rare configurations. The deeper analysis shows that problem is even more complicated since the classification whether the configuration is typical or rare can depend on the physical frame. Also Kozlov discussed the similar problem by employing a reggeon diagram technique to calculate the set of enhanced diagrams which are responsible for the fluctuations. The issue of the fluctuations and the validity of the BK equation will certainly be a subject intensive research in the near future. A very preliminary conclusion from these studies would be that the unitarity corrections can play quite a big role even if the actual value of the amplitude is small.

Apart from fluctuations other topics connected with the saturation have been discussed. Lipatov studied the relation between the BFKL in the dipole picture and in the usual momentum representation. He has shown that the two approaches are completely equivalent once the Pomeron Green's functions in the configuration space are redefined using the so called Möbius representation. This representation is defined by appropriate choice of the Green's functions. Roughly speaking one has certain freedom in defining these functions due to the presence of the Möbius invariance of the BFKL equation and additionally due to the fact that these functions couple to impact factors of colorless external probes. The procedure can be also applied to the evolution equation which sums the fan diagrams. This equation can be written using the $2 \to 4$ transition vertex for the reggeized gluons. The equivalence of this equation to the BK equation has been demonstrated again, by using the property of the Möbius representation for the Green's functions and by restriction to the large N_c limit. New equation has been derived which goes beyond this approximation and takes into account the four-gluon Green's function.

Kovchegov discussed the equation for the odderon in the dipole picture. The Odderon is the counterpart of the Pomeron in the case of the 3 gluon exchange. The Bartels-Kwieciński-Praszałowcz equation [6] gives the C-odd amplitude - the Odderon. It turns out that in the dipole picture the Odderon is described by the same equation as the Pomeron, the difference being in the initial conditions which have to be asymmetric with respect to the dipole coordinate exchange. In other words the initial conditions single out either Pomeron or Odderon depending on their C-parity. The solution of the Odderon equation in the dipole picture is found to coincide with the one found by Bartels-Lipatov and Vacca [7] that is $\alpha_{Odd}-1=0$ - the Odderon amplitude is constant as a function of energy. Kovchegov also showed how to include saturation corrections for the Odderon and get the corresponding nonlinear evolution equation. The exact solution to this equation is not known, however since the additional terms which emerge due to saturation are negative, one can expect that the intercept of the Odderon can only be decreased.

An interesting possibility of the instanton-driven saturation has been discussed by Utermann. It is very appealing since the instanton gauge field $A_\mu^{(I)}$, which is a non-perturbative and topologically non-trivial gluon fluctuation, has the strength $\propto 1/\sqrt{\alpha_s}$. This is the same behaviour as the classical gluon field in the color glass condensate theory. It turns out that within the I-perturbation theory supplemented by the lattice results the resulting dipole cross section has the saturating behaviour for dipole sizes which are larger than the instanton size $r \geq \langle\rho\rangle$. One has to stress that the important ingredient in this calculation is the lattice result about the distribution of the instanton size which is sharply peaked at an average value $\sim$ 0.5fm. The inverse of this average instanton size can be identified with the saturation scale $\frac{1}{\langle\rho\rangle} = Q_s$. There are of course remaining questions which have to be addressed, like the x_{Bj} dependence of the saturation scale, which is not included in the presented approach.

Saturation can be studied in various high energy processes. The deep inelastic process is the one where the formalism is best understood, however hadron-hadron collisions also offer an interesting possibility, especially at the very high energies to be accessible by LHC. Marquet presented a method to study the saturation effects in the two scale processes in the hadronic collisions. By extending the Golec-Biernat and Wüsthoff model [8] to dipole-dipole scattering he calculated the cross section for the production of two Mueller-Navelet jets in the dipole approach. There are several questions which arise, like the validity of the k_T factorisation used in this approach, which might not hold in the complicated environment of hadronic collision in the presence of saturation. Nevertheless these problems certainly deserve further theoretical and phenomenological studies.

Saturation can be also successfully studied in the diffractive processes. Munier presented a very elegant formulation of the diffractive dissociation in the scattering of the virtual photon off the proton. It is applicable in the kinematical regime where the photon virtuality is of the order of the saturation scale and much smaller than the invariant mass of the produced partonic system: $Q^2 \sim Q_s^2 \ll M_x^2$. This theoretical framework employs the dipole picture and makes use of the Good-Walker formalism in which the cross section for the diffractive dissociation can be expressed as follows

$$\frac{d\sigma_{\rm dissoc}}{d^2b} = \langle T^2(\omega)\rangle_{\omega(Y)} - \langle T(\omega)\rangle^2_{\omega(Y)} ,$$

where $\langle\ldots\rangle_{\omega(Y)}$ means integration over possible dipole configurations (dipole sizes and their positions) $\omega(Y)$ at a given value of rapidity Y. By using a Golec-Biernat and Wüsthoff model [8] for the dipole cross section one can obtain a very good description of the HERA data on diffractive dissociation as a function of Q^2, W and in bins of M_x^2. The formulation can be generalised to the arbitrary number of gluons in the final state. By improving it by the inclusion of the impact parameter dependence it could be possible to study the t - dependence as well, which might be helpful in discriminating various models and shed more light on the issue of saturation.

Rogers presented a model for the dipole-proton amplitude which includes the t-dependence. Based on this model he showed that about 20% of the cross section comes from the region which is close to the black disc limit. Also, a careful analysis of the effect of the quark mass in the low Q^2 region has been performed.

The deeply virtual Compton scattering is an excellent exclusive process in which one can test saturation ideas. Favart performed an analysis of the recent DVCS data from HERA using dipole picture together with different saturation models. He showed that an excellent description of the data can be obtained. The data at highest value of Q^2 are best described by the modified saturation model which incorporates correctly DGLAP evolution. Also, a study of the skewedness effects and the Q^2 dependence of the slope B has been performed. The possible t-slope parameter B measurement would allow to discriminate among various theoretical predictions.

The saturation effects at low x can be additionally enhanced if one considers large nucleus instead of a nucleon. Schäfer discussed the problem of the definition of k_T factorisation in case of deep inelastic scattering process of the nucleus. He showed that actually one can still use the k_T factorisation formalism as for the nucleon. The usual unintegrated gluon structure function of the free nucleon has to be replaced by the collective nuclear Weizsäcker - Williams unintegrated nuclear glue. It is an expansion over the collective gluon structure functions of overlapping nucleons in the Lorentz contracted nucleus. This formalism was then applied to estimate the angular distribution of the dijet pair produced in the DIS off nuclei. The key result of this calculation is that it predicts a stronger decorrelation between the two dijets as a result of the nuclear broadening in k_T of the unintegrated gluon structure function at small x. This effect is enhanced in the presence of saturation and leads to complete decorrelation for jet-jet system when their transverse momenta are below the saturation scale Q_s.

6 Unintegrated parton distributions functions

The standard approach to describe various hard processes in QCD is to use the collinear factorisation together with scale dependent integrated quark and gluon densities which depend upon longitudinal momentum fraction and the hard scale relevant for the process. On the other hand if one wants to describe some less inclusive quantities which are sensitive to the transverse momentum of the parton, one needs to use the alternative approach of the k_T factorisation [9] together with the unintegrated parton densities. Apart from the usual dependence on the longitudinal momentum fraction x, they possess the information about the transverse momentum k_T. When integrated over k_T the usual integrated gluon distribution is recovered $xg(x, Q^2)$ The unintegrated gluon distribution can be obtained from the CCFM equation [10] which embodies BFKL and DGLAP evolutions at the same time. This equation can be solved analytically in the 'single loop' approximation [11] in which the angular ordering constraint is replaced by the strong ordering and the non-Sudakov form factor is set equal to unity. In this approximation the CCFM equation is diagonalised in the transverse coordinate representation (using coordinate variables conjugated to transverse momenta of the gluons). Broniowski presented solution to the Kwieciński-CCFM equations which are extended to unintegrated quarks and contain full LO splitting functions. He considered parton distributions for the pion and utilised chiral quark models as a initial conditions for the evolution. These type of conditions provide with the proper normalisation, correct support in Bjorken variable as well as the momentum sum rule. The solution of the Kwieciński-CCFM evolution equations is found analytically in Mellin space and then inverted numerically into x space.

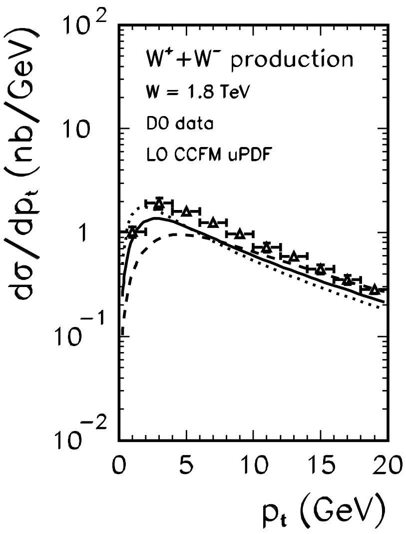

Figure 4. Transverse momentum distribution of $W^+ + W^-$ at $W = 1.8$TeV within the formalism of the CCFM unintegrated parton distributions. The data are from $D0$ experiment. Three curves correspond to different choices of the nonperturbative impact factor.

Szczurek talked about the phenomenological applications of the k_T formalism and the unintegrated gluons in various hard processes in hadronic collisions. In particular he considered the transverse momentum distribution of the gauge bosons $W^{\pm}$ produced at Tevatron, see Fig. 4. It turns out that the formalism which employs unintegrated parton distributions gives equally good description of the cross section as the standard resummation of large logarithms $\ln k_T^2/M_W^2$ in the collinear formalism [12].

Lonnblad presented results on the unintegrated parton distributions which are calculated from the Linked Dipole Chain model. He studied various effects like non-singular splitting functions and the addition of quarks into the evolution. The unintegrated parton distributions have been fitted to obtain best description of the inclusive structure function data, and then used to describe various exclusive processes, like the forward jet production in DIS and heavy quark production at the Tevatron. These processes turn out to be quite sensitive to the particular form of the unintegrated gluon distribution function. Also the predictions for the exclusive Higgs production can vary by an order of magnitude when using different forms of the unintegrated gluon. This is connected with the fact that in the cross section for this process is proportional to the fourth power of the unintegrated distribution.

The application of the k_T formalism to momentum distribution of the Higgs produced at LHC was discussed by Jung. He used full CCFM formalism to determine the unintegrated gluon and then used off-shell matrix elements to calculate the p_T spectrum of the Higgs. There are some differences with respect to the NLO calculation in the collinear approach, also this process is sensitive to the particular choice of unintegrated gluon distribution function.

To summarize, the approach based on unintegrated partons and k_T factorisation opens new direction in the description of hard processes in the hadronic collisions. It is quite promising and certainly deserves further studies, which would concentrate on some remaining issues like the unintegrated quark densities and the inclusion of NLO effects.

Acknowledgments

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