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THE INCLUSION OF HIGHER-ORDER QCD CORRECTIONS INTO PARTON SHOWER MONTE CARLOS

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Substantial progress has been made recently in the formalisms which form the basis of event generators. I discuss some of the ideas involved, emphasizing the role of higher-order QCD corrections and their interplay with parton showers

1 Introduction

Event Generators (denoted as EvG's henceforth) have been the workhorses of all modern experiments in high-energy physics. For good reasons: in spite of being conceptually simple, they provide fairly good descriptions of the real events occurring in detectors, allowing experimenters to perform a variety of tasks, from computing efficiencies to design strategies for achieving given measurements or searches. On the other hand, EvG's may not be the ideal tools for predicting the physical observables with high accuracy, something that is needed in order to – say – extracting the non-computable parameters of the theory from data; traditionally, this task is performed by a class of codes that can be called *cross section integrators* (CSI's). In a loose sense, CSI's can also output events; however, such events can be used only to predict a limited number of observables (for example, the transverse momentum of single-inclusive jets) and are not a faithful description of actual events taking place in real detectors.

Although complementary in nature, EvG's and CSI's are based on the same simple description of an elementary process (the *hard subprocess*), which doesn't even need to be a physically-observable one. To clarify this point, let us consider a gedanken experiment which, at an imaginary accelerator that collides 45 GeV u-quarks with 45 GeV $\bar{u}$-quarks, observes a $d\bar{d}$ quark pair produced through the decay of a Z^0. The process of interest is therefore $u\bar{u} \to Z^0 \to d\bar{d}$ at 90 GeV. Any theoretical model describing this process must start from the knowledge of its cross section

$$d\sigma(u\bar{u} \to Z^0 \to d\bar{d}) \;=\; \frac{1}{2\hat{s}}\,|\mathcal{M}(u\bar{u} \to Z^0 \to d\bar{d})|^2\, d\Phi_2\,, \qquad (1)$$

where $d\Phi_2$ is $d\bar{d}$ phase space, $\mathcal{M}$ is the relevant matrix element and $\hat{s}$ is the centre-of-mass energy squared. Equation (1) can be used to write an EvG or a CSI. After sampling the phase space, i.e. choosing a point in $d\Phi_2$, one has a complete description of the $u\bar{u} \to d\bar{d}$ kinematics – a *candidate event*. The candidate event's differential cross section (or *event weight*) $d\sigma$ is calculated from eq. (1) and is directly related to the probability of this event occurring. The information on such a probability can be exploited in two ways to get the distributions of the physical observables: (A) the event weights may be used to create histograms representing physical distributions, or (B) the events may be *unweighted* such that they are distributed according to the theoretical prediction. Procedure (A) is very simple and is what is done for CSI's. A histogram of some relevant distribution (e.g. the transverse momentum of the d quark) is filled with the event weights from a large number of candidate events. The individual candidate events do *not* correspond to anything observable but, in the limit of an infinite number of candidate events, the distribution is exactly the one predicted by eq. (1). Procedure (B) is a bit more involved, has added advantages, and is what is done in EvG's. It produces events with the frequency predicted by the theory being modelled, and the individual events represent what might be observed in a trial experiment—in this sense unweighted events provide a genuine simulation of an experiment. Strictly speaking, it would be desirable to talk about events only in the case of unweighted events; it is important to keep in mind that CSI's, no matter what their specific nature is, cannot output unweighted events.

What done so far is theoretically well defined, but scarcely useful, the process in eq. (1) being non physical. In fact: *a)* The kinematics of the process is trivial; the Z^0 has transverse momentum equal to zero. *b)* Quark beams cannot be prepared and isolated quarks cannot be detected. Items *a)* and *b)* have a common origin. In eq. (1) the number of both initial- and final-state particles is fixed, i.e. there is no description of the radiation of any extra particles. This radiation is expected to play a major role, especially in QCD, given the strength of the coupling constant.

In the case of item *a)*, the extra radiation taking place on top of the hard subprocess corresponds to considering higher-order corrections in perturbation theory. In the case of item *b)*, it can be viewed as an effective way of describing the dressing of a bare quark which ultimately leads to the formation of the bound states we observe in Nature (*hadronization*). Thus, any EvG or CSI which aims at giving a realistic description of collision processes must include: *i)* A way to compute exactly or to estimate the effects of higher-order corrections in perturbation theory. *ii)* A way to describe hadronization effects. Different strategies have been devised to solve these problems. They can be quickly summarized as follows. For higher orders: **HO.1)** Compute exactly the result of a given (and usually small) number of emissions. **HO.2)** Estimate the dominant effects due to emissions at all orders in perturbation theory. For hadronization: **HAD.1)** Use the QCD-improved version of Feynman's parton model ideas (the *factorization theorem*) to describe the parton $\leftrightarrow$ hadron transition. **HAD.2)** Use phenomenological models to describe the parton $\leftrightarrow$ hadron transition at mass scales where perturbation techniques are not applicable.

The simplest way to implement strategy **HO.1)** is to consider only those diagrams corresponding to the emission of real particles. Basically, the number of emissions coincides with the perturbative order in α_S. This choice forms the core of *Tree Level Matrix Element* generators. These codes can be used either within a CSI or within an EvG. A more involved procedure aims at computing all diagrams contributing to a given perturbative order in α_S, which implies the necessity of considering virtual emissions as well as real emissions. Such *N^kLO computations* are technically quite challenging and satisfactory general solutions are known only for the case of one emission (i.e., NLO). Until recently, these computations have been used only in the context of CSI's; their use within EvG's is a brand new field, and I'll deal with it in what follows.

Strategy **HO.2)** is based on the observation that the dominant effects in certain regions of the phase space have almost trivial dynamics, such that extra emissions can be recursively described. There are two vastly different classes of approaches in this context. The first one, called *resummation*, is based on a procedure which generally works for one observable at a time and, so far, has only been implemented in cross section integrators. The second procedure forms the basis of the *Parton Shower* technique and is, by construction, the core of EvG's. This procedure is not observable-specific, making it more flexible than the first approach, but it cannot reach the same level of accuracy as the first, at least formally.

At variance with the solutions given in **HO.1** and **HO.2**, solutions to the problem posed by hadronization always involve some knowledge of quantities which cannot be computed from first principles (pending the lattice solution of the theory) and must be extracted from data. The factorization theorems mentioned in **HAD.1** are the theoretical framework in which CSI's are defined. Parton shower techniques, on the other hand, are used to implement strategy **HAD.2** in the context of EvG's.

2 Event Generators at TeV Colliders

As discussed in the previous section, EvG's and those CSI's which are based upon strategy **HO.2** for the description of higher-order corrections (i.e. those that implement some kind of resummation) give exactly the same description for the observables for which the analytical computations required by the CSI's are feasible[a], provided that the logarithmic accuracy of the shower and of the resummation is the same. This is basically never the case; analytical resummations are more accurate than parton showers. In practice, some of the (formally uncontrolled) higher logarithms sneak in the showers, and the effective resummation performed by EvG's is seen to give, in many cases, results which are very close to those obtained with analytical resummation techniques. For this reason, the so-far unknown solution of the interesting and fairly challenging problem of improving the logarithmic accuracy of the showers would presumably give only marginal effects in phenomenological predictions. On the other hand, the improvement in the treatment of soft emissions at large angles would have a more visible effect, although on a more restricted class of observables.

The multiple emissions of quarks and gluons performed by the showers change the kinematics of the hard subprocess. The Z^0 of eq. (1) acquires a non-zero transverse momentum p_T by recoiling against the emitted partons. Since the parton shower is based upon a collinear approximation, one must expect the predictions of an EvG for, say, $p_T(Z^0) > 100$ GeV to be completely unreliable. Fortunately, the bulk of the cross section occurs at much smaller values of p_T, where EvG's

[a] An alternative approach to resummation, based on numerical methods, has been recently proposed in ref. [1].

do provide a sensible description of the production process. In the energy range involved in the collider physics program up to now, this was sufficient for the vast majority of the experimenters' needs.

The situation has now changed considerably. Tevatron Run II and especially LHC will feature very high-energy, high-luminosity collisions, and the events will have many more energetic well-separated particles/jets than before. An accurate description of these is necessary, especially in view of the fact that signals for many beyond-the-SM models involve in fact a large number of jets, resulting from the decay chains of particles of very high mass. The complexity of the LHC environment will be such that an incorrect description of the hard processes *may* even jeopardize the discovery potential of the machine, and will certainly prevent the experiments from performing detailed studies of the collision processes.

The collinear nature of the parton shower implies that EvG's cannot do well in predicting high-p_T processes. The fact that the description of the hard process is achieved using a leading-order picture, as outlined in the previous sections, has also a second implication: estimates of the rates (i.e., of the number of particles to be detected by the experiments) will be largely underestimated, since many processes have large K factors. Troubles arise when not only the K factors are large, but differ sizably between the various processes, since this complicates enormously the task of normalizing the signal using the background. It should be clear that the K factors needed here are those relevant to the visible regions of the detectors. It is usually assumed that the ratios of these is equal to ratios of the fully inclusive K factors. This crude approximation usually works decently, but may fail dramatically when a complex kinematics is at play.

The bottom line is that the EvG's, which have been one of the fundamental building blocks of the very successful collider physics program of the 80's and the 90's, will not perform well with the new generation of experiments. They will need either to be improved, or to be replaced.

The emphasis on large-p_T emissions implies that the only candidates for the replacement of EvG's are the CSI's that implement exactly the kinematics of the higher-order QCD corrections, thus performing N^{k}LO computations (strategy **HO.1**). Unfortunately, it is at present unknown how to cancel systematically, and without any reference to a specific observable, the infrared and collinear singularities beyond NLO. Besides, the description of the hadronization phenomena in such computations in very crude, and cannot match the sophistication of the hadronization model implemented in EvG's. Furthermore, as already mentioned, N^{k}LO computations cannot output events, which is what is absolutely needed.

Barring the possibility of replacing EvG's with something else, the only solution left is to improve them; the improved EvG's will be able to predict sensibly the large-p_T emissions, without losing their capability of treating fairly the low-p_T region, performing resummations there. Clearly, since the large-p_T region is associated with higher-order diagrams, the improvement of EvG's will be equivalent to answering the following question: *How can we insert higher-order QCD corrections into EvG's?* As I will soon discuss, there are two different, largely complementary ways, to solve this problem.

3 Matrix Element Corrections and CKKW

Since the large-p_T emissions are due to the real emission diagrams, the first strategy (denoted as Matrix Element Corrections, MEC henceforth) is that of considering only these diagrams among those contributing to higher-order QCD corrections, in this way neglecting all the diagrams with one or more virtual loops. In doing so, the possibility is given up of including the K factor consistently in the computations.

The starting point for including real emission diagrams in EvG's is that of computing them efficiently, which includes efficient samplings of very complex final-state phase spaces. Fortunately, techniques are known to highly automatize such computations, which are nowadays performed by specialized codes (the Tree Level Matrix Element generators), external to proper EvG's and interfaced to them in a standardized way for FORTRAN-based event generators by the Les Houches Accord (LHA) event record [2] (the LHA standard is supported in C++ by the HepMC [3] event record). Tree-level matrix element generators can be divided into two broad classes, which I will briefly review below; the interested reader can find more information in ref. [4].

The codes belonging to the first class feature a pre-defined list of partonic processes. Multi-leg amplitudes are strongly and irregularly peaked; for this reason the phase-space sampling has typically been optimized for the specific process. The presence of phase space routines implies that these codes are always able to output partonic events (weighted or unweighted). Popular packages are AcerMC [5], AlpGEN [6], Gr@ppa [7], MadCUP [8].

The codes belonging to the second class may be thought of as automated matrix element generator authors. The user inputs the initial and final state particles for a process. Then the program enumerates Feynman diagrams contributing to that process and writes the code to evaluate the matrix element. The programs are able to write matrix elements for *any* tree level SM process. The limiting factor for the complexity of the events is simply the power of the computer running the program. Typically Standard Model particles and couplings, and some common extensions are known to the programs. Many of the programs include phase space sampling routines. As such, they are able to generate not only the matrix elements, but to use those matrix elements to generate partonic events (some programs also include acceptance-rejection routines to unweight these events). Codes belonging to this class are AMEGIC++ [9], CompHEP [10], Grace [11], MadEvent [12].

The use of one of the codes listed above allows one to generate a final-state configuration made of hard quarks, gluons, and other non-coloured particles such as Higgs or gauge bosons. This final state is thus not directly comparable to what is observed in a detector. A drastic simplification is that of assuming that there is a one to one correspondence between hard partons and physical jets.

However, this assumption may cause problems when interfacing these codes to EvG's such as HERWIG [13] or PYTHIA [14]; a step which is necessary in order to obtain more sensible descriptions of the production processes. In fact, a kinematic configuration with n final-state partons can be obtained starting from $n-m$ partons generated by the tree-level matrix element generator, with the extra m partons provided by the shower. This implies that, although the latter partons are generally softer than or collinear to the former, there is always a non-zero probability that the *same* n-jet configuration be generated starting from *different* $(n-m)$-parton configurations. Since tree-level matrix elements do have soft and collinear singularities, a cut at the parton level is necessary in order to avoid them[b]; I will symbolically refer to this cut as y_{cut} in what follows. Physical observables should be independent of y_{cut}, but they are not; the typical dependence is of leading-log nature (i.e., $\alpha_S^k \log^{2k} y_{cut}$).

To clarify this issue with a simple example, let me consider again the hard subprocess of eq. (1), $q\bar{q} \to Z^0$. One of the NLO real contributions to this process is $q\bar{q} \to Z^0 g$. Events from these two processes should never be blindly combined, since a fraction of the latter events are already included in the former process via gluon radiation in the parton shower. Combining the two processes without special procedures amounts to *double counting* some portion of phase space.

The first approaches to the technique of MEC, which allows one to solve the double counting problem, limited themselves to the case of at most one extra hard parton wrt those present at the Born level [15,16]. These MEC can be implemented either as a strict partition of phase space between two processes, or as an event reweighting (re-evaluation of the event probability using the matrix element) using the higher order tree level matrix element for the related process. In either case the effect is the same: the event shapes are dominated by the parton shower in the low-p_T region, the shapes are NLO-like in the high-p_T region, and the total cross section remains leading order (i.e. for our example the total cross section will be the same as that for $q\bar{q} \to Z^0$). The trouble with such versions of MEC is that they can be applied only in a very limited number of cases, which are relatively simple in terms of radiation patterns and colour connections.

The way in which MEC can be achieved in the general case of n_E extra hard partons, with $n_E \geq 1$, has been clarified in ref. [17] for the case of e^+e^- collisions (referred to as CKKW after the names of the authors). The idea is the following: *a)* Integrate all the $\gamma^* \to 2 + n_E$ ME's by imposing $y_{ij} > y_{cut}$ for any pairs of partons i, j, with $y_{ij} = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})/Q^2$ the interparton distance defined according to the k_T-algorithm. *b)* Choose statistically an n_E, using the rates computed in *a)*. *c)* Generate a $(2+n_E)$-parton configuration using the exact $\gamma^* \to 2 + n_E$ ME, and reweight it with a suitable combination of Sudakov form factors (corresponding to the probability of no other branchings). *d)* Use the configuration generated in *c)* as initial condition for a *vetoed* shower. A vetoed shower proceeds as the usual one, except that it forbids all branchings $i \to jk$ with $y_{jk} > y_{cut}$ without stopping the scale evolution. Although the selection of an n_E value has a leading-log dependence on y_{cut}, it can be proved that this dependence is cancelled up to next-to-next-to-leading logs in physical observables (i.e., $\alpha_S^k \log^{2k-2} y_{cut}$), plus terms suppressed by powers of y_{cut}. It is clear that, in order to be internally consistent, matrix elements must be available for any value of $2+n_E$. In practice, $n_E \leq 3$ is a good approximation of $n_E < \infty$.

After CKKW proposed their implementation of MEC for e^+e^- collisions, an extension to hadronic collisions has been presented, without formal proof, in ref. [18];

[b] It is actually this cut that defines the "hardness" of the primary partons.

an alternative method for colour-dipole cascades has been presented in ref. [19]. There is a considerable freedom in the implementation of the CKKW prescription in the case of hadronic collisions. This freedom is used to tune (some of) the EvG's parameters in order to reduce as much as possible the y_{cut} dependence, which typically manifests itself in the form of discontinuities in the derivative of the physical spectra. A discussion on these issues, with practical examples of the implementation of CKKW in HERWIG and PYTHIA, can be found in ref. [20]. CKKW has also been implemented in SHERPA [21]; an alternative procedure, proposed by Mangano, is being implemented in AlpGEN.

I stress that the complete independence of y_{cut} cannot be achieved; this would be possible only by including all diagrams (i.e., also the virtual ones) contributing to a given order in α_S.

4 Adding virtual corrections: NLOwPS

The point made at the end of the previous section appears obvious; it is well known, and formally established by the BN and KLN theorems, that the infrared and collinear singularities of the real matrix elements are cancelled by the virtual contributions. One may in fact be surprised by the mild y_{cut} dependence left in the practical implementation of CKKW (see for example ref. [20]); however, we should keep in mind that parton showers do contain part of the virtual corrections, thanks to the unitarity constraint which is embedded in the Sudakov form factors. However, to cancel exactly the y_{cut} dependence there is no alternative way to that of inserting the exact virtual contributions to the hard process considered. In doing so, one is also able to include consistently in the computation the K factor. It is important to realize that this is *the only way* to obtain this result in a theoretically consistent way. The procedure of reweighting the EvG's results to match those obtained with CSI's for certain observables must be considered a crude approximation (since no CSI is able to keep into account all the complicated final-state correlations that are present when defining the cuts used in experimental analyses).

The desirable thing to do would be that of adding the virtual corrections of the same order as all of the real contributions to CKKW implementations. Unfortunately, this is unfeasible, for practical and principle reasons. The practical reason is that, at variance with real corrections, we don't know how to automatize efficiently the computations of loop diagrams in the Minkowskian kinematic regions. The principle reason is that there's no known way of achieving the cancellation of infrared and collinear divergences in an universal and observable-independent manner beyond NLO. We have thus to restrict ourselves to the task of including NLO corrections in EvG's; I'll denote the EvG improved in this way as NLO with Parton Showers (NLOwPS).

The fact that only one extra hard emission can be included in NLOwPS's is the reason why such codes must be presently seen as complementary to MEC. When one in interested in a small number of extra emissions, then NLOwPS's must be considered superior to MEC; on the other hand, for studying processes with many hard legs involved, such as SUSY signals or backgrounds, MEC implementations should be used. A realistic goal for the near future is that of incorporating the complete NLO corrections to all the processes with different n_E's in CKKW.

Before turning to a technical discussion on NLOwPS's, let me specify in more details the meaning of "NLO" in the context on an EvG. To do so, let me consider the case of SM Higgs production at hadron colliders, which at the lowest order, $\mathcal{O}(\alpha_S^2)$, proceeds through a loop of top quarks which is the only non-negligible contribution to the ggH effective vertex. When the p_T distribution of the Higgs is studied, we get what follows:

$$\frac{d\sigma}{dp_T} = \left(A\alpha_S^2 + B\alpha_S^3\right)\delta(p_T) + C(p_T)\alpha_S^3\,, \tag{2}$$

which means

$$\int_{p_T^{min}}^{\infty} dp_T \frac{d\sigma}{dp_T} = \mathcal{C}_3\alpha_S^3, \qquad\qquad p_T^{min} > 0 \tag{3}$$

$$= \mathcal{D}_2\alpha_S^2 + \mathcal{D}_3\alpha_S^3, \quad p_T^{min} = 0\,. \tag{4}$$

In the language of perturbative computations, the result for $p_T^{min} > 0$ would be denoted as LO, that for $p_T^{min} = 0$ as NLO. This is not appropriate for EvG's, since such a naming scheme depends on the observable considered, and EvG's produce events without any prior knowledge of the observable(s) which will eventually be reconstructed. Thus, in the context of EvG's, we generally define N^{k}LO accuracy with k the number of extra (real or virtual) gluons or light quarks wrt those present at the Born level.

Apart from this, there is a certain freedom in defining NLOwPS's. I follow here the definitions given in ref. [22], were the NLOwPS MC@NLO was first introduced:

- Total rates are accurate to NLO.
- Hard emissions are treated as in NLO computations.
- Soft/collinear emissions are treated as in MC.
- NLO results are recovered upon expansion of NLOwPS results in α_S.
- The matching between hard- and soft/collinear-emission regions is smooth.
- The output is a set of events, which are fully exclusive.
- MC hadronization models are adopted.

The fourth condition above defines the absence of double counting in NLOwPS's. In other words: *An NLOwPS is affected by double counting if its prediction for any observable, at the first order beyond the Born approximation in the expansion in the coupling constant, is not equal to the NLO prediction.* According to this definition, double counting may correspond to either an excess or a deficit in the prediction, at any point in phase space. This includes contributions from real emission and virtual corrections.

Let me now consider a generic hard production process, whose nature I don't need to specify, except for the fact that its LO contribution is due to $2 \to 2$ subprocesses, which implies that real corrections will be due to $2 \to 3$ subprocesses;

these conditions are by no means restrictive, and serve only to simplify the notation. Let O be an observable whose value can be computed by knowing the final-state kinematics emerging from the hard processes. At the NLO, we can write the distribution in O as follows:

$$\left(\frac{d\sigma}{dO}\right)_{subt} = \sum_{ab}\int dx_1\, dx_2\, d\phi_3\, f_a(x_1) f_b(x_2) \Big[\delta(O - O(2\to 3))\mathcal{M}_{ab}^{(r)}(x_1,x_2,\phi_3) + \delta(O - O(2\to 2))\Big(\mathcal{M}_{ab}^{(b,v,c)}(x_1,x_2,\phi_2) - \mathcal{M}_{ab}^{(c.t.)}(x_1,x_2,\phi_3)\Big)\Big]. \quad (5)$$

Here, $\mathcal{M}_{ab}^{(r)}$ is the contribution of the real matrix elements, whereas $\mathcal{M}_{ab}^{(b,v,c,c.t.)}$ are the contributions of the Born, virtual, collinear reminders and collinear counterterms; $O(2\to n)$, with $n = 2, 3$, is the value of the observable O as computed with 2- and 3-body final states. The form of eq. (5) is borne out by the universal formalism for cancelling the infrared and collinear divergences proposed in refs. [23,24], upon which MC@NLO is based. Other equivalent forms could be used at this point, without changing the conclusions.

In order to predict the distribution of O using an EvG, one computes the value of O for each event generated by the shower. The most compact way of describing how an EvG works is through the generating functional, which is basically the incoherent sum of all possible showers

$$\mathcal{F}_{\rm MC} = \sum_{ab}\int dx_1\, dx_2\, d\phi_2\, f_a(x_1) f_b(x_2)\, \mathcal{F}_{\rm MC}^{(2\to 2)}\mathcal{M}_{ab}^{(b)}(x_1,x_2,\phi_2), \quad (6)$$

where $\mathcal{F}_{\rm MC}^{(2\to 2)}$ is the generating functional for parton-parton scattering, with a $2\to 2$ configuration as a starting condition for the showers.

In the attempt of merging NLO and EvG, we observe that in eqs. (5) and (6) the short distance matrix elements serve to determine the normalization of the results, and the hard process kinematics. Such kinematics configurations are evolved by the showers $\mathcal{F}_{\rm MC}^{(2\to 2)}$ in eq. (6), and the resulting final states eventually used to compute the value of O. A similar "evolution" is performed in the context of the NLO computations by the δ functions appearing in eq. (5); clearly, the evolution is trivial in this case. However, this suggests that the incorporation of NLO results into EvG's may simply amount to replacing $\delta(O - O(2\to n))$ with $\mathcal{F}_{\rm MC}^{(2\to n)}$, i.e. with the generating functionals of the showers whose initial conditions are $2\to 2$ and $2\to 3$ hard kinematics configurations. It should be stressed that this strategy, that I'll call the *naive NLOwPS prescription*, actually works at the LO, since eq. (6) can be obtained from eq. (5) following this prescription, if terms beyond LO are dropped from the latter equation.

Unfortunately, things are more complicated than this. Basically, when $\mathcal{F}_{\rm MC}^{(2\to 2)}$ acts on $\mathcal{M}_{ab}^{(b)}$ in the analogue of eq. (5) obtained by applying the naive NLOwPS prescription, it generates terms that contribute to the NLO prediction of O, which are not present in eq. (5). According to the definition given above, this amounts to double counting. Furthermore, the weights associated with $\mathcal{F}_{\rm MC}^{(2\to 2)}$ and $\mathcal{F}_{\rm MC}^{(2\to 3)}$ (i.e., the coefficients multiplying $\delta(O-O(2\to 2))$ and $\delta(O-O(2\to 3))$ in eq. (5) respectively) are separately divergent. These divergences are known to cancel thanks to the KLN theorem and the infrared safeness of O; however, this happens efficiently in the case of the NLO computations, thanks to the fact that the final-state configurations with which the values of O are computed coincide with the hard configurations. This is not the case when the showers are attached, since the evolutions implicit in $\mathcal{F}_{\rm MC}^{(2\to 2)}$ and $\mathcal{F}_{\rm MC}^{(2\to 3)}$ are not correlated (and must not be so). This means that the naive prescription outlined above, apart from double counting, requires an infinite amount of CPU in order for the cancellation of the infrared divergences to occur.

I'll now show how these problems are solved in the context of MC@NLO [22,25]. We observe that, if the shower evolution attached to the Born contribution in the naive prescription results in spurious NLO terms, one may try to remove "by hand" such terms. Denoting by $\mathcal{M}_{\mathcal{F}(ab)}^{\rm (MC)}$ the terms that we'll actually remove, the following equation holds:

$$\mathcal{M}_{\mathcal{F}(ab)}^{\rm (MC)} = \mathcal{F}_{\rm MC}^{(2\to 2)} \mathcal{M}_{ab}^{(b)} + \mathcal{O}(\alpha_S^2 \alpha_S^b), \tag{7}$$

where α_S^b is the perturbative order corresponding to the Born contribution. Clearly, eq. (7) leaves lot of freedom in the definition of $\mathcal{M}_{\mathcal{F}(ab)}^{\rm (MC)}$ (which we denote as *MC counterterms*), in that all terms of NNLO and beyond are left unspecified. In MC@NLO, we defined the MC counterterms using eq. (7), and requiring all terms beyond NLO to be zero. With this, we define the MC@NLO generating functional as follows:

$$\begin{aligned}
\mathcal{F}_{\rm MC@NLO} = \sum_{ab} \int & dx_1\, dx_2\, d\phi_3\, f_a(x_1) f_b(x_2) \\
& \Big[\mathcal{F}_{\rm MC}^{(2\to 3)} \Big(\mathcal{M}_{ab}^{(r)}(x_1,x_2,\phi_3) - \mathcal{M}_{ab}^{\rm (MC)}(x_1,x_2,\phi_3) \Big) + \\
& \mathcal{F}_{\rm MC}^{(2\to 2)} \Big(\mathcal{M}_{ab}^{(b,v,c)}(x_1,x_2,\phi_2) - \mathcal{M}_{ab}^{(c.t.)}(x_1,x_2,\phi_3) + \mathcal{M}_{ab}^{\rm (MC)}(x_1,x_2,\phi_3) \Big) \Big].
\end{aligned} \tag{8}$$

Eq. (8) is identical to what one would have got by applying the naive NLOwPS prescription discussed above to eq. (5), except for the fact that the short-distance coefficient have been modified by adding and subtracting the MC counterterms; for this reason, MC@NLO is said to be based upon a modified subtraction method. At the first glance, it may appear surprising that the MC counterterms have been added twice, with different signs, since their role is that of eliminating the spurious terms arising from the evolution of the Born term. However, this is what they do indeed. In fact, the evolution of the Born term also includes a contribution due to the so-called non-branching probability, i.e. the probability that nothing happens. This corresponds to a would-be deficit of the naive NLOwPS prediction, which is taken into account by our definition of double counting.

Remarkably, the solution of the problem of double counting also solves the problem of the cancellation of the infrared and collinear divergences in a finite amount of time. In fact, the weights attached to the two generating functionals on the r.h.s.

of eq. (8) are now separately finite locally in the phase space. This is so since the showers are constructed to reproduce the behaviour of the collinear emissions as predicted by perturbation theory, and this in turn implies that the MC counterterms locally match the singular behaviour of the real matrix elements, hence the name "counterterms" (there are subtleties due to the peculiar treatment of soft emissions in showers, which are technically too involved to be discussed here; the interested reader may find all the details in ref. [22]). This fact also implies that MC@NLO produces events identical in nature to those of standard EvG's, since unweighting can be performed at the level of short-distance contributions. As a consequence, the convergence properties (i.e., the smoothness of the physical distributions) are much better than those of the corresponding NLO codes; typically, to achieve the same level of fluctuations, MC@NLO has to sample the phase space about 50 times less than the NLO code from which it is derived. This pattern is followed by all of the processes implemented in MC@NLO, whose (growing) list can be found with the package at `http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/`.

An important point to stress is that the computation of the MC counterterms requires a detailed knowledge of what the EvG does when performing the shower. This means that the MC counterterms are specific to a given Monte Carlo implementation: those corresponding to HERWIG differ from those corresponding to PYTHIA. Presently, MC@NLO can only be interfaced to HERWIG, since only the MC counterterms relevant to HERWIG have been computed. It is also worth mentioning that the form of the MC counterterms doesn't depend on the hard process considered; thus, their computation is performed once and for all. A second point is that NLOwPS's are in general not positive definite, i.e. a fraction of the generated events will have negative weights. Fortunately, this fraction is fairly small, and future work may lead to its further reduction.

In spite of attracting a considerable amount of theoretical interest in the past few years, at the moment there are only a couple of codes, plus MC@NLO, that can be used to produce actual events in hadronic collisions. Phase-space veto has been introduced in ref. [26], elaborating on an older idea presented in ref. [27], and applied to Z^0 production. The approach is interesting since no negative-weight events are produced. However, as shown in ref. [22], this is obtained at the price of double counting in certain regions of the phase space. Although the practical impact of such double counting seems to be modest for the physical process considered, it remains to be seen how the method can be generalized in order to treat processes more complicated from the point of view of kinematics and colour configurations. The code grcNLO [28] is characterized by the numerical computation of all the matrix elements involved. In order not to do double counting, the short distance cross sections have to be interfaced with an *ad-hoc* shower, i.e., the interfacing with HERWIG or PYTHIA does produce double counting. The method has so far been applied to Z^0 production, and efforts are being made in order to implement $Z^0 + 1$ jet production.

5 Conclusions

In the past few years, many significant advances have been achieved in the theory and implementation of the event generators. Although a considerable amount of work remains to be done, it is fair to say that the codes of the new generation will be up to the challenge posed by Tevatron Run II and LHC physics.

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