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# PROTON SPIN STRUCTURE AND INTRINSIC MOTION OF THE CONSTITUENTS

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The spin structure of the system of quasifree fermions having total angular momentum $J = 1/2$ is studied in a consistently covariant approach. Within this model the relations between the spin functions are obtained. Their particular cases are the sum rules Wanzura - Wilczek, Efremov - Leader - Teryaev, Burkhardt - Cottingham and also the expression for the Wanzura - Wilczek twist 2 term $g_2^{WW}$. With the use of the proton valence quark distributions as an input, the corresponding spin functions are obtained. The resulting structure functions $g_1$ and $g_2$ are well compatible with the experimental data. Comparison with the basic formulas following from the standard quark-parton model reveals the importance of the quark intrinsic motion inside the target for the correct evaluation of the spin structure functions.

## 1 Introduction

In this talk some results following from the covariant quark-parton model (QPM) will be shortly discussed, details of the model can be found in Refs. [1], [2]. In this version of QPM valence quarks are considered as quasifree fermions on mass shell. Momenta distributions describing the quark intrinsic motion have spherical symmetry corresponding to the constraint $J = 1/2$, which represents the total angular momentum - generally consisting of spin and orbital parts. I shall mention the following items: i) What sum rules follow from this approach for the spin structure functions $g_1$ and $g_2$? ii) How can these structure functions be obtained from the valence quark distributions $u_V$ and $d_V$ - if the $SU(6)$ symmetry is assumed? The results are compared with the existing experimental data. iii) Why the first moment $\Gamma_1$ calculated in this approach can be substantially less, than the corresponding moment calculated within the standard, non covariant QPM, which is based on the infinite momentum frame?

Recently, this model was generalized to include also the transversity distribution, for details I refer to [3].

## 2 Model

The model is based on the set of distribution functions $G_{k,\lambda}(\frac{pP}{M})$, which measure probability to find a quark in the state:

$$u\left(p,\lambda\mathbf{n}\right)=\frac{1}{\sqrt{N}}\begin{pmatrix}\phi_{\lambda\mathbf{n}}\\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}}\end{pmatrix};\qquad \frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}}=\lambda\phi_{\lambda\mathbf{n}},\qquad \lambda=\pm\frac{1}{2},$$

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where **n** coincides with the direction of the proton polarization **J**. Correspondingly, $m$ and $p$ are quark mass and momentum, similarly $M$ and $P$ for the proton. With the use of these distribution functions one can define the function $H$, which in the target rest frame reads:

$$H(p_0)=\sum_{k=1}^{3} e_k^2 \Delta G_k(p_0); \qquad \Delta G_k(p_0)=G_{k,+1/2}(p_0)-G_{k,-1/2}(p_0), \tag{1}$$

where $e_k$ represent the charges of the proton valence quarks. In the paper [1] I shown, how from the generic function $H$ the spin structure functions can be obtained. If one assume $Q^2 \gg 4M^2x^2$, then:

$$g_1(x)=\frac{1}{2}\int H(p_0)\left(m+p_1+\frac{p_1^2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0}; \quad x=\frac{Q^2}{2M\nu},$$

$$g_2(x)=-\frac{1}{2}\int H(p_0)\left(p_1+\frac{p_1^2-p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0},$$

which implies

$$g_T(x)\equiv g_1(x)+g_2(x)=\frac{1}{2}\int H(p_0)\left(m+\frac{p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0}.$$

Let me remark, that procedure for obtaining the functions $g_1, g_2$ from the distribution $H$ is rather complex, nevertheless the task is well-defined and unambiguous. Resulting structure functions are related to a naive QPM, in which the relativistic kinematics and spheric symmetry (which follows from the requirement $J=1/2$) are consistently applied. Both these requirements are very important.

## 3 Sum rules

One can observe, that the functions above have the same general form

$$\int H(p_0)f(p_0,p_1,p_T)\delta\left(\frac{p_0+p_1}{M}-x\right)d^3p \tag{2}$$

and differ only in kinematic term $f$. This integral, due to spheric symmetry and presence of the $\delta-$function term, can be expressed as a combination of the momenta:

$$V_n(x)=\int H(p_0)\left(\frac{p_0}{M}\right)^n\delta\left(\frac{p_0+p_1}{M}-x\right)d^3p. \tag{3}$$

One can prove [2], that these functions satisfy

$$\frac{V_j'(x)}{V_k'(x)}=\left(\frac{x}{2}+\frac{x_0^2}{2x}\right)^{j-k}; \qquad x_0=\frac{m}{M}.$$

This relation then gives possibility to obtain integral relations between different functions having form (3) or (2), in particular for $g_1(x)$ and $g_2(x)$ one gets:

$$g_2(x)=-\frac{x-x_0}{x}g_1(x)+\frac{x\,(x+2x_0)}{(x+x_0)^2}\int_x^1\frac{y^2-x_0^2}{y^3}g_1(y)dy,$$

$$g_1(x) = -\frac{x}{x-x_0} g_2(x) - \frac{x+2x_0}{x^2-x_0^2} \int_x^1 g_2(y) dy$$

and for limiting case $m \to 0$:

$$g_2(x) = -g_1(x) + \int_x^1 \frac{g_1(y)}{y} dy,$$

$$g_1(x) = -g_2(x) - \frac{1}{x} \int_x^1 g_2(y) dy.$$

Obviously, the first relation is the known expression for Wanzura - Wilczek twist-2 term for $g_2$ approximation [4].

Further, if one define

$$\langle x^\alpha \rangle = \int_0^1 x^\alpha V_0(x) dx,$$

then one can prove that

$$\int_0^1 x^\alpha \left[ g_1(x) + g_2(x) \right] dx = \langle x^\alpha \rangle \frac{\alpha+1}{(\alpha+2)(\alpha+3)},$$

$$\int_0^1 x^\alpha g_2(x) dx = -\langle x^\alpha \rangle \frac{\alpha(\alpha+1)}{(\alpha+2)(\alpha+3)}$$

for *any* $\alpha$, for which the integrals exist. Apparently these relations imply

$$\int_0^1 x^\alpha \left[ \frac{\alpha}{\alpha+1} g_1(x) + g_2(x) \right] dx = 0,$$

which for $\alpha = 2, 4, 6, ...$ corresponds to the Wanzura - Wilczek sum rules [4]. Other special cases correspond to the Burkhardt - Cottingham ($\alpha = 0$) [5] and the Efremov - Leader - Teryaev (ELT, $\alpha = 1$) [6] sum rules. Let me point out, that all these rules here were obtained only on the basis of covariant kinematics and requirement of rotational symmetry.

## 4 Valence quarks

Now I shall try to apply the suggested approach to the description of the real proton. For simplicity I assume:

1) Spin contribution from the sea of quark-antiquark pairs and gluons can be neglected, so the proton spin is generated only by the valence quarks.

2) In accordance with the non-relativistic *SU(6)* approach, the spin contribution of individual valence terms is given by fractions:

$$s_u = 4/3, \qquad s_d = -1/3. \tag{4}$$

If the symbols $h_u$ and $h_d$ denote momenta distributions of the valence quarks in the proton rest frame, which are normalized as

$$\frac{1}{2} \int h_u(p_0) d^3p = \int h_d(p_0) d^3p = 1, \tag{5}$$

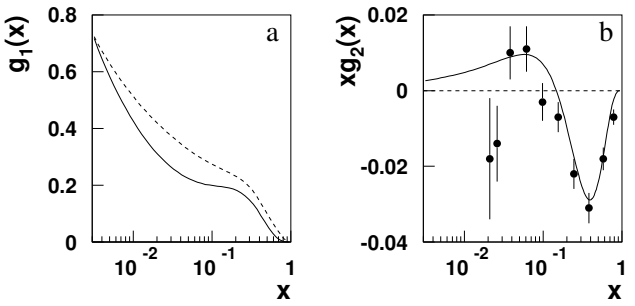


Figure 1. Proton spin structure functions. My calculation, which is represented by the full lines, is compared with the experimental data: dashed line ($g_1$) and full circles ($g_2$), see text.

then the generic distribution (1) reads

$$H(p_0) = \sum e_j^2 \Delta h_j(p_0) = \left(\frac{2}{3}\right)^2 \frac{2}{3} h_u(p_0) - \left(\frac{1}{3}\right)^2 \frac{1}{3} h_d(p_0). \tag{6}$$

In the paper [7], using a similar approach, I studied also the unpolarized structure functions. Structure function $F_2$ can be expressed as

$$F_2(x) = x^2 \int G(p_0) \frac{M}{p_0} \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p; \qquad G(p_0) = \sum_q e_q^2 h_q(p_0). \tag{7}$$

On the other hand, for proton valence quarks one can write

$$F_2(x) = \frac{4}{9} x u_V(x) + \frac{1}{9} x d_V(x), \tag{8}$$

so combination of the last two relations gives:

$$q_V(x) = x \int h_q(p_0) \frac{M}{p_0} \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p; \qquad q = u, d. \tag{9}$$

Since this is again the integral having the structure (2), one can apply the technique of integral transforms and (instead of relation between $g_1$ and $g_2$) obtain the relations between $g_j^q$ and $q_V$. For $m \to 0$ these relations read:

$$g_1^q(x) = \frac{1}{2}\left[q_V(x) - 2x^2 \int_x^1 \frac{q_V(y)}{y^3} dy\right],$$

$$g_2^q(x) = \frac{1}{2}\left[-q_V(x) + 3x^2 \int_x^1 \frac{q_V(y)}{y^3} dy\right].$$

Now, taking quark charges and corresponding $SU(6)$ factors as in Eq. (6), one can directly calculate $g_1, g_2$ only using the input on the valence quark distribution $q_V = u_V, d_V$. In Fig. 1 the results of $g_1$ and $g_2$ calculation is shown.

Experimental data on $g_1$ are represented by the new parameterization of the world data [8] and the $g_2$ points are data of the E155 Collaboration [9]. More detailed discussion of these figures is done in [2], in this talk I want concentrate on the discussion and explanation, why intrinsic quark motion substantially reduces the first moment of the spin function $g_1$. In [1] it is shown, that

$$\Gamma_1 \equiv \int g_1(x)dx = \frac{1}{2}\int H(p_0)\left(\frac{1}{3}+\frac{2m}{3p_0}\right)d^3p, \tag{10}$$

which, in the $SU(6)$ approach gives

$$\frac{5}{18} \geq \Gamma_1 \geq \frac{5}{54},$$

where left limit is valid for the static ($p_0 \to m$) and right one for massless quarks ($m \to 0$). In other words, it means:

$$\textit{more intrinsic motion} \Leftrightarrow \textit{less spin}$$

This is a mathematical result, but how to understand it from the point of view of physics?

First, forget structure functions for a while and calculate completely another task. Let me remind general rules concerning angular momentum in quantum mechanics:

1) Angular momentum consist of orbital and spin part: **j=l+s**

2) In the relativistic case **l** and **s** are not conserved separately, only total angular momentum **j** is conserved. So, one can have pure states of $j(j^2, j_z)$ only, which are for fermions with $s = 1/2$ represented by the relativistic spheric waves, see e.g. [10]:

$$\psi_{jlj_z}(\mathbf{p}) = \frac{1}{\sqrt{2p_0}}\begin{pmatrix} i^{-l}\sqrt{p_0+m}\,\Omega_{jlj_z}\left(\frac{\mathbf{p}}{p}\right) \\ i^{-l'}\sqrt{p_0-m}\,\Omega_{jl'j_z}\left(\frac{\mathbf{p}}{p}\right)\end{pmatrix}; \qquad j = l \pm \frac{1}{2}, \qquad l' = 2j - l,$$

$$\Omega_{l+1/2,l,j_z}\left(\frac{\mathbf{p}}{p}\right) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}Y_{l,j_z-1/2} \\ \sqrt{\frac{j-j_z}{2j}}Y_{l,j_z+1/2}\end{pmatrix},$$

$$\Omega_{l'-1/2,l',j_z}\left(\frac{\mathbf{p}}{p}\right) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}Y_{l',j_z-1/2} \\ \sqrt{\frac{j+j_z+1}{2j+2}}Y_{l',j_z+1/2}\end{pmatrix}.$$

This wavefunction is simplified for the state with total angular momentum (spin) equal 1/2:

$$j = j_z = \frac{1}{2}, \qquad l = 0 \qquad \Rightarrow \qquad l' = 1,$$

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = i\sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{11} = -i\sqrt{\frac{3}{8\pi}}\sin\theta\exp(i\varphi),$$

which gives

$$\psi_{jlm}\left(\mathbf{p}\right)=\frac{1}{\sqrt{8\pi p_0}}\left(\begin{array}{c}\sqrt{p_0+m}\begin{pmatrix}1\\0\end{pmatrix}\\-\sqrt{p_0-m}\begin{pmatrix}\cos\theta\\\sin\theta\exp\left(i\varphi\right)\end{pmatrix}\end{array}\right).$$

Let me remark, that $j=1/2$ is minimum angular momentum for particle with $s=1/2$. Now, one can easily calculate the average contribution of the spin operator to the total angular momentum:

$$\Sigma_3=\frac{1}{2}\begin{pmatrix}\sigma_3&\cdot\\\cdot&\sigma_3\end{pmatrix}\Rightarrow$$

$$\psi^{\dagger}_{jlm}\left(\mathbf{p}\right)\Sigma_3\psi_{jlm}\left(\mathbf{p}\right)=\frac{1}{16\pi p_0}\left[\left(p_0+m\right)+\left(p_0-m\right)\left(\cos^2\theta-\sin^2\theta\right)\right]$$

If $a_p$ is the probability amplitude of the state $\psi_{jlm}$, then

$$\left\langle\Sigma_3\right\rangle=\int a^{\star}_p a_p\psi^{\dagger}_{jlm}\left(\mathbf{p}\right)\Sigma_3\psi_{jlm}\left(\mathbf{p}\right)d^3p=\frac{1}{2}\int a^{\star}_p a_p\left(\frac{1}{3}+\frac{2m}{3p_0}\right)p^2dp, \qquad (11)$$

which means, that:

*i)* For the fermion at rest ($p_0=m$) we have $j=s=1/2$, which is quite comprehensible, since without kinetic energy no orbital momentum can be generated.

*ii)* For the state in which $p_0\geq m$, we have in general:

$$\frac{1}{3}\leq\frac{\left\langle s\right\rangle}{j}\leq 1.$$

where left limit is valid for the energetic fermion, $p_0\gg m$. In other words, in the states $\psi_{jlm}$ with $p_0>m$ part of the total angular momentum $j=1/2$ is *necessarily* created by orbital momentum. This is a simple consequence of quantum mechanics.

Now, one can compare integrals (10) and (11). Since both integrals involve the same kinematic term, the interpretation of dependence on ratio $m/p_0$ in (11) is valid also for (10). Otherwise, the comparison is a rigorous illustration of the statement, that $\Gamma_1$ measures contributions from quark spins (and not their total angular momenta).

In which point the present approach differ from standard QPM? Standard approach is closely connected with the preferred reference frame - infinite momentum frame. The basic relations like

$$g_1(x)=\frac{1}{2}\sum e_j^2\Delta q_j(x), \qquad F_2(x)=x\sum e_i^2 q_i(x)$$

are derived with the use of approximation

$$p_\alpha=xP_\alpha.$$

In the covariant formulation this relation is equivalent to the assumption, that the quarks are static. In the presented covariant approach quarks are not static, so this approximation cannot be used. As a result, different relations between the distribution and structure functions and also different behavior of $\Gamma_1$ are obtained.

## 5 Summary

I have studied spin functions in system of quasifree fermions having fixed effective mass $x_0 = m/M$ and total spin $J = 1/2$ - representing a covariant version of naive QPM. The main results are:

1) Spin functions $g_1$ and $g_2$ depend on intrinsic motion. In particular, the momenta $\Gamma_1$ corresponding to the static (massive) fermions and massless fermions, can differ significantly: $\Gamma_1(m \ll p_0)/\Gamma_1(p_0 \approx m) = 1/3$. It is due to splitting of angular momentum into spin and orbital part, as soon as intrinsic motion is present.

2) $g_1$ and $g_2$ are connected by a simple transformation, which is for $m \to 0$ identical to Wanzura - Wilczek relation for twist-2 term of the $g_2$ approximation. Relations for the $n-th$ momenta of the structure functions have been obtained, their particular cases are identical to known sum rules: Wanzura - Wilczek ($n = 2, 4, 6...$), Efremov - Leader - Teryaev ($n = 1$) and Burkhardt - Cottingham ($n = 0$).

3) Model has been applied to the proton spin structure, assuming proton spin is generated only by spins and orbital momenta of the valence quarks with *SU(6)* symmetry and for quark effective mass $m \to 0$. As an input I used known parameterization of the valence terms, then without any other free parameter, the functions $g_1$, $g_2$ were obtained. Comparison with the proton data demonstrates a good agreement.

4) Comparison with the corresponding relations for the structure functions following from the usual naive QPM was done. Both the approaches are equivalent for the static quarks. Differences for quarks with internal motion inside the proton are result of the conflict with the assumption $p_\alpha = xP_\alpha$, which is crucial for derivation of the relations in the standard QPM.

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