Proceedings of the DIS'2004, Štrbské Pleso, Slovakia

CHARGE ASYMMETRY IN 2π AND 3π ELECTROPRODUCTION ON PROTON AT HIGH ENERGIES

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Abstract
The charge asymmetry induced by interference of amplitudes with σ and ρ mesons decaying in the $\pi^+\pi^-$ pair created in the fragmentation region of proton is suggested to be a test of the degeneration hypothesis of ρ and σ mesons. Some numerical estimations are given.

Charge asymmetry in 3π production provides the measurement of $2\gamma \to 3\pi$ anomaly of Wess-Zumino-Witten effective lagrangian.

1 Introduction

Theoretical reasons for the existence of scalar neutral meson (σ-meson) were formulated in the late 60s [1]. It was recognized that the chiral and scaling symmetry of strong interactions were violated, which can be realized within an effective lagrangian by including a scalar field $\sigma(x)$. In the frameworks of QCD it was shown that breaking of scale invariance was related to the trace of the energy-momentum tensor [2]. In the papers by Schechter, Ellis, and Lanik [3] the effective QCD lagrangian with broken scale and chiral symmetries was constructed where a scalar gluonic current was related with $\sigma(x)$: $G^2_{\mu\nu} \sim m^4_\sigma \sigma(x)^4/G_0, G_0 =< 0|(\alpha_s/\pi G^2_{\mu\nu}|0 >= 0.017GeV^4$- is the gluonic condensate. Besides the widths of 2 pion and 2 gamma decay channels it was obtained

$$\Gamma(\sigma \to \pi^+\pi^-) = \frac{m^5_\sigma}{48\pi G_0}, \quad \Gamma(\sigma \to \gamma\gamma) = \frac{3}{4}\left(\frac{R\,\alpha}{8\pi^2}\right)^2 \Gamma(\sigma \to \pi^+\pi^-), \tag{1}$$

with $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$. Experimental evidence of possible existence of $\sigma(750)$ was obtained at CERN experiments for the process $\pi^-p \to \pi^-\pi^+n$ [4][5] with a polarized target. In S. Weinberg's paper [6] itwas shown that the relation $m_\rho = m_\sigma$ is a consequence of broken chiral symmetry. A similar statement follows from the analysis of superconvergent sum rules for helicity amplitudes, as was shown in the paper by Gillman and Harari [7]. Really, taking this value for the σ meson we obtain for its total (mainly 2 pion) width $\Gamma(\sigma \to 2\pi) = 150MeV = \Gamma_\rho$.

To obtain independent evidence of σ and the validity of the degeneracy $m_\rho = m_\sigma$, $\Gamma_\rho = \Gamma_\sigma$, we suggest measuring the charge asymmetry of two pion production at electron-proton collisions

$$e(p_1) + p(p) \to e'(p'_1) + p'(p') + \pi^+(q_+) + \pi^-(q_-), \tag{2}$$

which is defined as follows:

$$A_c = \frac{d\sigma(q_1, q_2) - d\sigma(q_2, q_1)}{d\sigma(q_1, q_2) + d\sigma(q_2, q_1)} = \frac{N(\pi^+(q_1), \pi^-(q_2)) - N(\pi^+(q_2), \pi^-(q_1))}{N(\pi^+(q_1), \pi^-(q_2)) + N(\pi^+(q_2), \pi^-(q_1))}, \tag{3}$$

where $d\sigma(q_1, q_2)$ means the inclusive cross section with π^- meson with momentum q_1 and π^+ with momentum q_2, and $N(\pi^+(q_1), \pi^-(q_2))$ is the number of corresponding events.

Asymmetry in the case of electroproduction does not depend on the total center of mass energy ($\sqrt{s}$) due to a dominant contribution of amplitudes with photon exchange in the scattering channel.

In the case of pion pair photoproduction on proton the asymmetry is decreasing function of the center-of-mass energy, which is contrary to the case of electroproduction.

Interference of 2 photon mechanism and bremsstrahlung mechanism of electroproduction contains the anomalous vertex $\gamma\gamma \to 3\pi$ of effective Wess-Zumino-Witten lagrangian. It can be measured as a charge-odd asymmetry of 3π electroproduction.

2 Calculation of asymmetry

Charge asymmetry can be more pronounced at invariant mass of pions close to the ρ meson mass where due to the Breit-Wigner enhancement of the cross section the counting rate is expected to be large.

The matrix element in this region can be put in the form $\mathcal{M} = \mathcal{M}_\rho + \mathcal{M}_\sigma$ (see fig.1). Then the charge asymmetry will have the form

$$A_c = \frac{2\mathcal{M}_\rho(\mathcal{M}_\sigma)^*}{|\mathcal{M}_\sigma|^2 + |\mathcal{M}_\rho|^2}. \tag{4}$$

To obtain the realistic estimation magnitude of the effect we take into account only Feynman amplitudes containing the σ, ρ intermediate states.

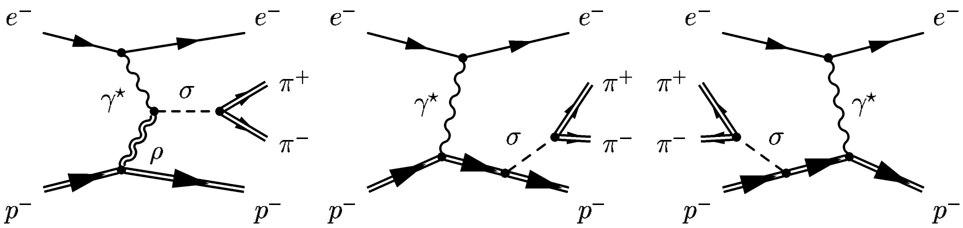

Figure 1. Feynman Diagrams (FD) for the matrix element $\mathcal{M}_\sigma$. By exchange $\sigma \to \rho, \rho \to \sigma$ one can obtain FD for $\mathcal{M}_\rho$

The matrix elements have the form

$$\mathcal{M}_\sigma = J_\mu \bar{u}(p') T^\mu u(p) R_\sigma g_{\sigma\pi\pi} g_{\sigma nn}, \tag{5}$$
$$\mathcal{M}_\rho = J_\mu \bar{u}(p') T^{\mu\nu} u(p)(q_- - q_+)^\nu R_\rho g_{\rho\pi\pi} g_{\rho nn},$$

with (for kinematics see (2))

$$J_\mu = \frac{4\pi\alpha}{Q^2}\bar{u}(p_1')\gamma_\mu u(p_1), \quad R_{\sigma,\rho} = \frac{1}{s_1 - m^2_{\sigma,\rho} + i(m\Gamma)_{\sigma,\rho}}, \tag{6}$$
$$Q^2 = -q^2, \quad s_1 = q_2^2, \quad q_2 = q_+ + q_-, \quad q = p_1 - p_1'.$$

Besides, $g_{\rho\pi\pi} = 4\sqrt{3\pi/\beta_\rho^3}\sqrt{\Gamma_\rho/m_\rho}$ and $g_{\sigma\pi\pi} = 4\sqrt{\pi/\beta_\sigma}\sqrt{m_\sigma\Gamma_\sigma}$ are the coupling constants, $\beta_{\rho,\sigma} = \sqrt{1-(4m_\pi^2/m^2_{\rho,\sigma})}$ is the velocity of pions in the rest frame of decaying σ or ρ mesons.

The hadronic currents obey the current conservation condition

$$\bar{u}(p')T_\mu u(p) q^\mu = 0, \quad \bar{u}(p')T^{\mu\nu}u(p) q_\mu = 0, \quad \bar{u}(p')T^{\mu\nu}u(p) q_{2\nu} = 0. \tag{7}$$

The expressions for T we have used are:

$$T^\mu = \Lambda\frac{1}{q_1^2 - m_\rho^2}R_1^{\mu\nu}\gamma_\nu + \frac{\hat{p}+\hat{q}+m_p}{(p+q)^2 - m_p^2}\gamma^\mu + \gamma_\mu\frac{\hat{p}'-\hat{q}+m_p}{(p'-q)^2-m_p^2}, \tag{8}$$

and

$$T^{\mu\nu} = \frac{1}{\Lambda}\frac{1}{q_1^2 - m_\sigma^2}R_2^{\mu\nu} + \gamma_\nu\frac{\hat{p}+\hat{q}+m_p}{(p+q)^2 - m_p^2}\gamma_\mu + \gamma_\mu\frac{\hat{p}'-\hat{q}+m_p}{(p'-q)^2-m_p^2}\gamma_\nu, \tag{9}$$

with $\Lambda = g_{\rho nn}/g_{\sigma nn}$, $q_1 = p - p'$. The vertex $\gamma^*\sigma\rho$ we parameterize as

$$R^{\mu\nu}_{1,2} = \frac{g\,m}{m^2+Q^2}(q^\nu q^\mu_{1,2} - g^{\mu\nu} q q_{1,2}), \tag{10}$$

which is an anzatz inspired by low-order triangle Feynman diagram calculation, and $m = 300MeV$ is the constituent quark mass. The coupling g is chosen in order to reproduce the $(g_{\rho\pi\pi}/e)^2\Gamma(\sigma\to\gamma\gamma)$. The factor $(g_{\rho\pi\pi}/e)^2$ is introduced to take into account the replacement of one of the photons by the vector meson. Our estimate gives $g \approx 2$.

By calculating the matrix element which is given above we obtain charge asymmetry in the form

$$A_c = \frac{2[(s_1-m_\rho^2)(s_1-m_\sigma^2) + m_\rho m_\sigma \Gamma_\rho\Gamma_\sigma]Q^{\mu\lambda}(q_- - q_+)^\nu Z_{\mu\nu\lambda}\Lambda g_{\rho\pi\pi}g_{\sigma\pi\pi}}{Q^{\eta\sigma}[|R_\rho|^{-2}Z_{\eta\sigma}g^2_{\sigma\pi\pi} + \Lambda^2|R_\sigma|^{-2}Z_{\eta\mu_1\sigma\nu_1}g^2_{\rho\pi\pi}(q_- - q_+)_{\mu_1}(q_- - q_+)_{\nu_1}]} \tag{11}$$

with

$$Q^{\mu\nu} = 2p_1^\mu p_1^\nu + (Q^2/2)g^{\mu\nu}, \tag{12}$$
$$Z_{\mu\nu\lambda} = Tr(\hat{p}'+m_p)T_\mu(\hat{p}+m_p)\tilde{T}_{\lambda\nu},$$
$$Z_{\eta\sigma} = Tr(\hat{p}'+m_p)T_\eta(\hat{p}+m_p)\tilde{T}_\sigma,$$
$$Z_{\eta\mu_1\sigma\nu_1} = Tr(\hat{p}'+m_p)T_{\eta\mu_1}(\hat{p}+m_p)\tilde{T}_{\sigma\nu_1}.$$

For inclusive set-up (pions as well as the scattered electron are detected) the numerator and the denominator must be integrated by phase volume of the scattered proton.

3 Discussion

We neglect above the final-state pion and pion-nucleon interaction which causes the pion-pion scattering phases [8]. It can be justified within 5% accuracy for the case of rather large effective mass of pions in the final state.

In Tables 1-4[a] we present the x_+, x_- distribution (which means energy fraction of π^+ and π^- in the case of the energies of experiment H1) for the equal masses $m_\sigma = m_\rho = 769$ MeV and different masses.

The integration of asymmetry was made over variables $\vec{q}_\pm$ in the region $0.01 - 0.9$ GeV for each x, y component of the vector $\vec{q}_\pm$:

$$Ac_{int}(x_+, x_-, Q) = \frac{\int d^2 q_+ \int d^2 q_- 2\mathcal{M}_\rho (\mathcal{M}_\sigma)^*}{\int d^2 q_+ \int d^2 q_- (|\mathcal{M}_\sigma|^2 + |\mathcal{M}_\rho|^2)} \tag{13}$$

We can see that the case $m_\sigma = m_\rho$ can be unambiguously separated from the $m_\sigma \neq m_\rho$ case within 5% accuracy of experimental data.

Asymmetry has a magnitude of order 1 ($|A_c| \sim 1$) in the fragmentation region of the initial proton for small values of momentum transferred $Q \sim 0.5$ GeV $\ll \sqrt{s} \sim 100$ GeV (DESY, H1). At higher Q it decreases rapidly: for $Q \sim 3$ GeV $|A_c| < 0.1$.

In Tables 5-7 we give asymmetry as a function of angles θ_+, θ_- for the HERMES energies, $\theta_\pm = \widehat{p_e q_\pm}$, integrated over $\varepsilon_+, \varepsilon_-$ (energies of π_+, π_-) in the region $0.5 - 20$ GeV, and over the angles ϕ_+, ϕ_- ($\phi_\pm = \widehat{q_\perp q_{\pm\perp}}$) in the region $0 - 2\pi$, $Q = 0.2$ GeV

$$Ac_{int}(\theta_+, \theta_-, Q = 0.2) = \frac{\int\limits_0^{2\pi} \int\limits_0^{2\pi} d\phi_+ d\phi_- \int \int d\varepsilon_+ d\varepsilon_- 2\mathcal{M}_\rho (\mathcal{M}_\sigma)^*}{\int\limits_0^{2\pi} \int\limits_0^{2\pi} d\phi_+ d\phi_- \int \int d\varepsilon_+ d\varepsilon_- (|\mathcal{M}_\sigma|^2 + |\mathcal{M}_\rho|^2)} \tag{14}$$

Note that asymmetry in Tables 5-7 turns to zero not only when $\theta_+ = \theta_-$, but also at the points $\theta_+ = -\theta_-$.

In all numerical estimates given in Tables we used the following values:

$$\Lambda = \frac{g_{\rho nn}}{g_{\sigma nn}} = 1, \quad m = 0.300\ GeV, \quad m_\rho = 0.769\ GeV, \tag{15}$$
$$\Gamma_\rho = \Gamma_\sigma = 150\ GeV, \quad M_p = 0.98\ GeV, \quad m_\pi = 0.139\ GeV.$$

4 Acknowledgements

One of us (V.V.B) is grateful to the HERMES Collaboration for valuable discussions and warm hospitality during May 2003. We acknowledge the support of RFBR, grant 03-02-17077, and two of us E.K. and V.B. the support INTAS, grant 00-00-366.
E.A.K. is grateful to L.Shimanovski and O.Teryaev for critical discussions.

[a] Tables are available in the hep/ph preprint

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