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THE INS AND OUTS OF $g-2$

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We review the Standard Model prediction of $g-2$, with particular emphasis on recent improvements of the hadronic vacuum polarization contributions. The result, when confronted with the latest experimental world average from BNL, reveals a $2.7\,\sigma$ discrepancy. A new evaluation of the hadronic contributions to $\alpha(M_Z^2)$ is also briefly discussed.

1 Standard Model contributions to $g-2$; recent developments

The anomalous magnetic moments of electrons and muons, $a_{e,\mu} \equiv (g-2)/2$, are among the most precisely measured quantities. While a_e is dominated by electromagnetic interactions and serves as a test of QED and a measurement of the electromagnetic coupling α, the anomaly of the much heavier muon, a_μ, obtains also non-negligible contributions from electroweak (EW) and strong interactions. For the sensitivity to effects due to 'New Physics' beyond the Standard Model (SM), the muon is typically favoured by $m_\mu^2/m_e^2 \sim 43000$, which by far exceeds the better precision of the measurement of a_e compared to a_μ ($\sim$ a factor 140).

The dominant contribution to the anomaly comes from the one-loop QED correction, $a^{1-\mathrm{loop}} = \alpha/(2\pi) \sim 11614097 \cdot 10^{-10}$, calculated first by Schwinger. Higher order QED corrections are known up to four-loop accuracy with estimates of five-loop contributions. Kinoshita and Nio recently have improved and corrected their earlier result, and we quote their latest result for the total QED contribution [1]: $a_\mu^{\mathrm{QED}} = (116584719.35 \pm 1.43) \cdot 10^{-11}$. EW contributions to a_μ are known to two-loop accuracy. Recent evaluations [2,3], although slightly different conceptually, agree very well numerically, and we quote the result of [3], $a_\mu^{\mathrm{EW}} = (152 \pm 2) \cdot 10^{-11}$. The third part of the corrections to $g-2$ within the SM, the hadronic contributions, are usually divided into three classes: leading order vacuum polarization (VP), next-to-leading order VP corrections and the so-called light-by-light scattering contributions, $a_\mu^{\mathrm{had}} = a_\mu^{\mathrm{had,LO}} + a_\mu^{\mathrm{had,NLO}} + a_\mu^{\mathrm{had,l-by-l}}$. As a_μ^{had} is dominated by the low energy part of the QCD spectrum, it cannot be calculated reliably in perturbation theory. First 'first principles' estimates from lattice gauge theory have become available (in quenched approximation) but are plagued with very large uncertainties. However, the vacuum polarization induced corrections, $a_\mu^{\mathrm{had,LO}}$, $a_\mu^{\mathrm{had,NLO}}$, can be predicted by using dispersion relations and experimental data for $e^+e^- \to$ *hadrons*. Alternatively, spectral functions for hadronic τ decays can be used, after corrections for the Iso-spin breaking are taken into account. We will discuss $a_\mu^{\mathrm{had,LO,NLO}}$ in the next section. For the hadronic light-by-light scattering contributions, $a_\mu^{\rm had,l-by-l}$, no dispersion relation exists and one relies on models for low energy hadronic physics. Various estimates have been made, and the early works predicted a negative contribution, before in 2001 sign errors were discovered. As a conservative estimate *after the sign change* we quote the prediction from Nyffeler [4], $a_\mu^{\rm had,l-by-l} = (+80 \pm 40) \cdot 10^{-11}$. Recently Melnikov and Vainshtein have recalculated the light-by-light contributions, applying additional constraints from perturbative QCD. This leads to an increase of the value and a decrease of the error in their prediction, $a_\mu^{\rm had,l-by-l} = (136 \pm 25) \cdot 10^{-11}$.[5]

2 Recent progress for the hadronic VP contributions

The dispersion integral to calculate the leading order VP contributions reads

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} {\rm d}s\, \sigma^0_{\rm had}(s) K(s). \tag{1}$$

$K(s) = m_\mu^2/(3s) \cdot (0.63\ldots 1)$ is a monotonic kernel function which gives strong weight to the lowest energy part of the hadronic cross section. $\sigma^0_{\rm had}$ stands for the total cross section $\sigma(e^+e^- \to hadrons)$ without vacuum polarization (running coupling) effects, but which has to include final state photon radiation. The treatment of data w.r.t. both these radiative corrections introduces uncertainties which — in most cases apart from very recent experiments — have to be estimated in addition to the statistical and systematic errors as given by the experiments.

The most important channels contributing to $\sigma^0_{\rm had}$ are $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, K^+K^-, $K^0_S K^0_L$, $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^+\pi^-$, but in total ~ 24 channels have to be combined to arrive at a sufficiently accurate prediction. For some (higher multiplicity pionic) final states not enough data exist and one has to rely on Iso-spin relations to estimate their contributions. Also details like the behaviour very close to thresholds have to be taken care of. Where no sufficient data is available we have estimated the threshold behaviour using chiral perturbation theory. For large energies (above ~ 11 GeV) data is not very accurate, but the prediction from perturbative QCD — if state-of-the-art higher order mass corrections are included — is very reliable. With the high quality recent data from CMD-2 and SND (at VEPP in Novosibirsk) even the narrow resonances ω, ϕ can be taken into account by direct integration of data. This has the advantage that no resonance parametrization has to be imposed, that there are no complications from resonance tails or non-resonant backgrounds and that automatically all interference effects are taken into account.

Our result for the leading order hadronic VP contribution is[a]

$$a_\mu^{\rm had,LO} = (692.4 \pm 5.9_{\rm exp} \pm 2.4_{\rm rad}) \cdot 10^{-10}. \tag{2}$$

Using a similar dispersion relation for the next-to-leading order VP induced corrections we update the earlier result from [7], obtaining [6]

$$a_\mu^{\rm had,NLO} = (-9.79 \pm 0.09_{\rm exp} \pm 0.03_{\rm rad}) \cdot 10^{-10}. \tag{3}$$

[a] For many more details and a complete list of references see [6].

3 The new experimental result from BNL, a sign of New Physics?

Combining the hadronic VP contributions (2, 3) with the QED, EW and hadronic light-by-light contributions [5] as discussed above we arrive at the SM prediction for $(g-2)/2$ of the muon:

$$a_\mu^{\rm SM} = (11659183.5 \pm 6.7) \cdot 10^{-10}. \tag{4}$$

Recently the Brookhaven $g-2$ experiment has published their final analysis using μ^- data from 2001. [8] The result is well compatible with their earlier results using μ^+ data, and the new world average is [8]

$$a_\mu^{\rm exp} = (11659208 \pm 6) \cdot 10^{-10}. \tag{5}$$

From the above numbers it is clear that $g-2$ of the muon is sensitive to all sectors of the SM. Despite all recent changes, the theoretical prediction is still smaller than the experimental measurement. We display the current situation in Fig. 1. The

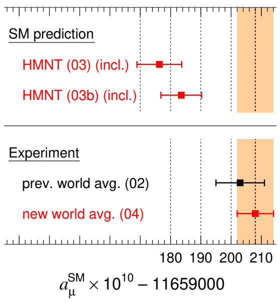

Figure 1. $a_\mu^{\rm SM}$ compared to the new world average from BNL [8].

shift upwards of the SM prediction from 'HMNT(03)' to 'HMNT(03b)', which is due to the recent changes in the QED and hadronic light-by-light contributions as discussed above, is practically 'compensated' by a similar shift of $a_\mu^{\rm exp}$ from the latest BNL result [8]. The discrepancy, still $2.7\,\sigma$, could easily be explained by extensions of the SM like the minimal supersymmetric SM (MSSM). In turn and maybe even more important, the smallness of the deviation already constrains the parameter space of the MSSM and other models beyond the SM considerably. But how trustworthy is the deviation? Both QED and EW contributions to $a_\mu^{\rm SM}$ are very well under control, and a further large shift upwards in $a_\mu^{\rm had,l-by-l}$ seems hard to believe given the model constraints. For the hadronic contributions, all depends on the input data. The e^+e^- data based predictions for $a_\mu^{\rm had,LO,NLO}$ crucially rely on the most accurate $\pi^+\pi^-$ data from CMD-2 [9]. It is puzzling that the use of Isospin corrected τ spectral functions leads to a result for a_μ which is larger than and incompatible with the evaluation based on e^+e^- data alone, see [10]. Nevertheless, as has been pointed out in [11], this puzzle may be due to incomplete consideration of a possible $\rho^0 - \rho^\pm$ mass difference, and further studies are required here. On the other hand, the e^+e^- data from CMD-2 are confirmed by a very recent analysis from KLOE which use radiative return to measure the $e^+e^- \to \pi^+\pi^-(\gamma)$ cross section for energies below the ϕ resonance.[12] With the same technique, BABAR (and hopefully also BELLE) will soon be able to provide independent measurements of several hadronic channels like $\pi^+\pi^-$ and 4π. Together with upgrades of e^+e^- machines, e.g. in Novosibirsk (VEPP2000), Beijing (BES) and Cornell (CLEO-C), these new developments and further theoretical work will help to make the prediction of $a_\mu^{\rm SM}$ more accurate in the future, and the comparison with the proposed new and improved measurements of $g-2$ even more exciting.

4 A new evaluation of $\alpha(M_Z^2)$

Even at high energies, the low energy hadronic spectrum leads to a sizeable change in the 'running' QED coupling, $\alpha(s) = \alpha/(1-\Delta\alpha(s))$. Similar to the VP contributions to $g-2$, the hadronic contributions $\Delta\alpha_{\rm had}^{(5)}$ (from the five light flavours not including the top quark) can be calculated via a dispersion integral, $\Delta\alpha_{\rm had}^{(5)}(s) = -\alpha s/(3\pi)\,{\rm P}\int_{s_{\rm th}}^\infty {\rm d}s'\,R(s')/(s'(s'-s))$. Compared to (1), which is dominated by low energies, here also higher energies are important. With the same comprehensive e^+e^- data compilation as for $g-2$ we arrive at the new prediction, $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.02755 \pm 0.00023$. [6] After adding the perturbatively calculable leptonic and top quark contributions, this corresponds to $\alpha(M_Z^2)^{-1} = 128.954 \pm 0.031$. This new, e^+e^- data based result compares well with other determinations but has a smaller error. It is very imortant in global fits of EW precision observables, which indirectly determine the Higgs mass. For a more detailed discussion see [6].

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