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NEW PREDICTIONS FOR ELECTROWEAK $\mathcal{O}(\alpha)$ CORRECTIONS TO NEUTRINO–NUCLEON SCATTERING — IMPLICATIONS FOR NUTEV?

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We calculate the $\mathcal{O}(\alpha)$ electroweak corrections to charged- and neutral-current deep-inelastic neutrino scattering off an isoscalar target. The full one-loop-corrected cross sections, including hard photonic corrections, are evaluated and compared to an earlier result which is the basis of the NuTeV analysis. In particular, we compare results that differ in input-parameter scheme, treatment of real photon radiation and factorization scheme. The associated shifts in the theoretical prediction for the ratio of neutral- and charged-current cross sections can be larger than the experimental accuracy of the NuTeV result. This work is described in more detail in a recently published paper[1].

1 Introduction

Deep-inelastic neutrino scattering has been analyzed in the NuTeV experiment [2] with a rather high precision. In detail, the neutral- (NC) to charged-current (CC) cross-section ratios [3]

$$R^{\nu} = \frac{\sigma^{\nu}_{NC}(\nu_\mu N \to \nu_\mu X)}{\sigma^{\nu}_{CC}(\nu_\mu N \to \mu^- X)}, \; R^{\bar\nu} = \frac{\sigma^{\bar\nu}_{NC}(\bar\nu_\mu N \to \bar\nu_\mu X)}{\sigma^{\bar\nu}_{CC}(\bar\nu_\mu N \to \mu^+ X)}$$

have been measured to an accuracy of about 0.2% and 0.4%, respectively. In addition, the quantity

$$R^{-} = \frac{\sigma^{\nu}_{NC}(\nu_\mu N \to \nu_\mu X) - \sigma^{\bar\nu}_{NC}(\bar\nu_\mu N \to \bar\nu_\mu X)}{\sigma^{\nu}_{CC}(\nu_\mu N \to \mu^- X) - \sigma^{\bar\nu}_{CC}(\bar\nu_\mu N \to \mu^+ X)},$$

as proposed by Paschos and Wolfenstein [4], has been considered. As a central result, the NuTeV collaboration has translated their measurements of $R^{\nu/\bar\nu}$ and R^- into values for the on-shell weak mixing angle, $\sin^2\theta_W = 1 - m_W^2/m_Z^2$, which can be viewed as independent (but rather indirect) determinations of the W- to Z-boson mass ratio. The NuTeV result on $\sin^2\theta_W$ is, however, about 3σ away from the result obtained from the global fit [5] of the Standard Model (SM) to the electroweak precision data.

It was pointed out [6] that the inclusion of electroweak radiative corrections, which influences the result significantly, is based on a single calculation [7] only[a] and that a careful recalculation of these corrections would be desirable. In this work we summarize a recent publication [1] describing such a calculation of the $\mathcal{O}(\alpha)$ electroweak corrections to NC and CC deep-inelastic neutrino scattering off an

[a] Electroweak radiative corrections were also investigated in Refs. [8], where in the numerical evaluation several approximations were made. Although the input used in Refs. [8] is obsolete, the prediction made in these references is relatively close to the results of our analysis.

isoscalar target. Apart from a different set of parton densities and input parameters the most important difference between our calculation and the result of Ref. [7] lies in the treatment of mass singularities due to collinear radiation of a photon from external charged particles.

2 Lowest-Order Results

We consider the NC and CC parton processes

$$\begin{aligned} \text{NC:}\quad \nu_\mu(p_l) + q(p_q) &\;\to\; \nu_\mu(k_l) + q(k_q), & (1)\\ \text{CC:}\quad \nu_\mu(p_l) + q(p_q) &\;\to\; \mu^-(k_l) + q'(k_q), & (2) \end{aligned}$$

where in Eq. (1) the generic label q stands for all light quark and antiquark flavours (including charm) and in Eq. (2) it stands for the quarks $\mathrm{d}, \mathrm{s}, \bar{\mathrm{u}}, \bar{\mathrm{c}}$ (q' represents all CKM-allowed light final-state quarks). Additionally, we consider the processes with all particles replaced by their antiparticles. With the usual Bjorken scaling variable x, neglecting all fermion masses where it is consistently possible, the squared partonic centre of mass energy s is given by

$$s = 4E^2 = 2xM_N E_\nu^{\mathrm{LAB}},$$

up to terms of higher order in the nucleon mass. Hadronic cross sections are obtained by convoluting parton-level cross sections with iso-averaged parton density functions (PDFs) which account for the isoscalar composition of the nuclear target.

At leading order the approximate relation

$$R^\nu \sim \frac{1}{2} - \sin^2\theta_{\mathrm{W}} + \frac{20}{27}\sin^4\theta_{\mathrm{W}} \tag{3}$$

and similar relations for $R^{\bar\nu}$ and R^- permit the translation of an experimental determination of the quantities $R^{\nu/\bar\nu}$, R^- into an indirect measurement of $\sin^2\theta_W$.

3 Higher-Order Corrections

The inclusion of quantum effects in the theoretical prediction can be incorporated in Eq. 3 in terms of a small variation, ultimately relating the relative higher-order corrections to the NC and CC cross sections to a shift in the predicted value of the weak mixing angle:

$$\Delta\sin^2\theta_W = \frac{\frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W}{1 - \frac{40}{27}\sin^2\theta_W}\left(\frac{\delta\sigma^\nu_{NC}}{\sigma^\nu_{NC}} - \frac{\delta\sigma^\nu_{CC}}{\sigma^\nu_{CC}}\right). \tag{4}$$

All parts of our calculation of $\mathcal{O}(\alpha)$ corrections to the leading order matrix elements have been performed in two independent ways, resulting in two completely independent computer codes. Both loop calculations are carried out in ‘t Hooft–Feynman gauge and are based on the standard techniques for one-loop integrations as, e.g., described in Refs. [9,10]. Ultraviolet divergences are treated in dimensional regularization and eliminated using the on-shell renormalization scheme [9,11] in the formulation of Ref. [9]. Infrared (i.e. soft and collinear) singularities are regularized by an infinitesimal photon mass and small fermion masses. The artificial photon-mass dependence of the virtual and (soft) real corrections cancels in the sum of both contributions, according to Bloch and Nordsieck [12].

The calculation of virtual and real corrections is by now a standard exercise and was in part performed with the help of suitable computer algebra programs like FeynArts [13], FormCalc [14] and FeynCalc [15] and in part carried out with independent computer-algebra routines or with recourse to related published work [16]. Among the virtual corrections, there are two contributions to the NC processes that become numerically delicate in the limit of small momentum transfer in the Mandelstam variable t: the $\gamma\nu_\mu\bar{\nu}_\mu$ vertex correction and the γZ mixing self-energy. The limit $t \to 0$ is physically well-defined in both cases, but the numerical treatment of the corresponding amplitudes deserves some care to ensure proper cancellation of powers of t in the form factors against the t-channel photon propagator.

Initial-state mass singularities due to collinear photon radiation were subtracted from the real corrections with a suitably defined $\overline{\mathrm{MS}}$ counterterm (see, e.g., Refs. [16,17]), as it is standard procedure in perturbative QCD. For the numerical evaluation of the cross sections, however, we adopted leading order CTEQ4L [18] parton densities, which is formally inconsistent with the aforementioned subtraction. Nevertheless, this procedure is acceptable as a full incorporation of $\mathcal{O}(\alpha)$ effects in the DGLAP evolution of PDFs and a corresponding fit to experimental data has not yet been performed and the overall effect of QED initial-state collinear radiation largely cancels in the radiative corrections to the quantities $R^{\nu,\bar{\nu}}$ and R^- (see Eq. 4). It should be mentioned that our method of initial-state mass factorization is fundamentally different from the technique employed in Ref. [7] (called BD below), where the initial-state mass dependence is left unsubtracted and $m_q = x\, m_N$, the scaled nucleon mass, is chosen for the initial-state mass value.

There are also $\alpha \ln m_{q'}$ and (in the CC case) $\alpha \ln m_\mu$ terms from final-state radiation. According to the Kinoshita–Lee–Nauenberg (KLN) theorem [19], these terms drop out if the final state is treated sufficiently inclusive, i.e. if the cones for quasi-collinear photon emission around the charged outgoing fermions are fully integrated over. This condition is, in general, not fulfilled if phase-space cuts at the parton level are applied. In the NuTeV analysis an event is discarded unless the energy deposited in the calorimeter lies within certain bounds, which imposes a cut on the final-state particles' energies. We have implemented this final-state energy cut in three different ways in our theoretical analysis, namely by imposing it either (1) on the final-state quark alone, (2) on the final-state quark and final-state real photon energy added together or (3) on the final-state quark alone except if the real photon is emitted within a cone of $5°$ (in the laboratory frame) around the final-state quark. For the first procedure the KLN theorem predicts a dependence of our numerical results on the final-state quark mass, in the second case there is some residual dependence on the muon mass. The third method, imposing the cut on the recombined quark and photon energy, renders our results independent of any final-state mass. Although this seems to be a strong argument in favor of the recombination technique, the cut on the sum of hadronic and photonic energies seems to be most appropriate for a fixed-target experiment where all but the energy of neutrinos and muons is deposited in a calorimeter. To demonstrate the effect of different implementations of the final-state energy cut we present numerical results for all three procedures described above.

4 Numerical Results

The authors of Ref. [7] advocate the on-shell scheme with G_F, m_Z, m_W, m_H and the fermion masses as independent input parameters. To compare our results, we adopt their numerical values for G_F, $\alpha(0)$, the Z- and Higgs-boson mass, the top-quark mass and the electroweak mixing angle $\sin^2\theta_W$. We supplement these parameters by the missing quark and lepton masses from Refs. [20,21] which are quoted within Ref. [7]. Unfortunately, it is not completely clear, whether, given this set of input parameters, the W-boson mass used in Ref. [7] is calculated from m_Z and $\sin^2\theta_W$ or by iterative solution of the relation

$$m_W^2\left(1-\frac{m_W^2}{m_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\frac{1}{1-\Delta r(\alpha,m_W,m_Z,m_H,m_f)}$$

from $G_F, \alpha(0)$ and all particle masses except m_W. The two alternative parameterizations are denoted "$G_F, \sin^2\theta_W$" and "$G_F, \alpha(0)$", respectively.

A compilation of our results for $\Delta\sin^2\theta_W \cdot 10^4$ as given in Eq. 4 for different input-parameter- and initial-state mass factorization schemes is given in the following table:

input parameters:	factorization scheme:	final-state energy cut: (1)	(2)	(3)
$G_F, \sin^2\theta_W$	$\overline{\text{MS}}$	-90	-130	-94
$G_F, \alpha(0)$	$\overline{\text{MS}}$	-95	-132	-99
$G_F, \sin^2\theta_W$	BD	-98	-138	-102
$G_F, \alpha(0)$	BD	-103	-139	-106

Different treatments of the final-state energy cut as explained in the previous section are labelled 1 through 3. Of course, a comparison of our results with $\Delta\sin^2\theta_W = -114\cdot 10^{-4}$ quoted in Ref. [7] can neither prove nor disprove the correctness of the number by Bardin and Dokuchaeva. However, the table suggests significant differences in the correction $\Delta\sin^2\theta_\mathrm{W}$, no matter what final-state energy cut or what input-parameter- or initial-state mass factorization scheme we chose. In any case, the variations in the corrections that are due to the different factorization schemes ($\overline{\text{MS}}$ versus BD) and due to different ways of including the final-state photon in the hadronic energy in the final state can be as large as the accuracy in the NuTeV experiment, which is about $16\cdot 10^{-4}$ in $\sin^2\theta_W$ (if statistical and systematic errors are combined quadratically).

5 Conclusions

A new calculation of electroweak $\mathcal{O}(\alpha)$ corrections to NC and CC neutrino deep-inelastic scattering has been presented and compared to an older work [7]. We have given a detailed account of the issue of collinear fermion-mass singularities. Drawing a comparison to the results of Ref. [7], which were used in the NuTeV data analysis, is not straightforward as the exact parameterization of the input data is not clear. However, a comparison based on an assumption for missing input parameters seems to suggest significant differences in the electroweak radiative corrections. Therefore, an update of the NuTeV analysis seems to be desirable. We provide a Fortran code for the electroweak radiative corrections that could be used in this task.

Specifically, our investigation of the factorization-scheme dependence for initial-state radiation and of different ways to treat photons in the final state reveals that these effects can be as large as the 3σ difference between the NuTeV measurement and the Standard Model prediction in the on-shell weak mixing angle.

The NuTeV collaboration estimated higher-order induced relative uncertainties of $5 \cdot 10^{-5}$ and $11 \cdot 10^{-5}$ in δR^{ν} and $\Delta \sin^2\theta_W$, respectively. The results on electroweak corrections presented in this paper indicate that these numbers might be too optimistic.

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