Proceedings of the DIS'2004, Štrbské Pleso, Slovakia

CAESAR: COMPUTER AUTOMATED RESUMMATIONS

GAVIN P. SALAM
LPTHE, Universities of Paris VI & VII and CNRS, 75252 Paris 75005, France

This talk gives a brief discussion of the motivations and principles behind computer automated expert semi-analytical resummation (CAESAR) for QCD final states.

Global properties of energy-momentum flow in final-states, such as event shapes and jet-resolution thresholds, offer a good compromise between simplicity and sensitivity to the dynamics of QCD. Thanks to the former, it has been possible to make a wide range of theoretical predictions for them, including both fixed-order and resummed perturbative calculations as well as non-perturbative model calculations. This set of theoretical tools has been essential in helping us to learn about QCD from the extensive experimental data, having led in particular to numerous of measurements of the running coupling, α_s [1], tests of the colour factors of QCD [2] and highly instructive studies concerning hadronisation (for reviews, see [3,4]).

Most of the investigations so far have been carried out for observables that are sensitive to the deviation from two-jet (1+1-jet) structure in e^+e^- (DIS). This fairly straightforward environment has been of considerable value, especially in developing and refining our ideas about hadronisation. However the most stringent tests of the understanding thus gained, as well as possible novel extensions, are likely to be found in the context of multi-jet events, including those at hadron-hadron colliders.

A difficulty in such studies is that the next-to-leading logarithmic (NLL) resummed calculations, needed in the limit of only soft and collinear emissions, have up to now usually been carried out by hand, a tedious and error-prone procedure, which has to be repeated separately for each new observable in each process. The complexity of such calculations can increase substantially as one goes to multi-jet processes [5].

This is to be contrasted with fixed-order calculations, usually embodied in the form of a 'fixed-order Monte Carlo' program (FOMC, e.g. [6]), which given a subroutine for the observable, provides, numerically, the leading and next-to-leading order (LO, NLO) coefficients of the perturbative series. Though the internal complexity of FOMCs increases for processes with extra legs, the principles of their use remain identical regardless of the process. This makes it possible, even for those not expert in higher order QCD calculations, to obtain fixed-order predictions for arbitrary observables, in a wide range of processes.

It is natural therefore to ask whether analogous programs could be constructed for resummed calculations. To help explain a solution to this question, proposed in [7], it is useful to define the problem a little more precisely. We will consider a general event shape (or jet-resolution threshold), defined by some function $V(q_1, \ldots, q_N)$ of the momenta $q_1, \ldots, q_N$ of the N particles in an event. If the observable is such that it vanishes smoothly in the limit of n narrow jets, then we

© Institute of Experimental Physics SAS, Košice, Slovakia

call it an $(n+1)$-jet observable, and resummation will be needed in the n-jet limit. The resummed calculations should provide results for the probability $f(v)$ that the event shape has a value smaller than v. For small v, this probability can often (exceptions are discussed below) be written in the 'exponentiated' form

$$f(v) \simeq \exp\left[Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right], \qquad L = \ln\frac{1}{v}, \quad (1)$$

where $Lg_1(\alpha_s L)$ contains LL terms, $\alpha_s^n L^{n+1}$, $g_2(\alpha_s L)$ the NLL ones and so forth. Usually NLL accuracy represents the state of the art (though a related kind of resummation has recently been performed to NNLL accuracy ($\alpha_s g_3$) for the e^+e^- energy-energy correlation [8]).

In analogy with FOMCs, which calculate the LO and NLO contributions to $f(v)$, the aim of an automated resummation program should be the calculation (separately) of the resummed LL and NLL contributions, i.e. the $g_1(\alpha_s L)$ and $g_2(\alpha_s L)$ functions, free of any contamination from potentially spurious higher orders, and in a form such that they can be expanded, allowing matching to fixed-order calculations.[a]

The construction of an automated resummer, as least in our approach [7], proceeds however in quite a different fashion from an FOMC (usually just a Monte Carlo integrator with appropriate weights). This is because, in order to separate out the structure of (1) it is useful to have some analytical understanding of the observable — for example regarding its behaviour in the presence of a soft and collinear emission, nearly always of the form

$$V(\{\tilde{p}\}, k) = d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta}\, g_\ell(\phi), \quad (2)$$

where $\{\tilde{p}\}$ are the hard momenta in the event and k is a soft emission that is collinear to the hard momentum (leg) with index ℓ, with a transverse momentum, rapidity and azimuthal angle k_t, η and ϕ relative to leg ℓ. For a large class of observables, given the coefficients a_ℓ and b_ℓ, the function $g_1(\alpha_s L)$ has a known simple analytical form. So an automated resummer can determine the LL terms by verifying the parametrisation (2) and determining the coefficients.

This sounds remarkably simple, and indeed there is a 'catch': one has also to establish that a given observable belongs to the class for which the LL terms truly are just determined by the values of the a_ℓ, b_ℓ. Understanding precisely what that class is has been one of the significant developments of [7], with the introduction of a concept that we have called recursive infrared and collinear (rIRC) safety. Normal infrared and collinear safety (IRC) states that when an emission is very soft and/or collinear it should have no effect on the observable. The recursive variant essentially extends this condition to two-scale problems (as is relevant when resumming) and states that given an ensemble of arbitrarily soft and collinear emissions, the addition

[a] This can be contrasted with the output from exclusive Monte Carlo event generators like Herwig [9] or Pythia [10], which embody resummation for almost any observable via parton branching algorithms, and which provide an overall result for $f(v)$, generally correct to LL accuracy, sometimes to NLL, but for which it is difficult to guarantee the accuracy, extract separately the LL and NLL terms, perform matching to fixed order, and unambiguously separate the perturbative and non-perturbative effects.

of further emissions of similar softness or collinearity should not change the value of the observable by more than a factor of order one. Furthermore the addition of relatively much softer or more collinear emissions (whether with respect to the hard leg or one of the other emissions) should not change the value of the observable by more than some power of the relative extra softness or collinearity. While at first sight this seems similar to normal IRC safety, a number of observables have been found to be IRC safe but rIRC unsafe. Recursive IRC safety is a sufficient condition for the form (1) to hold. While in principle one could also imagine resumming rIRC unsafe observables, we are not aware of any cases where this has been done.

At NLL accuracy, it is not sufficient to know just the observable's behaviour when there is a single soft and collinear emission — additionally one needs to know how the observable behaves when there are multiple soft and collinear emissions of roughly similar hardness (its 'multiple-emission dependence'). Analytically this usually requires an in-depth analysis of the observable. However, as was shown in [11], there exists a general solution to the problem, namely that at NLL all multiple-emission dependence can be taken into account via a contribution $-\ln \mathcal{F}(R')$ to g_2,

$$\mathcal{F}(R') = \lim_{\epsilon\to 0} \frac{\epsilon^{R'}}{R'} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m+1} \sum_{\ell_i=1}^{n} R'_{\ell_i} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \right) \delta(\ln \zeta_1) \times \\ \times \exp\left(-R' \ln \lim_{\bar{v}\to 0} \frac{V(\{\tilde{p}\}, \kappa_1(\zeta_1 \bar{v}), \ldots, \kappa_{m+1}(\zeta_{m+1}\bar{v}))}{\bar{v}} \right), \quad (3)$$

where $R'(\alpha_s L) = -\partial_L L g_1(\alpha_s L)$ ($R'+1$ determines the average number of 'relevant' emissions), R'_ℓ is the part of R' associated with leg ℓ, and $\kappa_i(\bar{v})$ is any soft momentum collinear to leg ℓ_i that satisfies $V(\{\tilde{p}\}, \kappa_i(\bar{v})) = \bar{v}$. Since $\mathcal{F}(R')$ can be calculated by Monte Carlo integration, one eliminates the need for any analytical study of the observable (the various limits can be taken numerically), thus providing the main NLL element of an automated resummation.

One unsatisfactory issue that remains with the above approach is connected with the problem of globalness. It was shown in [12] that for observables that are sensitive to emissions only in a restricted angular region (for example a single jet), additional classes of NLL terms arise, termed non-global logarithms (NGLs), so far calculated only in the large-N_c limit (though see also [13]). Such NGLs depend non-trivially on the details of the boundaries of the restricted region in which the observable is sensitive to emissions, and these boundaries can be quite complex. Accordingly, for the time being, we have chosen to consider only the simpler case of (continuously) global observables, which have the characteristic that the d_ℓ are all non-zero and the a_ℓ are all equal. These observables' resummations are free of NGLs.

The resulting resummation program is called CAESAR, the Computer Automated Expert Semi-Analytical Resummer, and many results obtained with it can be found at `http://qcd-caesar.org`, for e^+e^-, DIS and hadron-hadron collisions.

Particularly topical are hadron-collider dijet event-shapes, the subject also of recent experimental [14] and theoretical fixed-order [15] work. These are of interest for a variety of reasons, ranging from the possibility of 'standard' measurements such as α_s, to more novel studies examining for example the underlying event with

techniques such as those discussed in [16], or the soft (colour evolution) anomalous dimension matrices of [17]. The event shape studied in [14,15] (a transverse thrust) has the drawback that its resummation involves NGLs, and so is beyond the current scope of CAESAR. To allow more straightforward compatibility with resummed calculations, it is instead preferable to specifically design a set of global dijet event shapes (and jet rates). This has been the subject of work in [18]. One of the main practical issues discussed there is the question of how to reconcile the requirement of globalness with the limited experimental detector coverage at large rapidities. Fortunately this problem can be circumvented in a variety of ways. This opens the road at hadron-colliders to full resummed studies matched with fixed order calculations [6], with the prospect of significantly extending the very fruitful studies carried out in e^+e^- and DIS over the past years.

Acknowledgements. The work discussed here has been carried out in collaboration with Andrea Banfi and Giulia Zanderighi. I with to thank them also for helpful comments on this write-up.

References

1. For a review, see S. Bethke, Nucl. Phys. Proc. Suppl. **121** (2003) 74 [hep-ex/0211012].
2. S. Kluth et al., Eur. Phys. J. C **21** (2001) 199 and references therein.
3. M. Beneke, Phys. Rept. **317** (1999) 1.
4. M. Dasgupta and G. P. Salam, J. Phys. G **30** (2004) R143 [hep-ph/0312283].
5. A. Banfi, G. Marchesini, Yu. L. Dokshitzer and G. Zanderighi, JHEP **0007** (2000) 002 [hep-ph/0004027]; JHEP **0105** (2001) 040 [hep-ph/0104162].
6. Z. Nagy, Phys. Rev. Lett. **88** (2002) 122003 [hep-ph/0110315].
7. A. Banfi, G. P. Salam and G. Zanderighi, Phys. Lett. B **584** (2004) 298 [hep-ph/0304148]; hep-ph/0407286.
8. D. de Florian and M. Grazzini, hep-ph/0407241.
9. G. Abbiendi *et al.*, Comput. Phys. Commun. **67** (1992) 465. G. Corcella *et al.*, JHEP **0101** (2001) 010 [hep-ph/0011363].
10. T. Sjöstrand, Comput. Phys. Commun. **82** (1994) 74; T. Sjöstrand *et al.*, Comput. Phys. Commun. **135** (2001) 238; [hep-ph/0010017].
11. A. Banfi, G. P. Salam and G. Zanderighi, JHEP **0201** (2002) 018 [hep-ph/0112156].
12. M. Dasgupta and G. P. Salam, Phys. Lett. B **512** (2001) 323 [hep-ph/0104277]; JHEP **0203** (2002) 017 [hep-ph/0203009].
13. H. Weigert, Nucl. Phys. B **685** (2004) 321 [hep-ph/0312050].
14. I. A. Bertram [D0 Collaboration], Acta Phys. Polon. B **33** (2002) 3141.
15. Z. Nagy, Phys. Rev. D **68** (2003) 094002 [hep-ph/0307268].
16. A. Banfi, G. Marchesini, G. Smye and G. Zanderighi, JHEP **0108** (2001) 047 [hep-ph/0106278].
17. J. Botts and G. Sterman, Nucl. Phys. B **325** (1989) 62; N. Kidonakis, G. Oderda and G. Sterman, Nucl. Phys. B **531** (1998) 365 [hep-ph/9803241]; G. Oderda, Phys. Rev. D **61** (2000) 014004 [hep-ph/9903240].
18. A. Banfi, G. P. Salam and G. Zanderighi, hep-ph/0407287.