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SUDAKOV LOGS AND POWER CORRECTIONS FOR SELECTED EVENT SHAPES

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I summarize the results of two recent studies analyzing perturbative and nonperturbative effects of soft gluon radiation on the distributions of the C-parameter and of the class of angularities, by means of dressed gluon exponentiation.

1 Introduction

Event shape distributions in high energy scattering processes such as e^+e^- annihilation and DIS have been the focus of many theoretical studies in recent years (for reviews, see for example [1] and [2]). From the viewpoint of a QCD theorist, event shape distributions are of considerable interest because they probe the dynamics of strong interactions at a wide range of scales, from a purely perturbative regime to configurations dominated by soft gluon emission. As a consequence, the theoretical description of these distributions requires a wide range of tools, from the computation of finite order perturbative corrections to resummation and finally to the analysis of power corrections.

To establish the tools required for the analysis, consider the case of the thrust T. Away from the two-jet limit, $T \to 1$, the thrust distribution is dominated by hard gluon emission and can be computed perturbatively. Such a computation, however, is unreliable as $\tau = 1 - T \to 0$, where the results diverge order by order in perturbation theory, while the physical distribution vanishes. The reason is well understood: as $\tau \to 0$ gluon radiation is forced to be soft or collinear to the primary partons, and thus the distribution is dominated by Sudakov logarithms, which need to be resummed in order to recover even the qualitative features of data.

Resummation of Sudakov logarithms leads to exponentiation of the Laplace transform of the distribution [3]. For the thrust,

$$\int_0^\infty d\,\tau \mathrm{e}^{-\nu\tau}\,\frac{1}{\sigma}\frac{d\sigma}{d\tau} \;=\; \exp\left[\int_0^1 \frac{du}{u}\left(\mathrm{e}^{-u\nu}-1\right)\left(B\left(\alpha_s\left(uQ^2\right)\right)+\right.\right.$$
$$\left.\left.+\;\int_{u^2Q^2}^{uQ^2}\frac{dq^2}{q^2}\,A\left(\alpha_s(q^2)\right)\right)\right]. \qquad (1)$$

Corrections to this formula are suppressed by powers of $\nu\Lambda/Q$, corresponding to

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powers of $\Lambda/(Q\tau)$ upon inversion of the transform. The functions A and B are known respectively to three and two loops, corresponding to a resummation up to NNLL accuracy.

Although Eq. (1) is sufficient, upon matching with finite order results, to provide a fit of the data for values of τ larger than those corresponding to the peak of the distribution, a complete description requires the inclusion of power-suppressed corrections. In fact, as τ becomes of the order of Λ/Q, all corrections proportional to powers of $\Lambda/(Q\tau)$ must be taken into account. Fortunately, Eq. (1) can be used to construct a perturbatively motivated parametrization of these corrections [4]. Introducing an IR cutoff μ, one can isolate the ambiguous contributions to Eq. (1), arising from the fact that the Landau pole of the strong coupling is on the integration contour. Power corrections thus exponentiate in the Sudakov region, and they can be expressed in terms of the anomalous dimension A, as

$$\begin{aligned} S_{\rm NP}(\nu/Q,\mu) &= \int_0^{\mu^2} \frac{dq^2}{q^2} A\left(\alpha_s(q^2)\right) \int_{q^2/Q^2}^{q/Q} \frac{du}{u} \left(\mathrm{e}^{-u\nu}-1\right) \\ &= \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\nu}{Q}\right)^n \lambda_n(\mu^2) + \mathcal{O}\left(\nu \left(\frac{\Lambda}{Q}\right)^2\right) \ , \end{aligned} \tag{2}$$

The moments $\lambda_n(\mu^2)$ can be organized into a "shape function", which can be modeled and folded with the perturbative distribution [5,6].

An especially compelling model for shape functions can be constructed by combining renormalon methods with Sudakov resummations. The basic idea is to start by computing the single gluon contribution to the relevant cross section , with an arbitrary number of quark bubble insertions in the gluon propagator. Integrating inclusively over the quark pairs emitted into the final state, and summing over the number of bubbles, this leads to the "characteristic function" of the dispersive method [7,8], *i.e.* the mass distribution of the single virtual gluon contribution to the desired cross section. Under the assumption of ultraviolet dominance [9] of power corrections, this function encodes information about their size, and can be used to parametrize them. Dressed gluon exponentiation [10] combines this renormalon calculation with Sudakov resummation by using the single dressed gluon cross section as kernel for the exponentiation, writing

$$\ln\left[\left(\frac{d\tilde{\sigma}}{d\nu}\right)_{DGE}\right] = \int_0^{\infty} d\tau \left(\frac{d\sigma}{d\tau}\right)_{SDG} \left(1-e^{-\nu\tau}\right) \ . \tag{3}$$

Dressed gluon exponentiation (DGE) has several nice features. First of all, it incorporates most of the current knowledge of the cross section in Sudakov limit, including in particular NL logarithms to all orders, provided the coupling is chosen appropriately. Furthermore, since the renormalon calculation is formally exact in the large-n_f limit, all subleading logs are also included in the same limit. One can then observe that the coefficients of formally subleading logs grow factorially, and use this information to gauge the range of applicability of the resummed formalism. Finally, by imposing the constraint of energy conservation on multi-gluon emission by means of the Laplace transform in Eq. (3), DGE generates a nontrivial pattern of exponentiated power corrections, and can be used to construct a model for the

shape function. DGE has been applied to several high energy QCD cross sections, ranging from event shapes [11] to inclusive DIS [12] and to radiative and semileptonic B decays [13]. In the following we will describe the results obtained with this method in applications to the C-parameter [14], and to the class of angularities [15].

2 The C-parameter

The C-parameter in e^+e^- annihilation has the nice feature of being a function of the final state momenta p_i defined without reference to any minimization procedure, such as the one required to determine the thrust axis. It is well studied, both perturbatively [16] and at the level of power corrections [17,18]. A covariant definition is

$$C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)\,(p_j \cdot Q)} \,. \tag{4}$$

At one loop, we will need an expression for C in the case of emission of a single gluon with virtuality $\xi = k^2/Q^2$. In terms of $x_i = 2p_i \cdot Q/Q^2$ one finds

$$\mathbf{c}(x_1, x_2, \xi) \equiv \frac{C}{6} = \frac{(1-x_1)(1-x_2)(1-x_3+2\xi) - \xi^2}{x_1 x_2 x_3} \,. \tag{5}$$

The characteristic function, corresponding to the cross section for the emission of a gluon with virtuality ξ, is then given by

$$\mathcal{F}(\xi, c) = \int dx_1 dx_2 \, \mathcal{M}(x_1, x_2, \xi)\, \delta\left(\mathbf{c}(x_1, x_2, \xi) - c\right) \,, \tag{6}$$

where the one-loop matrix element for virtual gluon emission is

$$\mathcal{M}(x_1, x_2, \xi) = \frac{(x_1+\xi)^2 + (x_2+\xi)^2}{(1-x_1)(1-x_2)} - \frac{\xi}{(1-x_1)^2} - \frac{\xi}{(1-x_2)^2} \,. \tag{7}$$

The characteristic function $\mathcal{F}(\xi, c)$ in Eq. (6) can be computed exactly in terms of elliptic integrals [14]. In order to perform DGE it is useful to turn to a Borel representation of the single dressed gluon cross section,

$$\left.\frac{1}{\sigma}\,\frac{d\sigma}{dc}\right|_{\rm SDG} = \frac{C_F}{2\beta_0} \int_0^\infty du \, \left(Q^2/\Lambda^2\right)^{-u} \, B(c, u) \,, \tag{8}$$

where $B(c,u)$ is obtained by integrating $d\mathcal{F}(\xi, c)/d\xi$ over phase space with a weight ξ^{-u}. This integral cannot be performed exactly, but one can get an analytic answer for the terms responsible for Sudakov logarithms, which are singular as $c \to 0$. Having determined the relevant contributions to $B(c,u)$, one can exponentiate and obtain the physical distribution by mean of an inverse Laplace transform, as

$$\left.\frac{1}{\sigma}\,\frac{d\sigma}{dc}\right|_{\rm DGE} = \int_{k-{\rm i}\infty}^{k+{\rm i}\infty} \frac{d\nu}{2\pi{\rm i}}\, {\rm e}^{\nu c} \, \exp\left[S\left(\nu, Q^2\right)\right] \,, \tag{9}$$

where

$$S\left(\nu, Q^2\right) = \int_0^\infty dc \, \left.\frac{1}{\sigma}\frac{d\sigma}{dc}\right|_{\rm SDG} \left({\rm e}^{-\nu c} - 1\right) \,. \tag{10}$$

The exponent admits a Borel representation

$$S\left(\nu,Q^2\right)=\frac{C_F}{2\beta_0}\int_0^\infty du\,\left(Q^2/\Lambda^2\right)^{-u}\,B_c(\nu,u)\,, \tag{11}$$

where, in the large-n_f limit, one finds

$$B_c(\nu,u)=2\,\mathrm{e}^{5u/3}\,\frac{\sin\pi u}{\pi u}\,\left[\Gamma(-2u)\left(\nu^{2u}-1\right)2^{1-2u}\frac{\sqrt{\pi}\Gamma(u)}{\Gamma(\frac{1}{2}+u)}\right.$$
$$\left.-\,\Gamma(-u)\left(\nu^{u}-1\right)\left(\frac{2}{u}+\frac{1}{1-u}+\frac{1}{2-u}\right)\right]\,. \tag{12}$$

Starting from Eq. (12) one can recover perturbative Sudakov logarithms (by expanding in powers of u), and one can quantify the strength of power corrections, by looking at the location of poles in u. Specifically, the second factor in Eq. (12) corresponds to collinear radiation, and it is identical to the one found for thrust [11]. The poles at $u=1,2$ correspond to power corrections of the form $\nu(\Lambda^2/Q^2)^p$, with $p=1,2$. The first factor, on the other hand, arises form soft radiation, and has poles at $u=m/2$, with m odd, corresponding to power corrections of the form $\nu(\Lambda/Q)^m$. The cancellation of the pole at $u=0$ expresses the IR-collinear safety of the C-parameter.

Comparing, for example, with the results for the thrust [11], one verifies that Sudakov logarithms are identical for the two observables up to NLL level, as observed in [16]. The pattern of power corrections is also similar, however one finds that both the coefficients of subleading logs and the residues of the poles corresponding to soft power corrections are smaller for the C-parameter than they are for the thrust. This can be traced back to the fact that the typical scale for soft emissions is $2Qc$ for the C-parameter, as opposed to $Q\tau$ for the thrust. If one takes this large-n_f result seriously, one is lead to conclude that the impact of subleading logarithms and of subleading power corrections should be smaller for C than it is for the thrust. The resummed perturbative prediction should thus be more reliable, and the approximation of the shape function by a shift of the perturbative distribution should work better in this case.

3 The class of angularities

Angularities are a one-parameter class of event shapes introduced in [19]. They are defined by

$$\tau_a=\frac{1}{Q}\sum_i (p_\perp)_i \mathrm{e}^{-|\eta_i|(1-a)}\,, \tag{13}$$

where transverse momenta and rapidities are defined with respect to the thrust axis. For $a=0$ one recovers the thrust ($\tau_0=1-T$), while $a=1$ corresponds to jet broadening. Resummation of Sudakov logarithms was worked out in [19] for $a<1$. The result has a nontrivial a dependence: for example at the LL level one finds

$$\ln\left[\tilde{\sigma_{\mathrm{LL}}}\left(\nu,a\right)\right]=2\int_0^1\frac{du}{u}\left[\int_{u^2Q^2}^{uQ^2}\frac{dp_T^2}{p_T^2}A\left(\alpha_s(p_T)\right)\left(\mathrm{e}^{-u^{1-a}\nu\left(\frac{p_T}{Q}\right)^a}-1\right)\right]\,. \tag{14}$$

Notwithstanding this complicated a dependence, a study of power corrections of the form $\nu(\Lambda/Q)^m$, using Eq. (14) as a starting point, showed a remarkable scaling behavior: the shape function suggested by the resummation for $d\sigma/d\tau_a$ depends on a only through an overall factor of $1/(1-a)$ [20,21]. This simple scaling arises, in the context of resummation, from boost invariance of the eikonal cross section responsible for logarithmic enhancements. Since DGE complements the resummation by including the effect of subleading logarithms in the large-n_f limit, and provides an explicit model of power corrections consistent both with the resummation and with renormalon calculus, it was interesting to check whether the scaling suggested in [20,21] would remain valid. The test is nontrivial also because boost invariance is broken in DGE by gluon virtuality, and it is interesting to see how it is eventually recovered in the Sudakov limit. This study was performed in [15].

The first step, as for the C-parameter, is to provide a definition of the observable at one loop for an emitted gluon with virtuality ξ. The definition adopted in [15] is

$$\tau_a = \frac{(1-x_i)^{1-a/2}}{x_i}\left[(1-x_j-\xi)^{1-a/2}(1-x_k+\xi)^{a/2}+(j\leftrightarrow k)\right] \ , \qquad (15)$$

which has the correct limit as $\xi \to 0$ and is simple enough to allow for analytic computations. It can be shown that leading power corrections are not affected by changes of Eq. (15) which are analytic in ξ.

In this case, it is not possible to compute the characteristic function in closed form. A detailed study of the limit of soft radiation leads anyhow to a simple expression for the soft contribution to the Borel function of DGE, corresponding to the first erm of Eq. (12). One finds

$$B_a^{\rm soft}(\nu,u) = \frac{1}{1-a}\left[2\,\mathrm{e}^{5u/3}\,\frac{\sin\pi u}{\pi u}\,\Gamma(-2u)\left(\nu^{2u}-1\right)\frac{2}{u}\right] \ , \qquad (16)$$

exactly the scaling behavior predicted by the resummation. DGE also provides a model for power corrections of collinear origin. Although these in principle may be affected by the choice of the massive definition of the observable, it is interesting to notice that they are suppressed by a power of Q which grows as a becomes large and negative. One finds that collinear power corrections are suppressed at least by $\nu(\Lambda/Q)^{2-a}$. Comparing the thrust distribution to the angularity distribution for a negative value of a should thus provide a simple and clean test of the scaling rule, largely unaffected by errors due to subleading power corrections.

4 Perspective

Studies of event shape distributions in and beyond perturbative QCD have reached a considerable degree of refinement, and provide robust theoretical predictions which in some cases should be fairly easy to test experimentally. Such tests are indeed desirable, because they would strongly constrain our current understanding of the transition between perturbative and nonperturbative QCD, and they might have practical consequences, for example on current determinations α_s [22]. The fact that some of the recent theoretical progress has taken place as the work of the LEP collaborations was winding down is a warning for the future: "old" data may well

contain a wealth of unexplored information, so it should continue to be possible to perform new analyses, as theory progresses or new viewpoints emerge.

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