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NONPERTURBATIVE EFFECTS IN QUARK FORM FACTOR AT HIGH ENERGY

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The detailed analysis of nonperturbative contributions to the electromagnetic quark form factor is performed within the framework of the instanton liquid model (ILM) of the QCD vacuum. The method of the path-ordered Wilson exponentials is applied to derive the evolution equation describing the high energy behaviour of the form factor and evaluate explicitly the instanton corrections. By using the Gaussian interpolation of the constrained instanton solution, it is shown that the instantons yield the logarithmic corrections to the amplitudes in high energy limit which are exponentiated in small instanton density parameter.

1 Introduction

The necessity of a nonperturbative input for effective explicit calculations of hadronic processes even in high energy domain is an important reason for development of nonperturbative methods in QCD [1]. The nonperturbative effects are naturally related to the nontrivial structure of the QCD vacuum. In the last decades, a great progress has been made in study of the QCD ground state and a number of important results have been obtained that connect the properties of the vacuum with the hadron characteristics treating the QCD vacuum in the framework of the instanton liquid model [2,3,4]. The idea that the nontrivial vacuum structure can be relevant as well in high energy hadronic processes was explicitly formulated in the context of the so-called soft Pomeron problem in Refs. [5,6,7], and further developed using the eikonal approximation and the method of Wilson path-ordered exponentials in Refs. [8,9]. In the present work, we report our recent results on investigation of the instanton induced effects in high energy regime for the electromagnetic (EM) quark form factor, *i.e.,* the amplitude of the quark elastic scattering in an external color singlet gauge field.

2 Evolution equation for the EM quark form factor

The electromagnetic quark form factors are determined via the elastic scattering amplitude of a quark in an external EM field:

$$\mathcal{M}_\mu = F_q\left[(p_1-p_2)^2\right]\bar{u}(p_1)\gamma_\mu v(p_2) - G_q\left[(p_1-p_2)^2\right]\bar{u}(p_1)\frac{\sigma_{\mu\nu}(p_1-p_2)_\nu}{2m}v(p_2) \quad , \tag{1}$$

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where $u(p_1)$, $v(p_2)$ are the spinors of outgoing and incoming quarks, and $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$. The kinematics of the process is described by two invariants:

$$s = (p_1+p_2)^2 = 2m^2(1+\cosh\chi) \ , \ \ t = -Q^2 = (p_1-p_2)^2 = 2m^2(1-\cosh\chi) \ , \quad (2)$$

where χ is the scattering angle. In this work we assume that both the momentum transfer $-t$ and the total center-of-mass energy s are large compared to the quark mass:

$$(p_1 p_2) \gg p_{1,2}^2 = m^2 \ \ , \ \ \text{or} \ \ \cosh\chi \gg 1 \ . \quad (3)$$

In this regime the Pauli form factor G_q is power suppressed and will be neglected.

The classification of the diagrams with respect to the momenta carried by their internal lines allows one to express the form factor F_q in the amplitude (1) in the factorized form [10,11]

$$F_q(q^2) = F_H(q^2/\mu^2, \alpha_s) \cdot F_S(q^2/m^2, \mu^2/\lambda^2, \alpha_s) \cdot F_J(\mu^2/\lambda^2, \alpha_s) \ , \quad (4)$$

where the hard, soft, and collinear (jet) part are separated by the scale parameters μ^2, λ^2. The following relations between these scales are assumed: $q^2 \gg \mu^2 \gg \lambda^2 \gg 1$. Note, that in the present paper all dimensional variables are expressed in units of the QCD scale Λ_{QCD}, so that $q^2 = Q^2/\Lambda_{QCD}^2$, *etc.* The arbitrary scale μ^2 is equal to the UV normalization point.

Within the eikonal approximation, the resummation of all logarithmic terms coming from the soft gluon subprocesses allows us to express F_S in terms of the vacuum average of the gauge invariant path-ordered Wilson exponential [12,13]

$$F_S(q^2/m^2, \alpha_s) = W(C_\chi, \alpha_s) = \frac{1}{N_c}\mathrm{Tr}\left\langle 0 \left| \mathcal{P} \exp\left\{ ig \int_{C_\chi} dx_\mu \hat{A}_\mu(x) \right\} \right| 0 \right\rangle \ . \quad (5)$$

In Eq. (5) the integration path corresponding to the considered process goes along the closed contour C_χ: the angle (cusp) with infinite sides. We parameterize the integration path $C_\chi = \{z_\mu(t); t = [-\infty, \infty]\}$ as follows

$$z_\mu(t) = \begin{cases} v_1 t, & -\infty < t < 0, \\ v_2 t, & 0 < t < \infty. \end{cases} \quad (6)$$

The Wilson integral (5) is multiplicatively renormalizable [14,15]:

$$W(C_\chi; \mu^2/\lambda^2, \alpha_s(\mu^2)) = Z_{cusp}(C_\chi; \bar{\mu}^2/\mu^2, \alpha_s(\mu^2)) \cdot W_{bare}(C_\chi; \bar{\mu}^2/\lambda^2, \alpha_s(\mu^2)) \ , \quad (7)$$

where $\bar{\mu}^2$ is the UV cutoff, μ^2 is the normalization point, and λ^2 is the IR cutoff. Therefore, we can define the cusp anomalous dimension Γ_{cusp}:

$$\frac{1}{2}\,\Gamma_{cusp}(C_\chi; \alpha_s(\mu^2)) = -\mu^2 \frac{d}{d\mu^2} \ln W(C_\chi; \mu^2/\lambda^2, \alpha_s(\mu^2)) \ . \quad (8)$$

Then we get the q^2-dependence of the total form factor at large q^2:

$$\ln \frac{F_q\left[q^2\right]}{F_q\left[q_0^2\right]} = -\int_{q_0^2}^{q^2} \frac{d\xi}{2\xi} \left[\ln\frac{q^2}{\xi}\ \Gamma_{cusp}(\alpha_s(\xi)) - NLO \right] -$$

$$- \ln\frac{q^2}{q_0^2} \cdot \int_{\lambda^2}^{q_0^2} \frac{d\xi}{2\xi}\ \Gamma_{cusp}(\alpha_s(\xi)) + \ln\frac{W_{np}(q^2)}{W_{np}(q_0^2)} \ . \quad (9)$$

In one-loop order of perturbative expansion, the cusp anomalous dimension is given by [11]: $\Gamma^{(1)}_{cusp}(\alpha_s) = \frac{\alpha_s}{\pi} C_F$. In what follows, we explicitly calculate the nonperturbative part W_{np} within the ILM [16]. The latter plays a role of initial conditions for perturbative evolution.

3 Instanton induced contributions

Let us consider the instanton contribution to the function $W_{np}(q^2)$ from Eq.(9). The instanton field has the general form

$$\hat{A}_\mu(x;\rho) = A^a_\mu(x;\rho)\frac{\sigma^a}{2} = \frac{1}{g}\mathtt{R}^{ab}\sigma^a\,\eta^{\pm\,b}_{\mu\nu}(x-z_0)_\nu\varphi(x-z_0;\rho), \tag{10}$$

where $\varphi(x)$ is the gauge dependent profile function, $\mathtt{R}^{ab}$ is the color orientation matrix, σ^a's are the Pauli matrices, $\eta^{\pm\,a}_{\mu\nu} = \varepsilon_{4a\mu\nu} \mp (1/2)\varepsilon_{abc}\varepsilon_{bc\mu\nu}$ are 't Hooft symbols, and $(\pm)$ corresponds to the instanton or antiinstanton solution. The averaging of the Wilson operator over the nonperturbative vacuum is performed by the integration over the coordinate of the instanton center z_0, the color orientation and the instanton size ρ. The measure for the averaging over the instanton ensemble reads $dI = d\mathtt{R}\; d^4z_0\; dn(\rho)$, where $d\mathtt{R}$ refers to the averaging over color orientation, and $dn(\rho)$ depends on the choice of the instanton size distribution. After averaging over all possible embeddings of $SU_c(2)$ into $SU_c(3)$ [17], we write the Wilson integral (5) over the contour (6) in the single instanton approximation in the form:

$$w_I(\chi) = \frac{1}{3}\mathrm{Tr}\Big\langle 0\Big|\frac{4}{9}\,\cos(\alpha(v_1,z_0))\cos(\alpha(v_2,z_0))\;\cdot(\mathtt{I}\times\mathtt{I}) +$$

$$\frac{1}{8}\left[\frac{1}{3}\cos(\alpha(v_1,z_0))\cos(\alpha(v_2,z_0)) - \sin(\alpha(v_1,z_0))\sin(\alpha(v_2,z_0))\hat{n}^a_1\hat{n}^a_2\right]\left(\lambda^A\times\lambda^A\right)\Big|0\Big\rangle\,, \tag{11}$$

where $(i=1,2)$

$$\hat{n}^a_i = \frac{(-1)^i}{s(v_i,z_0)}\eta^{\pm\,a}_{\mu\nu}v^\mu_i z^\nu_0\;, \tag{12}$$

$$\alpha(v_i,z_0) = s(v_i,z_0)\int_0^\infty d\lambda\varphi\left[(v_i\lambda+(-1)^i z_0)^2;\rho_c\right]\;, \tag{13}$$

and

$$s^2(v_i,z_0) = z_0^2 - (v_i z_0)^2\;, \tag{14}$$

where $(v_1v_2) = \cosh\chi$ in Minkowski geometry. After evaluating the traces, the resulting gauge invariant contribution to the Wilson loop of the single instanton reads [16]

$$w_I(\chi) = \frac{2}{3}\int dn(\rho)\left[w^I_c(\chi) + w^I_s(\chi) - w^I_c(0) - w^I_s(0)\right], \tag{15}$$

$$w^I_c(\chi) = \int d^4z_0\;\cos\,\alpha(v_1,z_0)\cos\,\alpha(v_2,z_0)\;, \tag{16}$$

$$w^I_s(\chi) = -\int d^4z_0\;(\hat{n}^a_1\hat{n}^a_2)\sin\,\alpha(v_1,z_0)\sin\,\alpha(v_2,z_0)\;,$$

where the normalized color correlation factor is

$$\hat{n}_1^a \hat{n}_2^a = -\frac{\eta^a_{\mu\nu} v_1^\mu z_0^\nu \eta^a_{\rho\sigma} v_2^\rho z_0^\sigma}{s(v_1,z_0)s(v_2,z_0)} \ . \tag{17}$$

In realistic instanton vacuum model there are two essential effects: stabilization of the instanton density with respect to unbounded expansion of instantons in size and screening of instantons by surrounding background fields. To take into account these features we approximate first the narrow instanton size distribution by the δ-function:

$$dn(\rho) = n_c \delta(\rho - \rho_c) d\rho \quad , \tag{18}$$

where the model parameters are [3]:

$$n_c \approx 1 fm^{-4}, \quad \rho_c \approx 1/3 fm \ , \tag{19}$$

and assume that the integration over the instanton size is already performed. The screening effect modifies the instanton shape at large distances leading to the constrained instantons [18]. To take into account this screening and to have also simpler analytical form for $w_I(\chi)$, we use the Gaussian Anzatz for the instanton profile function

$$\varphi_G(x^2) = \frac{1}{\rho_c^2} \mathrm{e}^{-x^2/\rho_c^2} \ . \tag{20}$$

The parameters in this expression are fixed by the requirement of reproducing the vacuum average $\left\langle g^2 A^a_\mu(0) A^a_\mu(0) \right\rangle = 12\pi^2 \rho_c^2 n_c$.

Performing tedious calculations (for details, see Ref. [19]), we find that the leading correction to the quark form factor is given by

$$w_G(q^2) = -1.0053 \ \frac{\pi^2 n_c {\rho_c}^4}{12} \cdot \ln q^2 = B^{LOG}_{inst} \cdot \ln q^2 \ . \tag{21}$$

Therefore, one concludes that the instanton induced effects (for the Gaussian simulation of instanton profile function) yield the logarithmic, *i.e.,* sub-leading, correction to the high energy behavior of quark EM form factor, with the numerical coefficient smaller then that of corresponding perturbative term $B^{LOG}_{inst} \approx 0.01 \ll B^{LOG}_{pert} = \frac{2C_F}{\beta_0} \approx 0.24$. Thus, the total expression for the quark form factor at large-q^2 with the one-loop perturbative contribution and the nonperturbative contribution found in the instanton model reads:

$$F_q\left[q^2\right] =$$

$$\exp\left[-\frac{2C_F}{\beta_0} \ln q^2 \ln\ln q^2 - \ln q^2 \ \left(B^{LOG}_{inst} - \frac{2C_F}{\beta_0}\right) + O(\ln\ln q^2)\right] \cdot F_q\left[q_0^2\right] \ . \tag{22}$$

It is clear, that while the asymptotic "double-logarithmic" behaviour is controlled by the perturbative cusp anomalous dimension, the leading instanton correction results in a finite renormalization of the subleading perturbative term.

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References

1. R. Brock, *et. al.* [CTEQ Collaboration], Rev. Mod. Phys. **67** (1995) 157; Yu. Dokshitzer, D. Dyakonov, S. Troyan, Phys. Reports **58** (1980) 269; A.H. Mueller, Phys. Reports **73** (1981) 237.
2. E. Shuryak, Nucl. Phys. **B203** (1982) 93, 116, 140.
3. T. Schäfer, E.V. Shuryak, Rev. Mod. Phys. **70** (1998) 323.
4. D. Diakonov, V. Petrov, Nucl. Phys. **B245** (1984) 259.
5. F.E. Low, Phys. Rev. **D12** (1975) 163.
6. S. Nussinov, Phys. Rev. Lett. **34** (1975) 1286.
7. P.V. Landshoff, O. Nachtmann, Z. Phys. **C35** (1987) 405; O. Nachtmann, Nucl. Phys. **B63** (1973) 237.
8. A. Bassetto, M. Ciafaloni, G. Marchesini, Phys. Reports **100** (1983) 201; O. Nachtmann, Ann. Phys. (N.Y.) **209** (1991) 436.
9. G. Korchemsky, Phys. Lett. **B325** (1994) 459; I. Korchemskaya, Nucl. Phys. **B490** (1997) 306.
10. A.H. Mueller, Phys. Rev. **D20** (1979) 2037; J.C. Collins, Phys. Rev. **D22** (1980) 1478; A. Sen, Phys. Rev. **D24** (1981) 3281.
11. G. Korchemsky, Phys. Lett. **B217** (1989) 330; G. Korchemsky, Phys. Lett. **B220** (1989) 629; G.C. Gellas, A.I. Karanikas, C.N. Ktorides, N.G. Stefanis, Phys. Lett. **B412** (1997) 95.
12. Yu.M. Makeenko, A.A. Migdal, Phys. Lett. **B88** (1979) 135; Nucl. Phys. **B188** (1981) 269.
13. A. Polyakov, Phys. Lett. **B82** (1979) 247; Nucl. Phys. **B164** (1980) 171; *Gauge Fields and Strings*, Harwood, 1987
14. G. Korchemsky, A. Radyushkin, Sov. J. Nucl. Phys. **45** (1987) 910; G. Korchemsky, A. Radyushkin, Nucl. Phys. **B283** (1987) 342.
15. V.S. Dotsenko, S.N. Vergeles, Nucl. Phys. **B169** (1980) 527; R.A. Brandt, F. Neri, M.-A. Sato, Phys. Rev. **D24** (1981) 879; R.A. Brandt, A. Gocksch, M.A. Sato, F. Neri, Phys. Rev. **D26** (1982) 3611.
16. A.E. Dorokhov, I.O. Cherednikov, Phys. Rev. **D66** (2002) 074009; Phys. Rev. **D67** (2003) 114017.
17. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B163** (1980) 46.
18. A.E. Dorokhov, S.V. Esaibegyan, A. E. Maximov, S.V. Mikhailov, Eur. Phys. J. **C13** (2000) 331.
19. A.E. Dorokhov, I.O. Cherednikov, Ann. Phys. (2004) in print; `hep-ph/0404040`.