Proceedings of the DIS'2004, Štrbské Pleso, Slovakia

UPDATE OF MRST PARTON DISTRIBUTIONS

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I present a brief summary of the most important updates in the MRST determinations of the NLO and NNLO parton distributions. I concentrate on uncertainties due to input assumptions and possible theoretical corrections, particularly isospin violation, QED corrections, NNLO and corrections at low Q^2 and small x.

1 Introduction

A knowledge of the partonic structure of the proton is an essential ingredient in the analysis of hard scattering data from pp or $\bar{p}p$ or ep high energy collisions. Much attention has recently been devoted to obtaining reliable uncertainties on the parton distributions obtained from global or semi-global fits. One obvious uncertainty is due to the systematic and statistical errors of the data, i.e the *experimental* errors. They have been estimated by several groups [1]–[10], working within a NLO (and sometimes NNLO) framework using a variety of different procedures. The general conclusion is that in all approaches the uncertainties are of the order of $2-5\%$, except for certain partons and regions of phase space, where the uncertainty can be bigger, particularly the high-x gluons and very high-x down quarks. However, when the partons and predictions for physical quantities using different parton sets are compared, the deviations can be significantly greater than the quoted errors. A recent summary can be found in [11]. This unfortunate discrepancy is likely to be due to different assumptions made by different groups when performing their fitting procedures, such as different data sets used, parameterizations for input sets, choices of heavy flavour prescriptions, ordering of perturbative series, etc. However, the different results for partons obtained when making different assumptions also suggest that the standard fixed order in the perturbative expansion is not completely adequate to describe the data, with different individual data sets making different and conflicting demands on the partons and on α_S. Indeed, MRST do see certain difficulties in obtaining the best possible fit to the data with the current theoretical treatment – the HERA and NMC data in the range $x \sim 0.01 - 0.001$ are too steep in Q^2; the gluon is negative (or at best valencelike) at small x and

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Q^2; and the Tevatron jet data prefer a different shape gluon from the DIS data. Hence, in order to understand the uncertainties in parton distributions, it is vital to consider theoretical corrections. These include the possibility of isospin violation; $s(x) \neq \bar{s}(x)$; higher orders (NNLO); QED (comparable to NNLO ? i.e. $\alpha_s^3 \sim \alpha$); large x ($\alpha_s^n \ln^{2n-1}(1-x)$); low Q^2 (higher twist); small x ($\alpha_s^n \ln^{n-1}(1/x)$). The uncertainties associated with many of these possible sources have recently been considered [12]. Here I will highlight some of the main conclusions and some additional results which postdate this analysis.

One of the issues we have examined is the possibility of isospin violation. This is particularly relevant because of the "NuTeV anomaly" [13]. NuTeV measure, with a $3-\sigma$ discrepancy,

$$R^- = \frac{1}{2} - \sin^2\theta_W + (1 - \frac{7}{3}\sin^2\theta_W)\frac{[\delta U_{\rm v}] - [\delta D_{\rm v}]}{2[V^-]}.$$

where $[\delta U_{\rm v}] = [U_{\rm v}^p] - [D_{\rm v}^n]$ and $[\delta D_{\rm v}] = [D_{\rm v}^p] - [U_{\rm v}^n]$, $[U_{\rm v}^p] = \int_0^1 x\, u_{\rm v}^p(x)\, dx$ etc.. We parameterize the isospin violation by

$$u_{\rm v}^p(x) = d_{\rm v}^n(x) + \kappa f(x), \qquad d_{\rm v}^p(x) = u_{\rm v}^n(x) - \kappa f(x),$$

where $f(x)$ is a number conserving function with valence like behaviour at large and small x [12]. The variation of the χ^2 for the global fit is shown in Fig. 1, and the best fit leads to $\kappa = -0.2$ (which agrees very well with a prediction in [14]), and to a reduction of the NuTeV anomaly to $\sim 1.5-\sigma$, i.e. $\Delta sin^2\theta_W \sim -0.002$. However, a wide variation in κ is possible, and the anomaly may be reduced further or enlarged. Other issues to which the anomaly is sensitive are the strangeness asymmetry in the proton recently investigated in [15] and electroweak corrections [16]. In each case the uncertainties may be large as the anomaly, though there is some disagreement regarding the strangeness asymmetry.

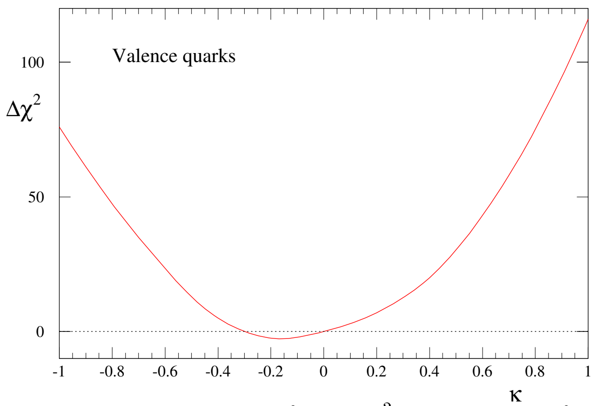

Figure 1. Variation of global χ^2 with the value of κ

We have also recently examined the effects due to QED. The corrections to DIS include those in Fig. 2 and lead to mass singularities when $\gamma \parallel q$. The QED collinear singularities are *universal* and can be absorbed into pdfs, exactly as for QCD collinear singularities, leaving finite (as $m_q \to 0$) $\mathcal{O}(\alpha)$ QED corrections in coefficient functions, and QED–improved DGLAP equations. The effect on the gluon distribution is negligible as is that on the quark distributions at small x where the gluon contribution dominates DGLAP evolution. Only at large x is the effect noticeable ($\mathcal{O}(1\%)$) as first noted in [17], and more recently verified in [18], and it is equivalent to a slight shift in α_S: $\Delta\alpha_S(M_Z^2) \simeq +0.0003$. This is much smaller than many sources of uncertainty.

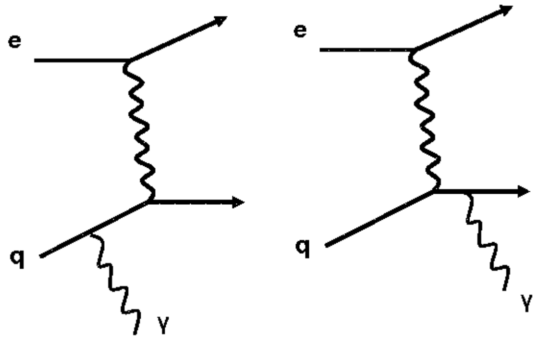

Figure 2. Diagrams for QED corrections to DIS

However, these QED effects, although small, lead to isospin violation. $u_V^p(x)$ quarks radiate more photons than $d_V^n(x)$ quarks. To a rough approximation, the photon distribution should be

$$\gamma(x,Q^2) = \sum_j e_j^2 \frac{\alpha}{2\pi} \ln(Q^2/m_q^2) \int_x^1 \frac{dy}{y} P_{\gamma q}(y) \;\; q_j(\frac{x}{y},Q^2).$$

So there is more photon momentum in the proton than in the neutron due to high-x up quarks radiating more than high-x down quarks. Momentum conservation leads to $u_V^p(x) < d_V^n(x)$ at high x. Hence, $[\delta U_{\rm v}] < 0$ as required by the NuTeV anomaly and as found by our best fit above. Our estimates imply that the QED effects lead to $\sim 3/4$ of the isospin violation observed by our best fit. Details of this and parton distributions involving QED corrections will soon appear [19].

Much more generally important are NNLO corrections. The coefficient functions for $F_2(x,Q^2)$ are known at NNLO [20]. Singular limits $x \to 0$ and large N_F limits are known for the NNLO splitting functions [21]–[23] as well as limited moments [24], and approximate NNLO splitting functions were devised [25] and used by MRST [26] (and Alekhin [3]) in NNLO fits. Compared to NLO they improved the quality of the fit very slightly, and reduced α_S to ~ 0.1155. The predictions lead to reasonable stability order by order for (quark-dominated) W and Z cross-sections. However, this fairly good convergence is largely guaranteed because the quarks are fit directly to data, and it is much worse for gluon-dominated quantities e.g. $F_L(x,Q^2)$ or Higgs production. Very recently, the full calculation of the NNLO splitting functions was completed [27]. We have confirmed that these alter the parton distributions obtained from the previous best estimates by only a very small amount. Full NNLO partons will appear soon, but also require the complete incorporation of NNLO coefficient functions for as many processes as possible, and a rigorous treatment of heavy flavour thresholds at NNLO [28]. For predictions it will be useful to have the exact NNLO ($\mathcal{O}(\alpha_S^3)$) longitudinal coefficient functions, and these, along with the $\mathcal{O}(\alpha_S^3)$ (NNNLO) coefficient functions for $F_2(x,Q^2)$ will soon be complete. We have estimated the most likely value, along with uncertainties, from the limited information, as had previously been done for the splitting functions. The central value and the two "extremes" for the dominant contribution, i.e. the part of C_{Lg}^3 which is $\propto N_f$ are respectively (the common factor of $N_f(\alpha_S/(4\pi))^3$ is taken out and $L=\ln(1/x)$, $L_1=\ln(1-x)$)

$$410\frac{L}{x}-\frac{1212}{x}-1685-54161x-47131L_1-11496L_1^2-867L_1^3,$$
$$410\frac{L}{x}-\frac{0}{x}+20298-106974x-67527L_1-16336L_1^2-1241L_1^3-14116L,$$
$$410\frac{L}{x}-\frac{2057}{x}+30895x^3+6220L_1-102L_1^2-66.6L_1^3+1378L^2.$$

The failure of the NNLO corrections to solve completely the problems with the quality of the fit leads us to try an alternative approach. In order to investigate the real quality of the fit and the regions with problems we vary kinematic cuts on data, i.e. change W^2_{cut}, Q^2_{cut} and x_{cut}, re-fit and see if the quality of the fit to the remaining data improves and/or the input parameters change dramatically. This is continued until the quality of the fit and the partons stabilize. This is discussed in detail in [12]. Briefly, our W^2 cut of 12.5GeV^2 seems reasonable, whereas the fit improves gradually until $Q^2_{cut}\sim 10\text{GeV}^2$ and $x_{cut}\sim 0.005$.[a] This is true at both NLO and NNLO, but the modification of the partons with changing cuts is much greater at NLO. For both NLO and NNLO we have defined "conservative" partons [12] which are obtained using only the data within the conservative cuts.

In order to investigate the possible theoretical improvements which could modify the partons at small x and low Q^2 (particularly at high x) we have tried including known $\ln(1-x)$ resummations, saturation corrections, and purely phenomenological higher twist and $\ln(1/x)$ corrections. At high x, higher orders in the coefficient functions and higher twist contributions can improve the fit, but are difficult to distinguish. At low x, there is no evidence for phenomenological higher twist. Saturation corrections [30] do not seem to help the NLO[b] or NNLO fits (they would help only at LO). There is a distinct improvement in the fit quality when (momentum conserving) $\ln(1/x)$ corrections are added to the splitting functions, increasing the small-x evolution. Hence, there is some reason for uncertainty about the correct approach, and hence the correct predictions at very small x. Indeed, a comparison of the LO, NLO, approximate NNLO and a resummed prediction [32] for $F_L(x,Q^2)$ is shown in Fig. 3, exhibiting the level of uncertainty. Also shown are potential data which could be obtained by low energy running at HERA. It is clear that such data would play a huge role in enabling us to understand and be confident of predictions involving small x partons.

In summary, as well as the uncertainties on partons due to experimental errors, there are uncertainties on partons, and related physical quantities, due to assumptions in the fit and having to work within an incomplete theoretical framework.

[a] We note that CTEQ do not seem to reproduce these conclusions [29].
[b] A recent joint analysis with diffractive data suggests some slight improvement [31].

Some of the uncertainties are small, but in some cases, e.g. QED corrections or $s(x) \neq \bar{s}(x)$, they can be important for specific quantities. Others, associated with low Q^2 and small x, may be potentially large, and more theoretical work is needed, along with specifically targeted data to guide and verify this work.

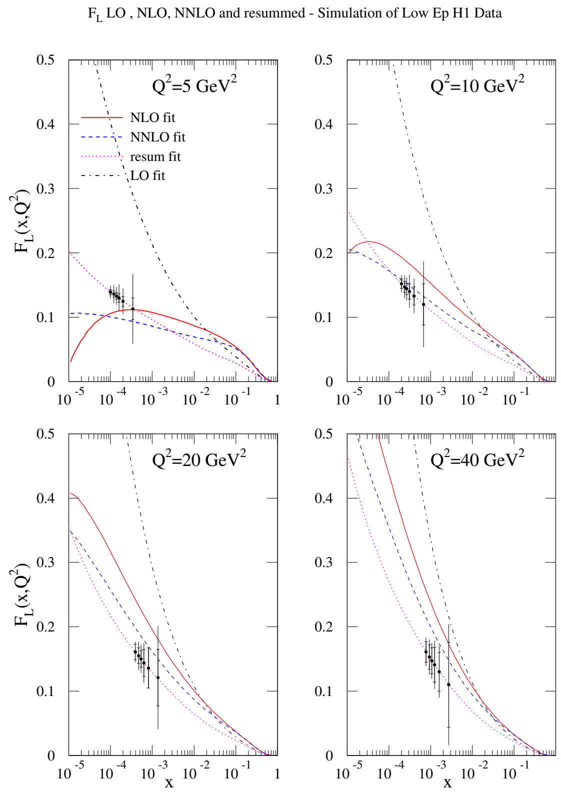

Figure 3. Predictions for $F_L(x, Q^2)$ at LO, NLO, NNLO and with a resummed calculation

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