Proceedings of the DIS'2004, Štrbské Pleso, Slovakia

COLOR DIPOLES AND K$_\perp$–FACTORIZATION FOR NUCLEI

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We discuss applications of the color dipole approach to hard processes on nuclei. We focus on the relation to $k_\perp$–factorisation and the role of a nuclear unintegrated gluon distribution in single– and two– particle inclusive spectra in γ^*A and pA collisions. Linear $k_\perp$ factorisation is broken for a wide class of observables, which we exemplify on the case of heavy quark $p_\perp$–spectra.

1. Color dipoles, the unintegrated gluon distribution and DIS

When studying the interactions of a highly energetic (virtual) photon it is of great help to think of its hadronic vacuum fluctuations as being components of its (light–cone–) wave function [1]. Deep inelastic scattering (DIS) can then be viewed as an interaction of frozen multi–parton Fock states of the virtual photon with the target nucleon or nucleus. The proper formalisation valid for inclusive, as well as diffractive deep inelastic processes results in the color–dipole approach to small–x–DIS [2]. Specifically, the total virtual photoabsorption cross section takes the well-known quantum mechanical form $\sigma_{tot}(\gamma^* p; x, Q^2) = \int_0^1 dz \int \ d^2\mathbf{r} \Psi^*_{\gamma^*}(z,\mathbf{r})\, \sigma_2(\mathbf{r}) \Psi_{\gamma^*}(z,\mathbf{r})\,$, with x, Q^2 the standard DIS–variables, Ψ_{γ^*} is the $q\bar{q}$–lightcone-wavefunction of the virtual photon, $z, 1-z$ are the photon's lightcone momentum fractions carried by the quark/antiquark, and finally $\sigma_2(\mathbf{r})$ is the dipole–nucleon cross section. The connection between color–dipole formulas and $\mathbf{k}_\perp$–factorization is provided by $\sigma_2(\mathbf{r}) = \sigma_0 \int d^2\boldsymbol{\kappa} [1 - e^{i\boldsymbol{\kappa}\mathbf{r}}] f(\boldsymbol{\kappa})$, where $f(\boldsymbol{\kappa})$ is directly related to the unintegrated gluon distribution $f(\boldsymbol{\kappa}) = \frac{4\pi}{N_c \sigma_0} \frac{1}{\boldsymbol{\kappa}^4} \partial G_N / \partial \log(\boldsymbol{\kappa}^2)$. Now, for DIS off nuclei, the dipole is coherent over the whole nucleus for $x \leq x_A = 1/m_N R_A$, where m_N is the nucleon mass, and R_A the nuclear radius. The dipole–nucleus cross section assumes the Glauber–Gribov form [2] $\sigma_A(\mathbf{r}) = 2 \int d^2\mathbf{b} \Gamma_A[\sigma_2(\mathbf{r}); \mathbf{b}]$, with $\Gamma_A[\sigma_2(\mathbf{r}); \mathbf{b}] = 1 - \exp[-\frac{1}{2}\sigma_2(\mathbf{r}) T_A(\mathbf{b})]$, $T_A(\mathbf{b})$ is the nuclear thickness. If we now write $\Gamma_A[\sigma_2(\mathbf{r}); \mathbf{b}] = \int d^2\boldsymbol{\kappa} [1 - e^{i\boldsymbol{\kappa}\mathbf{r}}] \phi(\boldsymbol{\kappa})$, then the function $\phi(\boldsymbol{\kappa})$(we suppress its dependence on $\mathbf{b}$) walks and talks like an unintegrated gluon distribution in inclusive as well as diffractive DIS on nuclei [3], hence its name 'nuclear unintegrated glue'. It includes multiple scatterings and the features of nuclear absorption, as well as $\mathbf{k}_\perp$ broadening of propagating partons, both controlled by the saturation scale Q^2_A. Its salient features, including a Cronin enhancement at intermediate $\boldsymbol{\kappa}$ and an explicit representation in terms of convolutions of its free nucleon counterpart can be found in [3,4]. Below we shall have a look at the role of the nuclear unintegrated glue in a broader class of hard, pQCD–observables than just DIS.

2.Single– and two particle–inclusive spectra, p$_\perp$–dependence of heavy quarks, and the breakdown of linear k$_\perp$–factorisation

We now present the essentials of the color–dipole formalism that allow us to calculate single– and two–particle spectra differential in the relevant transverse momenta, as well as e.g. associated azimuthal asymmetries. Here we think of a situation, where a highly energetic virtual particle (parton) a dissociates into two partons, $a \to bc$, in a collision with a heavy nucleus. The abc–coupling should be weak, so that to the first order in a perturbative coupling (which we absorb into the light–cone wave function $\Psi(\mathbf{r})$ for the $a \to bc$ transition), the free–particle state is $|a\rangle_{phys} = |a\rangle_0 + \Psi(\mathbf{r})|bc\rangle_0$, with $\mathbf{r}$ the transverse distance between b and c. The virtue of the impact parameter representation is the simplicity of the S–matrix action on the bare partons, namely we can write for the scattered wave

$$\begin{aligned}\mathsf{S}|a\rangle_{phys} &= S_a(\mathbf{b})|a\rangle_0 + gS_b(\mathbf{b}_+)S_c(\mathbf{b}_-)\Psi(\mathbf{r})|bc\rangle_0 \\ &= S_a(\mathbf{b})|a\rangle_{phys} + g\Big[S_b(\mathbf{b}_+)S_c(\mathbf{b}_-) - S_a(\mathbf{b})\Big]\Psi(\mathbf{r})|bc\rangle_{phys} \qquad (1)\end{aligned}$$

The meaning of the transverse coordinates $\mathbf{b}, \mathbf{b}_\pm$ is obvious from Fig 1. Here the terms in brackets represent the amplitude for the inelastic excitation $a \to bc$, and we may further identify $S_a S_b$ as a contribution from a scattering of the constituents, after the dissociation, and S_a as a contribution of scattering of the parton a before the dissociation vertex. Upon squaring the amplitude and using closure on the nuclear side, one obtains the following form of the differential, two–particle inclusive cross section for the process $a \to b(\mathbf{p}_+)c(\mathbf{p}_-)$:

$$\frac{(2\pi)^4 d\sigma}{dz d^2\mathbf{p}_+ d^2\mathbf{p}_-} = \int d^8\Big\{\mathbf{b}_i\Big\} e^{i\mathbf{p}_+(\mathbf{b}_+ - \mathbf{b}'_+) + i\mathbf{p}_-(\mathbf{b}_- - \mathbf{b}'_-)}\Psi(\mathbf{b}_+ - \mathbf{b}_-)\Psi^*(\mathbf{b}'_+ - \mathbf{b}'_-)$$
$$\Big\{S^{(4)}(\mathbf{b}_+, \mathbf{b}_-, \mathbf{b}'_+, \mathbf{b}'_-) + S^{(2)}(\mathbf{b}, \mathbf{b}') - S^{(3)}(\mathbf{b}_+, \mathbf{b}_-, \mathbf{b}') - S^{(3)}(\mathbf{b}'_+, \mathbf{b}'_-, \mathbf{b})\Big\}. \qquad (2)$$

Here the integration is over the impact parameters $\mathbf{b}_\pm, \mathbf{b}'_\pm$, $\mathbf{b} = z\mathbf{b}_+ + (1-z)\mathbf{b}_-$,

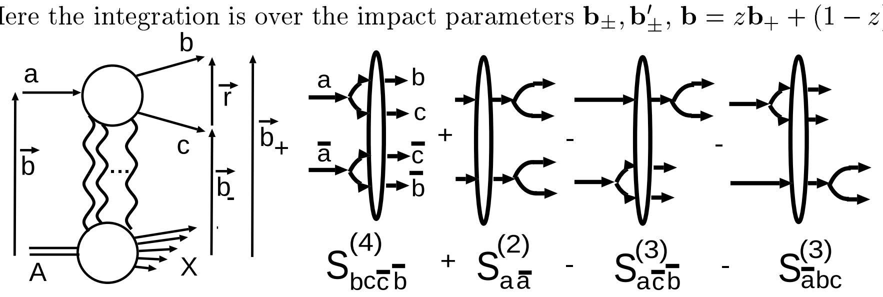

Figure 1. Left: Amplitude for the process $aA \to bcX$. Multiple gluon exchanges connect between the nuclear– and a–fragmentation regions. The relevant impact parameters, which are conserved in the high energy limit, are indicated. Right: Diagrammatic representation for the evolution operator of the four parton density–matrix. Particles from the complex conjugated amplitude become antiparticles in the four–body density matrix. Their impact parameters are the primed ones in the text.

and $\mathbf{b}' = z\mathbf{b}'_+ + (1-z)\mathbf{b}'_-$ where z is the fraction of a's light cone momentum carried by b. Here $S^{(4,3,2)}$ is an appropriate matrix element of the intranuclear evolution operator for a four(three,two)–particle system, coupled to an overall color–singlet state, cf.[4]. We stress that the intranuclear evolution operator is a matrix in the space of singlet four-parton dipole states $|R\bar{R}\rangle = |(bc)_R \otimes (\bar{b}\bar{c})_{\bar{R}}\rangle$, further details depend on the color representations of the partons involved, e.g. $R = 1,8$ for $bc = q\bar{q}$, $R = 1, 8_A, 8_S, 10 + \bar{10}, 27$ for $bc = gg$. A further evaluation of $S^{(4,3,2)}$ would involve the standard Glauber–Gribov approximation for a dilute gas nucleus of color–singlet nucleons. For the scattering off individual nucleons the two–gluon exchange approximation is certainly appropriate in a range of typical Bjorken x not much lower than x_A (i.e. the range $10^{-3} \leq x \leq 10^{-2}$ relevant for RHIC or a possible future electron–nucleus collider [6]). It is important to realize that the color coupled channel aspect of the intranuclear dipole evolution *cannot* be absorbed into a single, 'color–scalar' unintegrated gluon distributions of the nucleus. Hence, for two–particle–inclusive spectra *there is no* $\mathbf{k}_\perp$*–factorization.* Instead, depending again on the color multiplets of the multiparton system that interacts coherently with the nucleus, multigluon exchange effects call upon a whole density matrix of nuclear gluons in color space. Single–particle inclusive spectra (for a host of examples see e.g [3,7]) are in most relevant cases of abelian nature and transitions between color channels during intranuclear rescattering do not appear. Still, if the dissociating particle a interacts with the nucleus by gluon exchanges, $\mathbf{k}_\perp$–factorization is violated already in the single–particle spectra. We make our point on the example of the transverse–momentum spectrum of heavy quarks in pp and pA–collisions, thereby generalizing [5]. For the free nucleon target eq.(2) reduces to

$$\frac{2(2\pi)^2 d\sigma(g^*N \to Q\bar{Q}X)}{dz d^2\mathbf{p}} = \int d^2\mathbf{r} d^2\mathbf{r}' e^{i\mathbf{p}(\mathbf{r}-\mathbf{r}')} \Psi(\mathbf{r})\Psi^*(\mathbf{r}') \Big\{ \sigma_3(z\mathbf{r}',\mathbf{r}) + \sigma_3(z\mathbf{r},\mathbf{r}') - \sigma_{2,Q\bar{Q}}(\mathbf{r}-\mathbf{r}') - \sigma_{2,gg}(z(\mathbf{r}-\mathbf{r}')) \Big\}, \quad (3)$$

with the three–body dipole cross section $\sigma_3(\mathbf{x},\mathbf{r}) = \frac{C_A}{2C_F}[\sigma_2(\mathbf{x}) + \sigma_2(\mathbf{x}-\mathbf{r}) - \frac{1}{N_c^2}\sigma_2(\mathbf{r})] \equiv \mathcal{F}[\sigma_2]$, and $\sigma_{2,gg}(\mathbf{x}) = \frac{C_A}{C_F}\sigma_{2,Q\bar{Q}}(\mathbf{x})$. We indicated that for the free nucleon target σ_3 is a certain linear functional $\mathcal{F}$ of the two–body dipole cross section, and thus also of the unintegrated gluon distribution. Now, when going to the nuclear target, we utilize the Glauber–Gribov substitution $\sigma_2(\mathbf{r}) \to \sigma_{2A}(\mathbf{r}) = 2\int d^2\mathbf{b}\Gamma_A[\sigma_2(\mathbf{r});\mathbf{b}]$; $\sigma_3(\mathbf{x},\mathbf{r}) \to \sigma_{3A}(\mathbf{x},\mathbf{r}) = 2\int d^2\mathbf{b}\Gamma_A[\sigma_3(\mathbf{x},\mathbf{r});\mathbf{b}]$ and, obviously, σ_{3A} is not the same linear functional of σ_{2A} as its free–nucleon counterpart: $\sigma_{3A} \neq \mathcal{F}[\sigma_{2A}]$. Thus, the single–particle inclusive transverse momentum spectrum of heavy quarks in pA–collisions is necessarily a different functional of the nuclear unintegrated glue than the corresponding spectrum in pp collision is of the proton's unintegrated glue. In short: $\mathbf{k}_\perp$ factorization does not hold. This seemingly somewhat technical point is maybe best illustrated by a look at momentum space formulas. The free–nucleon cross section now becomes

$$\frac{2(2\pi)^2 d\sigma(g^*N \to Q\bar{Q}X)}{dz d^2\mathbf{p}} = \int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa}) \Big\{ \frac{C_A}{2C_F} \Big(|\Psi(\mathbf{p}) - \Psi(\mathbf{p}+z\boldsymbol{\kappa})|^2 + |\Psi(\mathbf{p}+\boldsymbol{\kappa}) - \Psi(\mathbf{p}+z\boldsymbol{\kappa})|^2 - |\Psi(\mathbf{p}) - \Psi(\mathbf{p}+\boldsymbol{\kappa})|^2 \Big) + |\Psi(\mathbf{p}) - \Psi(\mathbf{p}+\boldsymbol{\kappa})|^2 \Big\}, \quad (4)$$

where the linear dependence on the unintegrated glue $f(\boldsymbol{\kappa})$ is in clear evidence. If eq.(4) was a true factorization theorem, all the target dependence would be buried in f, and the nuclear cross section should just be given by properly substituting $f \to \phi$. Instead, in a strong absorption regime, say for central g^*–nucleus collisions, the nuclear cross section has a drastically different functional dependence on the (nuclear–) unintegrated glue, namely:

$$\frac{(2\pi)^2 d\sigma(g^* A \to Q\bar{Q}X)}{dz d^2\mathbf{p} d^2\mathbf{b}}\Big|_{\mathbf{b}\to 0} = \int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 \phi(\boldsymbol{\kappa}_1)\phi(\boldsymbol{\kappa}_2)|\Psi(\mathbf{p}+\boldsymbol{\kappa}_2) - \Psi(\mathbf{p}+z\boldsymbol{\kappa}_1+z\boldsymbol{\kappa}_2)|^2 \tag{5}$$

It is important to stress that the nonlinear (quadratic) dependence of the heavy quark spectrum on the target's unintegrated glue has nothing to do with matters of taste concerning our definition of a gluon distribution. Simply, with f and ϕ both defined by means of the same observable –the total DIS–cross section– equations (4,5) entail a very different *relation between two observables*–the total DIS cross section and the heavy quark spectrum–depending on whether the target is a single nucleon or a strongly absorbing nucleus. We conclude that phenomenologies which treat hard nuclear processes simply by substituting nuclear gluon distributions into linear $\mathbf{k}_\perp$–factorization formulas are not borne out by a consistent treatment, and certainly have nothing to say about a possible role of saturation/absorption/multiple scattering effects in hadron–nucleus collisions. Finally, we remark that in a limit where $\mathbf{p}$ becomes the hardest scale, our eq.(5) smoothly connects to the much cherished hard collinear factorisation theorems. A similar phenomenon is observed for gluon jets in $g^* \to gg$, where a cubic dependence on ϕ is obtained in the strong absorption limit. Violations of linear $\mathbf{k}_\perp$–factorisation had previously been discussed in the breakup of virtual photons into dijets [4] $\gamma^* \to q\bar{q}$, where the correct treatment of the color multichannel aspects is crucial. There we found a complete azimuthal decorrelation of semihard dijets with transverse momenta below the saturation scale, in which case the relation to the nuclear unintegrated glue is highly nonlinear. For hard dijets a linear dependence on the unintegrated glue emerges, with $\mathbf{k}_\perp$–factorisation however being violated. Still, sizeable jet decorrelation effects from intranuclear rescattering remain, especially in central DIS.

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