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TARGET SIZE EFFECTS IN NONLINEAR EVOLUTION

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A numerical solution of the Balitsky-Kovchegov nonlinear evolution equation for arbitrary impact parameter is presented. The transverse size of the interacting dipoles is restricted to be smaller than the transverse size of the target, and the effect of such kinematical restriction is discussed.

1 Introduction

The Balitsky-Kovchegov (BK) equation [1] is the simplest nonlinear equation describing dynamics in the saturation region of the Color Glass Condensate (CGC). This equation follows from a more general renormalization group equation [2] on assuming that all correlations between partons, both in the parton cascade and in the target, are negligible, except the annihilation of two partons into one parton.

Although being trustworthy only when the sizes of interacting dipoles are substantially different, the BK equation has been successfully applied to deep inelastic processes. Progress in developing numerical methods for its solution is one of the most important problems for DIS theory in the region of high parton density QCD.

The first attempt to solve the full BK equation including the impact parameter behavior [3] shows that to obtain the correct behavior at large values of b, it is necessary to alter the kernel of the equation.

In this contribution, our numerical solution [4] for the BK equation is presented. The method of the solution is briefly described in the following section, and representative examples of the evolution results as well as the properties of the obtained solution are presented in section 3.

2 Method of the Solution

The BK equation characterizes the low x behavior of the parton densities, while taking into account hdQCD effects, such that unitarity constraints are inherently obeyed. The equation is for $N(y,\vec{r};\vec{b})$, the ($\vec{b}$-dependent) imaginary part of the amplitude for a parent dipole of transverse size $\vec{r}$ and impact parameter $\vec{b}$ to scatter at a rapidity y off a nucleon.

In the procedure leading to the construction of the nonlinear evolution equation, only fan diagrams having multiple small x gluon emissions from the target were taken into account, while inverse fan diagrams (with multiple emissions from the parent dipole) were omitted [1]. In other words, the equation does not include contributions for $|\vec{r}_1|,|\vec{r}_2| > R$, where $\vec{r}_1,\vec{r}_2$ are transverse sizes of two dipoles produced by the parent dipole prior to the interaction, and R is the transverse size of the target.

The fact that large dipoles are not included in the BK equation is well known, but at the time the expectation was that for $r \ll R$ the large produced dipole contribution would be sufficiently small, so that no explicit exclusion of such contribution is needed. This assumption was based on the semiclassical solution in which large size dipole configurations are suppressed.

A priori, the maximal size of the dipoles that should be considered, is related to the color correlation length, which, in principle, does not depend on the size of the target. However, an attempt to include a model which takes into account non-perturbative effects relating to color correlation, will introduce non-perturbative corrections which are not under theoretical control. In our approach, we explicitly introduce a constraint that the interacting dipoles are smaller than the target, hence making our calculations valid only for short distances, compared to the size of the target. We have demonstrated, however, that contributions coming from large dipoles are relatively small (see [4] for further details).

Hence, we modify the BK kernel, and write the nonlinear evolution equation in the form:

$$N_y(y,\vec{r};\vec{b}) \equiv \frac{\partial N(y,\vec{r};\vec{b})}{\partial y} = \frac{C_F\alpha_s}{2\pi^2}\int d^2r_2\,\frac{r^2}{r_1^2r_2^2}\,\theta(R-|\vec{r}_1|)\,\theta(R-|\vec{r}_2|)$$
$$\Big[2N(y,\vec{r}_1;\vec{b}-\tfrac{1}{2}\vec{r}_2) - N(y,\vec{r}_1;\vec{b}-\tfrac{1}{2}\vec{r}_2)\,N(y,\vec{r}_2;\vec{b}-\tfrac{1}{2}\vec{r}_1) - N(y,\vec{r};\vec{b})\Big]\,. \quad (1)$$

To obtain a solution to (1) at arbitrary b, one needs to specify the initial conditions of N at $y = y_0$, the rapidity from which the evolution is commenced. To this end, we use the rescaled solution of [5] to construct the initial conditions, with a modification that N decreases for large r already at $b = 0$. This modification is consistent with the kinematical restriction imposed on the BK kernel. Specifically, our initial conditions have the form $N(y_0,\vec{r};\vec{b}) = 1 - \exp[-\kappa(y_0,\vec{r})\,S(\sqrt{b^2+r^2})/S(0)]$, where $\kappa(y_0,\vec{r})$ is taken from [5].

In the numerical evolution procedure, we have neglected the effect that the angular dependence has on the evolution itself, and have integrated out the angles at each step of the evolution. In the following section, the evolution results are presented in terms of the angle-integrated amplitude, defined as $N(y,r;b) \equiv \int d\hat{r}_{||}\,N(y,r;b;\hat{r}_{||})$, with $\hat{r}_{||} \equiv \vec{b}\cdot\vec{r}/(b\,r)$.

3 Evolution Results

Fig. 1 shows the evolution process from $y = 4.6$ to $y = 14$ at $b = 0$, 1 and 3 GeV^{-1} as a function of r/R. We have taken a target size of $R^2 = 3.1\,\text{GeV}^{-2}$ so as to compare with [5], where, using R^2 as a parameter, the fixed b solution was successfully fitted to the HERA experimental F_2 data. Note that at the beginning of the evolution process, the amplitude increases mildly with the dipole size, and does not exhibit saturation in the region of $r < R$. As y increases, the rise of N becomes steeper. This rise is tamed when the impact parameter becomes large.

The most troublesome contribution in the BK equation comes from the region

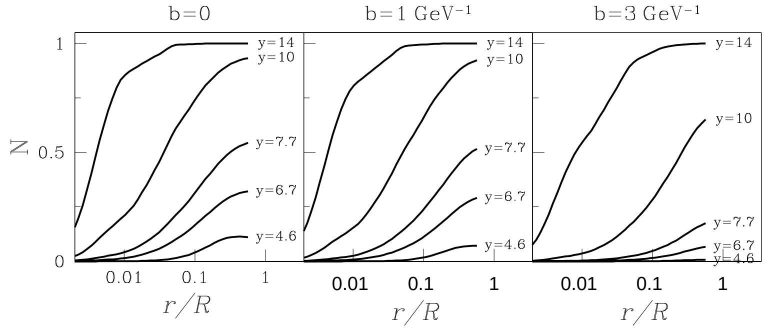

Figure 1. The evolution of N as a function of r/R, for fixed b.

of r_1 and/or r_2 of the order of $2b$, where the amplitude exhibits a b^{-4} tail at large b [3]. We claim that this contribution originates from interactions of dipoles whose transverse size is much larger than the size of the target, and therefore cannot be evaluated using the BK equation. We found that in our approach the impact parameter dependence of the solution is well described by $\exp(-m\,b)$ with $1\,\mathrm{GeV} \lesssim m \lesssim 2\,\mathrm{GeV}$. We consider this result as a support for the above claim.

The solution can be used for investigating saturation effects and scaling properties of the dipole-nucleon scattering amplitude. We define the saturation scale $r_s(y,b)$ as the value of the dipole size, at which the amplitude, $N(r_s,y,b)$, equals $1-e^{-1}=0.632$. Qualitatively, in the semiclassical solution for fixed α_s, the saturation momentum, $Q_s = 2/r_s$, is expected to grow rapidly for large y and decrease rapidly for large b, according to the parameterization $Q_s(y,b) \propto \exp(\lambda_F\,y - b/R)$, with $\lambda_F \approx 2\alpha_\mathrm{s}$. For running α_s, on the other hand, one expects different behavior for small and large b. Specifically, $Q_s(b < R\lambda_R\,y) \propto \exp(\sqrt{\lambda_R\,y})$ and $Q_s(b > R\lambda_R\,y) \propto \exp(-b/R)$, with $\lambda_R \approx 2.5$.

Generally, our solution qualitatively manifests the expected energy and impact parameter behavior of the saturation momentum. With respect to the y-dependence, for both fixed and running QCD coupling, our solution favors the fixed coupling semiclassical properties for low rapidities, and the running coupling semiclassical properties for large rapidities. With respect to the b-dependence, we found that, for large y, the behavior of Q_s at fixed α_s tends to the same distribution as in the running α_s case.

Fig. 2 demonstrates the scaling property of our solution. Geometrical scaling occurs due to the relationship between the y- and r-dependences of the amplitude in the saturation region, and implies that the amplitude is a function of one variable $\tau \equiv rQ_s(y,b)$. In Fig. 2 our solution is plotted as function of τ for $y = 11, 13$ and 14. The solution shows the scaling properties at large values of y, as was predicted by theory [6]. Note that the geometrical scaling behavior is more pronounced for running than for constant QCD coupling. The improved geometrical scaling which we observe for the running coupling may, at first sight, be confusing, since α_s contains a new dimensional parameter Λ_QCD, which in principle could spoil the scaling. However, in Ref. [7] it has been suggested that the running of the QCD coupling

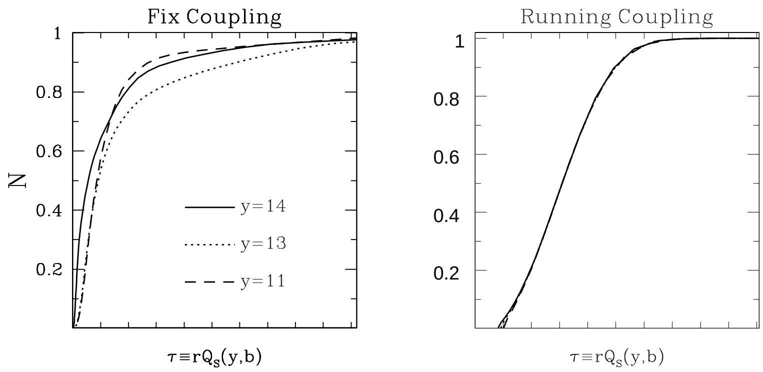

Figure 2. The evolution of N as a function of r/R, for fixed b.

freezes at the saturation momentum, thus recovering the broken geometrical scaling property. Our numerical solution seems to support this hypothesis.

4 Summary

A numerical solution of the BK nonlinear evolution equation for arbitrary b, was presented. The solution was obtained by imposing a restriction on the size of the interacting dipoles, resulting in an exponential b-dependence at large b. The solution qualitatively manifests the expected energy and impact parameter behavior of the saturation scale and features a geometrical scaling behavior, which is more striking for running α_s, than for fixed α_s.

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