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THE REAL CORRECTIONS TO THE γ^* IMPACT FACTOR: FIRST NUMERICAL RESULTS

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We have performed analytically the transverse momentum integrations in the real corrections to the longitudinal γ^*_L impact factor and carried out numerically the remaining integrations. I outline the analytical integration and present the numerical results: we have performed a numerical test and computed those parts of the impact factor that depend upon the energy scale s_0.

1 Introduction

The $\gamma^*\gamma^*$ total cross section is a very attractive observable to be calculated in the framework of NLO BFKL. To perform this calculation and also to approach e.g. the problem of the color dipole picture at NLO, the γ^* impact factor is needed at NLO. The NLO corrections of this impact factor are calculated from the photon-Reggeon vertices for $q\bar{q}$ and $q\bar{q}g$ production, respectively. The virtual corrections involve the vertex $\Gamma^{(1)}_{\gamma^*\to q\bar{q}}$ at one-loop level which has been calculated in [1,2]. As to the real corrections, the squared vertex $|\Gamma^{(0)}_{\gamma^*\to q\bar{q}g}|^2$ is needed at tree level; it has been computed in [3,4] (cf. also [2]). In [4] we have combined the infrared divergences of the virtual and of the real parts, and we have demonstrated their cancellation. What remains to complete the calculation of the NLO photon impact factor are the integrations over the $q\bar{q}$ and $q\bar{q}g$ phase space, respectively. A slightly different approach of calculating the NLO photon impact factor has been proposed in [5].

We have performed, for the case of longitudinal photon polarisation, the phase space integration in the real corrections [6]. After the introduction of Feynman parameters we carried out the integration over transverse momenta analytically. This will allow for further theoretical investigations of the photon impact factor. In particular, the Mellin transform of the real corrections w.r.t the Reggeon momentum can be calculated. This representation (together with an analogous representation of the virtual corrections) can be a starting point for the resummation of the next-to-leading logs(1/x) in the quark anomalous dimensions.

The procedure of arriving at finite NLO corrections to the γ^* impact factor has been described in [4]: $|\Gamma^{(0)}_{\gamma^*\to q\bar{q}g}|^2$ has to be integrated over the $q\bar{q}g$ phase space. But there are two restrictions. First, we have to exclude that region of phase space where the gluon is separated in rapidity from the $q\bar{q}$ pair (central region); this configuration belongs to the LLA and has to be subtracted. To divide the $q\bar{q}g$ phase space an energy cutoff s_0 is introduced which plays the role of the energy scale.

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The virtual corrections to the impact factor are independent of s_0. As a result, the integration of the real corrections already allows to study the s_0 dependence of the NLO impact factor. Second, we need to take care of the infrared infinities. The divergences in $|\Gamma^{(0)}_{\gamma^*\to q\bar{q}g}|^2$ due to the gluon being either soft or collinear to either of the fermions are regularised by subtracting the approximation of the squared vertex in the corresponding limit. These expressions are then re-added and integrated in $4-2\epsilon$ space-time dimensions, giving rise to poles in ϵ, which drop out in combination with the virtual corrections and to finite pieces. The subtraction of the collinear limit requires the introduction of a momentum cutoff parameter, Λ. The final NLO corrections must be independent of this auxiliary parameter. In our numerical analysis we performed this important test.

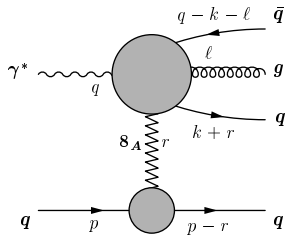

Figure 1. The process $\gamma^* + q' \to q\bar{q}g + q'$ to calculate $\Gamma^{(0)}_{\gamma^*\to q\bar{q}g}$

2 The integration

The $q\bar{q}g$ production vertex is calculated from the process $\gamma^* + q' \to q\bar{q}g + q'$ at tree level. The integration of $|\Gamma^{(0)}_{\gamma^*\to q\bar{q}g}|^2$ therefore implies the integration of a sum of expressions, corresponding to products of Feynman diagrams that differ in their denominator structure (for simplicity we will in the following use 'diagram' rather than 'product of diagrams'). In order to introduce Feynman parameters we therefore split this sum and treat each diagram (or pairs of them) independently. This gives rise to divergences, that in the sum of all diagrams cancel but show up in individual diagrams. The main task in the program of performing the integration analytically is the regularisation of these additional divergences in each individual diagram. We used the subtraction method: From each divergent diagram its approximation in the limit of divergence is subtracted and re-added. The integrations in the re-added pieces are carried out analytically in $4-2\epsilon$ space-time dimensions; the poles in ϵ are found to cancel.

As the result of the regularisation and of the analytical integration over transverse momenta we have obtained for each diagram (or pairs of them) a convergent integral over the Feynman parameters and the momentum fractions of the final state quark and gluon. We denote the sum of these integrals from all diagrams (including the finite pieces from the regularisation of the additional divergences) by Φ_A and Φ_F according to the separation w.r.t. the color factors C_A, C_F. The integrand for each diagram is a sum of expressions corresponding to the subtractions necessary to regularise all divergences of the diagram (see [6]).

3 Numerical results

We have carried out numerically the integrations over the Feynman parameters and the momentum fractions of the quark and gluon for all diagrams. As the result we have obtained values of the real corrections, Φ^{real}, to the photon impact factor as a function of $\Lambda/Q, s_0/Q^2$ and $\mathbf{r}^2/Q^2$, where $\mathbf{r}$ is the transverse part of the Reggeon momentum and Q^2 is the photon virtuality. In the following we suppress the scaling with Q and label the dimensionless variables $\Lambda, s_0, \mathbf{r}$. Besides Φ_A, Φ_F (see above) we include in Φ^{real} those finite pieces from the regularisation of the soft and collinear divergences that depend upon Λ or s_0 (Δ_Λ and Δ_{s_0}, respectively):

$$\Phi^{real} = e^2 e_f^2 \left(\Phi_A + \Phi_F + \Delta_\Lambda + \Delta_{s_0}\right), \quad \Delta_\Lambda = -3\, C_F\, \frac{\Phi^{(0)}}{(2\pi)^2}\, \ln\Lambda.$$

$\Phi^{(0)}$ denotes the LO γ^* impact factor, ee_f is the charge of the quark. The full dependence of the NLO impact factor on Λ and s_0 is now included in Φ^{real}. As said above, the NLO impact factor and hence Φ^{real} has to be independent of Λ. Fig.2 shows that, in fact, the Λ dependence of Φ_A is very weak. As to the C_F terms, Φ_F turns out to be proportional to $\ln\Lambda$. This growth with Λ, however, is fully compensated by Δ_Λ. Note that Φ_F is a sum of many Feynman diagrams, whereas $\Delta_\Lambda \sim \ln\Lambda$. The compensation of the Λ dependence, therefore, represents a rather stringent test of the calculation of the γ^* impact factor.

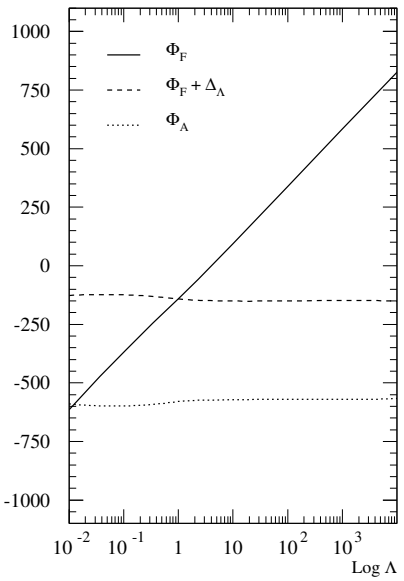

Figure 2. The dependence of the real corrections on Λ (with $\mathbf{r}^2 = s_0 = 1$)

Next we address the dependence of the NLO γ^* impact factor on the energy scale s_0. Since, at the moment, we only know the real corrections, we compute, as

a part of the full LO and NLO impact factor:

$$\Phi' = g^2 \Phi^{(0)} + g^4 \Phi^{real}$$

where $g^2 = 4\pi\alpha_s$. We set $e^2 e_f^2 = 1$. For the photon virtuality we choose $Q^2 = 15$ GeV^2 which leads $\alpha_s(Q^2) = 0.18$ or $g^2 = 1.5$. Fig.3 compares Φ' to the LO impact factor $g^2\Phi^{(0)}$ as function of $\mathbf{r}^2$ at different values of s_0. The real corrections are negative and rather large. More important, Φ' becomes, in absolute terms, more significant for smaller values of s_0. Since we included all s_0 dependent terms in Φ', this implies that the γ^* impact factor tends to become smaller with decreasing s_0. A physical scattering amplitude (e.g. for the $\gamma^*\gamma^*$ scattering process), when consistently evaluated in the framework of NLO BFKL has to be invariant under changes of s_0. Since a decrease of s_0 in the energy dependence $(\frac{s}{s_0})^\omega$ will enhance the scattering amplitude, the combined s_0 dependence of the impact factors and the BFKL Green's function has to compensate this growth. Our result for the s_0 behaviour of the γ^* impact factor is therefore, at least, consistent with the general expectation.

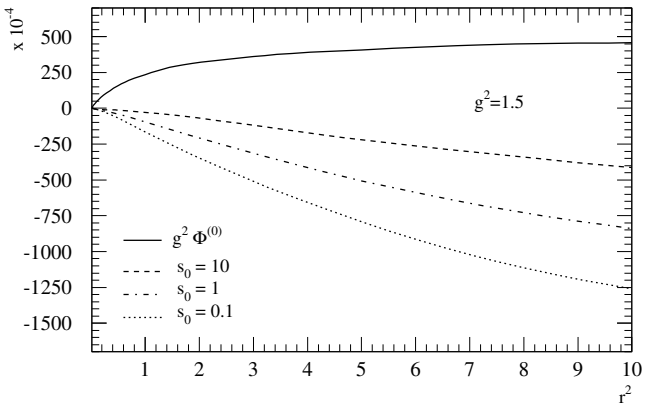

Figure 3. Φ' at different different values of s_0

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