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NUCLEAR PARTON DISTRIBUTION FUNCTIONS AND THEIR EFFECTS ON $\sin^2\theta_W$ ANOMALY

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Nuclear parton distribution functions (NPDFs) are investigated by analyzing the data on structure functions F_2^A and Drell-Yan cross sections σ_{DY}^{pA}. An important point of this analysis is to show uncertainties of the NPDFs by the Hessian method. The analysis indicates that the uncertainties are large for antiquark distributions at $x > 0.2$ and gluon distributions in the whole x region. We also discuss a nuclear effect on the NuTeV $\sin^2\theta_W$ anomaly as an application.

1 Introduction

We report an analysis on nuclear parton distribution functions (NPDFs) and its application to the NuTeV $\sin^2\theta_W$ issue. Although the PDFs in the nucleon are now known rather accurately, their nuclear modifications are not well determined. It is important to determine the NPDFs not only for establishing QCD in nuclei but also for applications to heavy-ion physics and neutrino reactions.

We proposed the optimum NPDFs in 2001 by a χ^2 analysis of nuclear data on deep inelastic lepton scattering [1]. In recent years, it becomes important to show reliability of the obtained PDFs. Actually, uncertainties of the PDFs have been investigated in unpolarized and polarized PDFs in the nucleon. Since there was no serious error estimate of the NPDFs, we investigated the uncertainties by the Hessian method in the recent version [2]. In addition, Drell-Yan and HERMES data are added into the data set, and charm-quark distributions are included in the new analysis.

The NuTeV collaboration reported that their neutrino scattering experiments indicate a significant deviation for the weak mixing angle $\sin^2\theta_W$ from other measurements. Because the target is the iron nucleus, nuclear effects need to be investigated before discussing any new physics mechanisms. In particular, we study effects due to the difference between u_v and d_v nuclear modifications [3,4].

This paper consists of the following sections. In Sec. 2, the χ^2 analysis method is explained for obtaining the optimum NPDFs. Results on the NPDFs are shown in Sec. 3, and their effects are discussed on the determination of $\sin^2\theta_W$ in Sec. 4.

2 Analysis method

The NPDFs are expressed in terms of parameters, which are determined by an analysis of the data on nuclear structure functions F_2^A and Drell-Yan processes. The NPDFs are taken as a nucleonic PDF multiplied by a weight function w_i

which indicates nuclear modifications:

$$f_i^A(x, Q_0^2) = w_i(x, A, Z)\, f_i(x, Q_0^2), \tag{1}$$

where A and Z are mass number and atomic number, and i denotes a parton-distribution type. The function w_i is expressed by parameters:

$$w_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^{1/3}}\right) \frac{a_i(A, Z) + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}. \tag{2}$$

The A dependence is motivated by a simple picture of nuclear volume and surface contributions to cross sections. The cubic functional form for the x dependence is motived by the shape of F_2^A/F_2^D data, and the factor $1/(1-x)^{\beta_i}$ is introduced to reproduce the Fermi-motion part.

For the NPDFs (f_i^A), we take u_v^A, d_v^A, $\bar{q}^A$, and g^A, and the initial point is chosen Q_0^2=1 GeV^2. These distributions are evolved to experimental Q^2 points to calculate $\chi^2 = \sum_j (R_j^{data} - R_j^{theo})^2/(\sigma_j^{data})^2$, where σ_j^{data} is an experimental error and R_j indicates the ratios $F_2^A/F_2^{A'}$ and $\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$. Leading-order expressions are used in the theoretical calculations. By the χ^2 analysis, we obtain the optimum NPDFs and a Hessian matrix H. Using this matrix, we calculate the uncertainty of the distribution $f^A(x)$ by

$$[\delta f^A(x)]^2 = \Delta\chi^2 \sum_{i,j} \left(\frac{\partial f^A(x,\xi)}{\partial \xi_i}\right)_{\xi=\hat{\xi}} H_{ij}^{-1} \left(\frac{\partial f^A(x,\xi)}{\partial \xi_j}\right)_{\xi=\hat{\xi}}, \tag{3}$$

where ξ_i is a parameter, and $\hat{\xi}$ indicates the parameter set for the minimum χ^2. The $\Delta\chi^2$ determines a confidence region, and it is taken $\Delta\chi^2$=10.427 for nine parameters [2]. It corresponds to the one-σ-error range.

3 Results on nuclear PDFs

The experimental data, which are used for the analysis, are taken for the nuclei: deuteron (D), helium-4 (^{4}He), lithium (Li), beryllium (Be), carbon (C), nitrogen (N), aluminum (Al), calcium (Ca), iron (Fe), copper (Cu), krypton (Kr), silver (Ag), tin (Sn), xenon (Xe), tungsten (W), gold (Au), and lead (Pb). The total number of the data is 951. The minimum χ^2 becomes χ^2_{min}=1489.8 in the current analysis.

Among many nuclear data sets, we show only two examples in Fig. 1 for comparing fit results with the data for the calcium nucleus. The parametrization results and uncertainties are calculated at Q^2=5 and 50 GeV^2 for F_2 and Drell-Yan, respectively. Because the data are taken at various Q^2 points, the curves cannot be really compared with the data due to the Q^2 difference. However, because the scaling violation is not a large effect, it is obvious in Fig. 1 that the analysis well reproduces the data. The F_2 data play a role of determining antiquark distributions at small x and valence-quark distributions in the medium- and large-x regions. On the other hand, the Drell-Yan data provide a constraint on the antiquark distributions at $x \sim 0.1$.

From the χ^2 analysis, we obtain the NPDFs for nuclei with A =2 to about 208. Among them, we show nuclear modifications of a medium size nucleus, calcium.

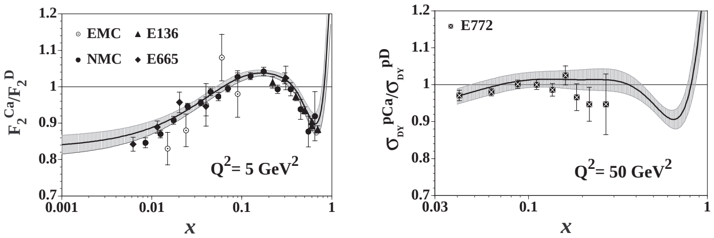

Figure 1. Fit results are compared with the data for F_2^{Ca}/F_2^D and $\sigma_{DY}^{pCa}/\sigma_{DY}^{pD}$. The curves and shaded areas indicate the fit results and their uncertainties at Q^2=5 or 50 GeV^2.

In Fig. 2, the weight functions are shown for the distributions u_v^{Ca}, $\bar{q}^{Ca}$, and g^{Ca} at Q^2=1 GeV^2. The shaded areas indicate uncertainties also at Q^2=1 GeV^2. The antiquark distributions at small x and the valence-quark distributions are well determined. However, the antiquark distributions at $x > 0.2$ and gluon distributions in the whole x region have large uncertainties. The valence-quark distributions are determined by the F_2 data at medium x, and the small-x part is constrained by the baryon-number and charge conservations. For fixing $\bar{q}^A$ at large x and g^A, we need new data which are sensitive to these distributions [5].

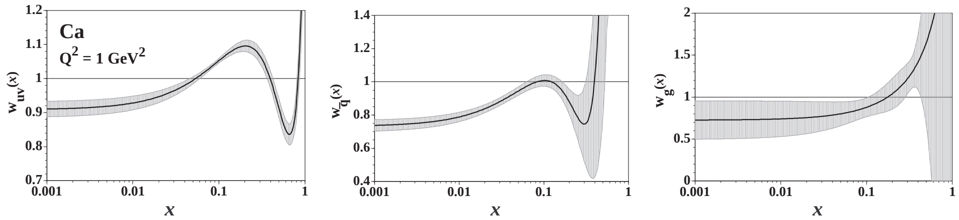

Figure 2. Weight functions w_{u_v}, $w_{\bar{q}}$, and w_g and their uncertainties are shown for the calcium nucleus at Q^2=1 GeV^2.

4 Effects on NuTeV $\sin^2\theta_W$ anomaly

The NuTeV collaboration suggested that their neutrino scattering data should indicate a significant deviation of $\sin^2\theta_W$ value from other measurements. Their value is $\sin^2\theta_W = 0.2277 \pm 0.0013\,(\text{stat}) \pm 0.0009\,(\text{syst})$ in comparison with a global analysis of other data: $\sin^2\theta_W = 0.2227 \pm 0.0004$. Because their experiments use the iron target, nuclear corrections have to be investigated. Among various nuclear corrections, it was shown in Ref. [3] that the difference between nuclear modifications of u_v and d_v should contribute to the determination of $\sin^2\theta_W$. We investigate more details by using χ^2 analysis results for nuclear valence-quark distributions.

For extracting $\sin^2\theta_W$, the following Paschos-Wolfenstein (PW) relation is useful: $R^- = (\sigma_{NC}^{\nu N} - \sigma_{NC}^{\bar{\nu}N})/(\sigma_{CC}^{\nu N} - \sigma_{CC}^{\bar{\nu}N}) = 1/2 - \sin^2\theta_W$, where $\sigma_{CC}^{\nu N}$ and $\sigma_{NC}^{\nu N}$ indicate deep inelastic neutrino-nucleon cross sections for charged-current and neutral-current processes, respectively. This relation is derived for the isoscalar nucleon without strange-antistrange asymmetry ($s - \bar{s} = 0$). We should note that the NuTeV experiments have been done for the iron nucleus which has a neutron excess. There could be various nuclear corrections to the PW relation. In fact, writing down the relation for a nuclear target, we obtain

$$R_A^- = \Bigg[\left(\frac{1}{2} - \sin^2\theta_W \right) \{1 + \varepsilon_v(x)\,\varepsilon_n(x)\} + \frac{1}{3}\,\sin^2\theta_W\,\{\varepsilon_v(x) + \varepsilon_n(x)\} \\ + \left(\frac{1}{2} - \frac{2}{3}\,\sin^2\theta_W \right) \varepsilon_s(x) + \left(\frac{1}{2} - \frac{4}{3}\,\sin^2\theta_W \right) \varepsilon_c(x) \Bigg] \Bigg/ \Bigg[1 + \varepsilon_v(x)\,\varepsilon_n(x) \\ + \frac{1+(1-y)^2}{1-(1-y)^2} \{\varepsilon_v(x) + \varepsilon_n(x)\} + \frac{2\{\varepsilon_s(x) - (1-y)^2 \varepsilon_c(x)\}}{1-(1-y)^2} \Bigg]. \tag{4}$$

Here, the correction factors ε_v, ε_n, ε_s, and ε_c are defined by $\varepsilon_v = (w_{d_v} - w_{u_v})/(w_{d_v} + w_{u_v})$, $\varepsilon_n = ((N-Z)/A)\,(u_v - d_v)/(u_v + d_v)$, $\varepsilon_s = s_v^A/[w_v\,(u_v + d_v)]$, and $\varepsilon_c = c_v^A/[w_v\,(u_v + d_v)]$, where N is the neutron number and q_v is defined by $q_v = q - \bar{q}$. The functions w_{u_v} and w_{d_v} indicate nuclear modifications for u_v and d_v distributions as explained in section 2. The function w_v is defined by the average, $w_v = (w_{d_v} + w_{u_v})/2$. We note that Eq. (4) becomes the PW relation in the limit $\varepsilon \to 0$.

In particular, we investigate ε_v effects on the $\sin^2\theta_W$ determination [3,4]. For calculating ε_v effects, we should take the NuTeV kinematical into account. Such a kinematical effect can be incorporated by using functionals provided by the NuTeV collaboration. The nuclear modification (ε_v) effects on the $\sin^2\theta_W$ are then calculated. Our research is still in progress [4]; however, the preliminary result indicates a large uncertainty for the $\sin^2\theta_W$ determination. It means that the nuclear modification could not be determined accurately and that there is still a possibility that the deviation is explained by the nuclear effect.

5 Summary

Nuclear parton distribution functions (NPDFs) have been obtained by analyzing the data for the structure function F_2 and Drell-Yan cross sections. Their uncertainties were calculated by the Hessian method. The uncertainties are especially large for the antiquark distributions at $x > 0.2$ and the gluon distributions, so that they need future experimental measurements. Such a determination of the NPDFs could contribute to the clarification of the NuTeV $\sin^2\theta_W$ anomaly.

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