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PERTURBATIVE QCD ODDERON IN THE DIPOLE MODEL

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We show that, in the framework of Mueller's dipole model, the perturbative QCD odderon is described by the dipole model equivalent of the BFKL equation with a C-odd initial condition. The eigenfunctions and eigenvalues of the odderon solution are the same as for the dipole BFKL equation and are given by the functions $E^{n,\nu}$ and $\chi(n,\nu)$ correspondingly, where the C-odd initial condition allows only for odd values of n. The leading high-energy odderon intercept is given by $\alpha_{odd} - 1 = \frac{2\,\alpha_s\,N_c}{\pi}\,\chi(n = 1, \nu = 0) = 0$ in agreement with the solution found by Bartels, Lipatov and Vacca. We also write down an evolution equation for the odderon at very high energy, where parton saturation/Color Glass Condensate sets in. We argue that saturation makes the odderon solution a decreasing function of energy.

The results presented here are based on the work performed in collaboration with Lech Szymanowski and Samuel Wallon [1].

1 Odderon Evolution Equation and its Solution

Soon after the BFKL equation had been derived in [2] for the evolution of a two-reggeon state it was generalized to the case of any arbitrary number of reggeons in [3]. The resulting BJKP equation [3] can, in particular, be written for the case of three reggeon exchange, corresponding to a C-odd amplitude. This is the perturbative QCD analogue of the odderon exchange [4].

The high energy asymptotics of the perturbative odderon has been unclear: while several solutions exist with negative [5] and zero [6] intercepts, no one has been able to prove that there is no other solution of the three-reggeon BJKP equation with higher intercept. This has become known as the odderon problem.

Here we are going to present a solution of this problem for the case of DIS using Mueller's dipole model [7]. We start by considering DIS in the rest frame of the target proton, where the incoming virtual photon splits into a quark-antiquark pair, which then interacts with the target. Single odderon exchange does not contribute to the forward DIS amplitude. The process where the odderon should contribute is diffractive production of a C-even meson, like a pion or η_C. In DIS this corresponds to $q\bar{q}$ pair diffractively interacting with the target via odderon exchange and then forming a meson. The odderon exchange amplitude, which we will denote by $\mathcal{O}(\underline{x}_0, \underline{x}_1; z, 1-z)$ for a dipole with transverse coordinates of the quark and antiquark $\underline{x}_0$ and $\underline{x}_1$ and longitudinal momentum fractions z and $1-z$ correspondingly, should indeed be C-odd. It should change sign under the charge conjugation, which interchanges the quark with the antiquark: $\mathcal{O}(\underline{x}_0, \underline{x}_1; z, 1-z) = -\mathcal{O}(\underline{x}_1, \underline{x}_0; 1-z, z)$.

The lowest order C-odd exchange diagram is given by a three-gluon exchange

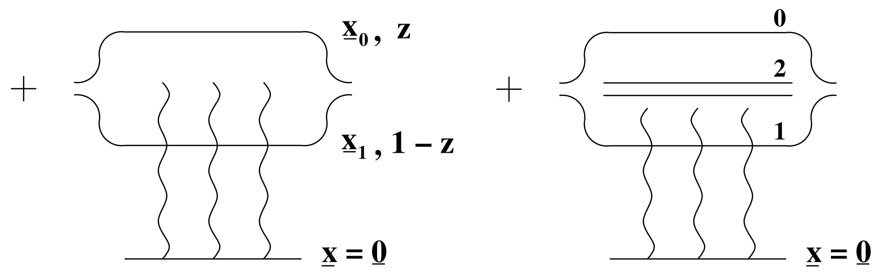

Figure 1. Three-gluon exchange initial conditions (left) and one step of dipole evolution with this initial conditions (right) for DIS on a target at rest.

and is shown on the left of Fig. 1. The corresponding lowest order amplitude is

$$\mathcal{O}_0(\underline{x}_0, \underline{x}_1) \; = \; c_0 \, \alpha_s^3 \, \ln^3 \frac{x_0}{x_1} \tag{1}$$

with

$$c_0 = \frac{(N_c^2 - 4)\,(N_c^2 - 1)}{4\, N_c^3}. \tag{2}$$

To understand what happens to the amplitude (1) at small-x let us include one rung of the dipole evolution in it. One step of the dipole evolution [7] corresponds to emitting a single s-channel gluon in the onium wave function (the target is at rest). In the large-N_c limit in which the dipole model [7] is constructed, the gluon line is represented by a double quark line, as shown in Fig. 1 on the right. The s-channel gluon can be emitted by either quark or anti-quark to the left and to the right of the interaction, which is implied by the disconnected gluon (double) line. Employing the dipole evolution kernel from [7] we obtain the following expression for the first step of dipole evolution with the initial condition (1) ($\underline{x}_{ij} = \underline{x}_i - \underline{x}_j$)

$$\frac{\alpha_s \, N_c}{2\, \pi^2} \int_{z_{init}}^{\min\{z, 1-z\}} \frac{d z_2}{z_2} \int d^2 x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{12}^2} \, [\mathcal{O}_0(\underline{x}_0, \underline{x}_2) + \mathcal{O}_0(\underline{x}_2, \underline{x}_1) - \mathcal{O}_0(\underline{x}_0, \underline{x}_1)] \tag{3}$$

where the first and the second terms correspond to real emissions with the second term pictured on the right side of Fig. 1, while the last term in (3) corresponds to the virtual correction [7]. As one can see explicitly by substituting Eq. (1) into Eq. (3), the amplitude given by Eq. (3) is C-odd. Therefore, the dipole evolution with a C-odd initial condition of Eq. (1) gives an amplitude which is also C-odd, i.e., C-parity is preserved by the dipole evolution [7].

We can, therefore, trivially generalize Eq. (3) to an arbitrary number of evolution steps. For the odderon evolution equation we thus write [1,7]

$$\frac{\partial}{\partial Y} \mathcal{O}(\underline{x}_0, \underline{x}_1, Y) \; = \; \frac{\alpha_s \, N_c}{2\, \pi^2} \int d^2 x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{12}^2} \Big[\mathcal{O}(\underline{x}_0, \underline{x}_2, Y) + \mathcal{O}(\underline{x}_2, \underline{x}_1, Y) - \mathcal{O}(\underline{x}_0, \underline{x}_1, Y) \Big] \tag{4}$$

where we changed the arguments of the dipole amplitude $\mathcal{O}(\underline{x}_i, \underline{x}_j; z_i, z_j)$ to $\mathcal{O}(\underline{x}_i, \underline{x}_j, Y)$ such that $Y = \ln(\min\{z_i, z_j\}/z_{init})$ is the rapidity variable. In the leading logarithmic approximation for gluon evolution the dependence of the dipole amplitude on the light cone momentum fractions of the quark z_i and the antiquark z_j comes in only through the quantity $\min\{z_i, z_j\}$ [7], which allowed us to reduce the number of arguments in $\mathcal{O}$. The initial condition for the Eq. (4) is given by Eq. (1).

Eq. (4) is the analogue of the three-reggeon BJKP equation in the dipole model [1]. The equation is identical to the dipole model equivalent of the BFKL equation [7]. Therefore, we conclude that in the dipole model, both the C-even two-reggeon evolution (BFKL) and the C-odd three-reggeon evolution (BJKP) are given by the same evolution equation (4). The difference is only in the initial conditions: C-even initial conditions pick out the BFKL pomeron contribution, while the C-odd initial conditions pick out the odderon contribution.

Eq. (4) is easy to solve: since it is equivalent to the BFKL evolution equation in the dipole model its kernel's eigenfunctions and their eigenvalues are known [7]. The eigenfunctions of Eq. (4) are given by (ρ_i's denote $\underline{x}_i$'s in the complex notation)

$$E^{n,\nu}(\rho_0, \rho_1) \;=\; \left(\frac{\rho_{01}}{\rho_0\,\rho_1}\right)^{\frac{1+n}{2}+i\nu} \left(\frac{\rho^*_{01}}{\rho^*_0\,\rho^*_1}\right)^{\frac{1-n}{2}+i\nu} \tag{5}$$

which are the eigenfunctions of the Casimir operators of the conformal algebra. Since $E^{n,\nu}(\rho_1, \rho_2) \,=\, (-1)^n\, E^{n,\nu}(\rho_2, \rho_1)$, the C-odd initial condition (1) allows only for odd values of the integer index n in the odderon solution. The corresponding eigenvalues of the eigenfunctions (5) are again the same as for the BFKL equation

$$\frac{2\,\alpha_s\,N_c}{\pi}\,\chi(n,\nu) \;=\; \frac{2\,\alpha_s\,N_c}{\pi}\,\left[\psi(1) - \frac{1}{2}\,\psi\left(\frac{1+|n|}{2} + i\nu\right) - \frac{1}{2}\,\psi\left(\frac{1+|n|}{2} - i\nu\right)\right], \tag{6}$$

where only odd n are allowed for the odderon. The leading odderon intercept is given by $n = 1$ eigenvalue taken at the $\nu = 0$ saddle point [1,6]

$$\alpha_{odd} - 1 \;=\; \frac{2\,\alpha_s\,N_c}{\pi}\,\chi(n=1, \nu=0) \;=\; 0. \tag{7}$$

One can show [1] that the solution in Eqs. (5) and (6) is equivalent to the solution of three-reggeon BJKP equation found recently by Bartels, Lipatov and Vacca in [6]. However, since the eigenfunctions of Eq. (5) form a complete basis in the space of functions contributing to the dipole amplitude, we have proven that there could be no other odderon solution as long as dipole scattering on any target is concerned.

2 Odderon Evolution in the Saturation Region

At very high energy the linear evolution of Eq. (4) breaks down due to nonlinear effects of parton saturation/Color Glass Condensate. Similar effects led to a nonlinear evolution equation written down in [8] for the C-even forward dipole scattering amplitude $N(\underline{x}_0, \underline{x}_1, Y)$. Similarly to [8] we can write down a nonlinear evolution equation for the odderon amplitude $\mathcal{O}$ generalizing Eq. (4) to higher energies. The equation is graphically represented in Fig. 2, which translates into [1]

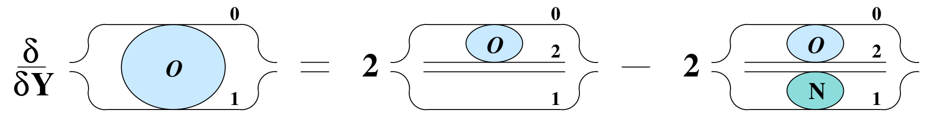

Figure 2. Evolution equation for the odderon exchange amplitude $\mathcal{O}$ in the presence of gluon saturation, which comes in through the C-even dipole ("pomeron") amplitude N [1].

$$\frac{\partial}{\partial Y}\mathcal{O}(\underline{x}_0,\underline{x}_1,Y) = \frac{\alpha_s\,N_c}{2\,\pi^2}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\Big[\mathcal{O}(\underline{x}_0,\underline{x}_2,Y)+\mathcal{O}(\underline{x}_2,\underline{x}_1,Y)$$
$$-\mathcal{O}(\underline{x}_0,\underline{x}_1,Y)\Big] - \frac{\alpha_s\,N_c}{2\,\pi^2}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\Big[\mathcal{O}(\underline{x}_0,\underline{x}_2,Y)\,N(\underline{x}_2,\underline{x}_1,Y)$$
$$+N(\underline{x}_0,\underline{x}_2,Y)\,\mathcal{O}(\underline{x}_2,\underline{x}_1,Y)\Big]. \tag{8}$$

The initial condition for Eq. (8) can be found in [1]. From Eq. (8) we conclude that saturation effects *decrease* the odderon intercept, making it less than 0 of Eq. (7).

Acknowledgments

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