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STRUCTURE FUNCTION F_2: HIGHER TWIST EFFECTS AT SMALL X

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Higher twist corrections to F_2 at small x are studied for the case of a flat initial condition for the twist-two QCD evolution at next-to-leading order approximation. We present an analytical parameterization of the contributions from the twist-two and higher twist operators of the Wilson operator product expansion. The results are in very good agreement with deep inelastic scattering data from HERA.

For more than a decade the various models of the behavior of quarks and gluons at small x has been confronted by a large amount of experimental data from HERA on the deep-inelastic scattering (DIS) structure function (SF) F_2 [1,2]. In the small x regime, non-perturbative effects are expected to give a substantial contribution to F_2. However, what is observed up to very low $Q^2 \sim 1$ GeV^2 values, traditionally explained by soft processes, is described reasonably by perturbative QCD evolution (see, e.g., [3]). Thus, it is important to find the kinematical region where the well-established perturbative QCD formalism can be safely applied.

At small x the Q^2 dependence of quarks and gluons is usually obtained from the numerical solution of the DGLAP equations. The x profile of partons at some initial Q_0^2 and the QCD energy scale Λ are determined from a fit to experimental data.

On the other hand, when analyzing exclusively the small x region, a much simpler analysis can be done by using some of the existing analytical approaches of DGLAP equations in the small x limit. In Refs. [4,5,6] it was pointed out that HERA small x data can be interpreted in terms of the so called doubled asymptotic scaling (DAS) phenomenon related to the asymptotic behavior of the DGLAP evolution discovered many years ago.

In the present talk we report the new results of [7] about the incorporation of the contribution from higher twist (HT) operators of the Wilson operator product expansion to our previous analysis [6]. The semianalytical solution of DGLAP equations obtained in Ref. II using a flat initial condition, is the next-to-leading order (NLO) extension of previous studies performed at the leading order (LO) in perturbative QCD [4,5]. The flat initial conditions correspond to the case when parton distributions tend to some constant value at $x \to 0$ and at some initial value Q_0^2.

In Ref. [6], both the gluon and quark singlet densities are presented in terms of the diagonal $'+'$ and $'-'$ components obtained from the DGLAP equations in the Mellin moment space. The $'-'$ components are constants at small x for any values

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of Q^2, whereas the '+' components grow for $Q^2 \geq Q_0^2$ as

$$\sim \exp\left(2\sqrt{\left[a_+ \ln\left(\frac{a_s(Q_0^2)}{a_s(Q^2)}\right) - \left(b_+ + a_+\frac{\beta_1}{\beta_0}\right)\left(a_s(Q_0^2) - a_s(Q^2)\right)\right]\ln\left(\frac{1}{x}\right)}\right) ,$$

where $a_+ = 12/\beta_0$, $b_+ = 412/(27\beta_0)$, $\beta_0 = 11 - 2f/3$ and $\beta_1 = 102 - 38f/3$ for f active quarks. Hereafter we use the notation $a_s = \alpha_s/(4\pi)$.

The analysis performed in [6] has shown very good agreement with HERA 1994 data [1,2] at $Q^2 \geq 1.5$ GeV^2. In Ref. [7] we added the contribution from HT operators and described also more modern 1996/97 data at lower Q^2.

1. The basic results are the twist-four [a] corrections for the SF F_2

$$F_2(x,Q^2) \;=\; F_2^{\tau 2}(x,Q^2) \;+\; \frac{1}{Q^2}\,F_2^{R\tau 4}(x,Q^2) \;. \tag{1}$$

We mark the parts of HT corrections proportional to the twist-two quark and gluon densities as $F_{2,q}^{R\tau 4}$ and $F_{2,G}^{R\tau 4}$, respectively.

Note that the parton distributions $f_a^{\tau 2}(x,Q^2)$ are already multiplied by x and that we neglect the nonsinglet quark density $f_\Delta(x,Q^2)$ and the valence part $f_V(x,Q^2)$ of the singlet quark distributions.

At the leading twist part we have (see [6]) at the LO and NLO approximations, respectively,

$$F_{2,\mathrm{LO}}^{\tau 2}(x,Q^2) \;=\; e\, f_{q,\mathrm{LO}}^{\tau 2}(x,Q^2) \;, \tag{2a}$$

$$F_2^{\tau 2}(x,Q^2) \;=\; e\left(f_q^{\tau 2}(x,Q^2) \;+\; \frac{4T_R f}{3}\,a_s(Q^2)\,f_G^{\tau 2}(x,Q^2)\right) \;, \tag{2b}$$

where $e = (\sum_1^f e_i^2)/f$ is the average charge square for f active quarks.

Let us to keep the NLO relation (2b) beyond the leading twist approximation. Then for the total SF F_2 (1) we obtain (see [7])

$$F_2(x,Q^2) \;=\; e\left(f_q(x,Q^2) \;+\; \frac{4T_R f}{3}\,a_s(Q^2)\,f_G(x,Q^2)\right) \;, \tag{3}$$

where $f_a(x,Q^2)$ are the total parton distributions containing both the twist-two part, presented in [6], and the twist-four contributions,

$$f_a(x,Q^2) \;=\; f_a^{\tau 2}(x,Q^2) \;+\; \frac{1}{Q^2}\,f_a^{R\tau 4}(x,Q^2) \;. \tag{4}$$

In the Ref. [7] we study the x and Q^2 dependences of the structures F_2, $\partial F_2/\partial \ln Q^2$ and $\partial \ln F_2/\partial \ln(1/x)$, that needs to define the parton densities in a proper way. So, we take quite *natural* choice

$$f_q^{R\tau 4}(x,Q^2) \;=\; \frac{1}{e}\,F_{2,q}^{R\tau 4}(x,Q^2) \;,\quad f_G^{R\tau 4}(x,Q^2) \;=\; \frac{3/4T_R f}{e a_s(Q^2)}\,F_{2,G}^{R\tau 4}(x,Q^2) \;, \tag{5}$$

i.e., the HT quark (gluon) part of F_2 relates only to the corresponding quark (gluon) twist-two density. This choice corresponds exactly to the Eq. (3), i.e. to generalization of the standard twist-two relation (2b) between F_2 and parton densities at the NLO approximation with the purpose to include the HT contributions.

[a] Here we restrict ourselves by the twist-four terms [8] in the renormalon formalism. The full consideration can be found in [7].

2. As it has been already noted above it is useful to split the parton distributions in two parts

$$f_a(x,Q^2) \;=\; f_a^+(x,Q^2) \;+\; f_a^-(x,Q^2)\;, \tag{6}$$

where the both $'+'$ and $'-'$ components contain twist-two and HT parts. The two component representation follows directly form the exact solution of DGLAP equation in the Mellin moment space at the leading twist approximation.

The twist-two contribution is presented in [6] and the twist-four parts can be expressed through the twist-two one as follows (here for simplicity we restrict our consideration by LO approximation):
for the (singlet) quark distribution

$$\begin{aligned}
\frac{f_q^{R\tau 4,+}(x,Q^2)}{f_{q,\mathrm{LO}}^{\tau 2,+}(x,Q^2)} &= \frac{128f}{45\beta_0^2}\,\mathrm{a}_q^{\tau 4}\left\{\frac{2}{\rho_{\mathrm{LO}}^2}+\ln\left(\frac{Q^2}{|\mathrm{a}_q^{\tau 4}|}\right)\frac{\widetilde{I}_0(\sigma_{\mathrm{LO}})}{\rho_{\mathrm{LO}}\,\widetilde{I}_1(\sigma_{\mathrm{LO}})}\right\} \;+\; \mathcal{O}\left(\rho_{\mathrm{LO}}\right)\;,\\
\frac{f_q^{R\tau 4,-}(x,Q^2)}{f_{q,\mathrm{LO}}^{\tau 2,-}(x,Q^2)} &= \frac{128f}{45\beta_0^2}\,\mathrm{a}_q^{\tau 4}\left\{\ln\left(\frac{1}{x_q}\right)\ln\left(\frac{Q^2}{x_q\,|\mathrm{a}_q^{\tau 4}|}\right)-p'(\nu_q)\right\} \;+\; \mathcal{O}\left(z\right)\;.
\end{aligned} \tag{7a}$$

for the gluon distribution

$$\begin{aligned}
\frac{f_G^{R\tau 4,+}(x,Q^2)}{f_{G,\mathrm{LO}}^{\tau 2,+}(x,Q^2)} &= \frac{8}{5\beta_0^2}\,\frac{\mathrm{a}_G^{\tau 4}}{a_s(Q^2)}\left\{\frac{2}{\rho_{\mathrm{LO}}}\frac{\widetilde{I}_1(\sigma_{\mathrm{LO}})}{\widetilde{I}_0(\sigma_{\mathrm{LO}})}+\ln\left(\frac{Q^2}{|\mathrm{a}_G^{\tau 4}|}\right)\right\} \;+\; \mathcal{O}\left(\rho_{\mathrm{LO}}\right)\;,\\
\frac{f_G^{R\tau 4,-}(x,Q^2)}{f_{G,\mathrm{LO}}^{\tau 2,-}(x,Q^2)} &= \frac{8}{5\beta_0^2}\,\frac{\mathrm{a}_G^{\tau 4}}{a_s(Q^2)}\,\ln\left(\frac{Q^2}{x_G^2\,|\mathrm{a}_G^{\tau 4}|}\right) \;+\; \mathcal{O}\left(z\right)\;,
\end{aligned} \tag{7b}$$

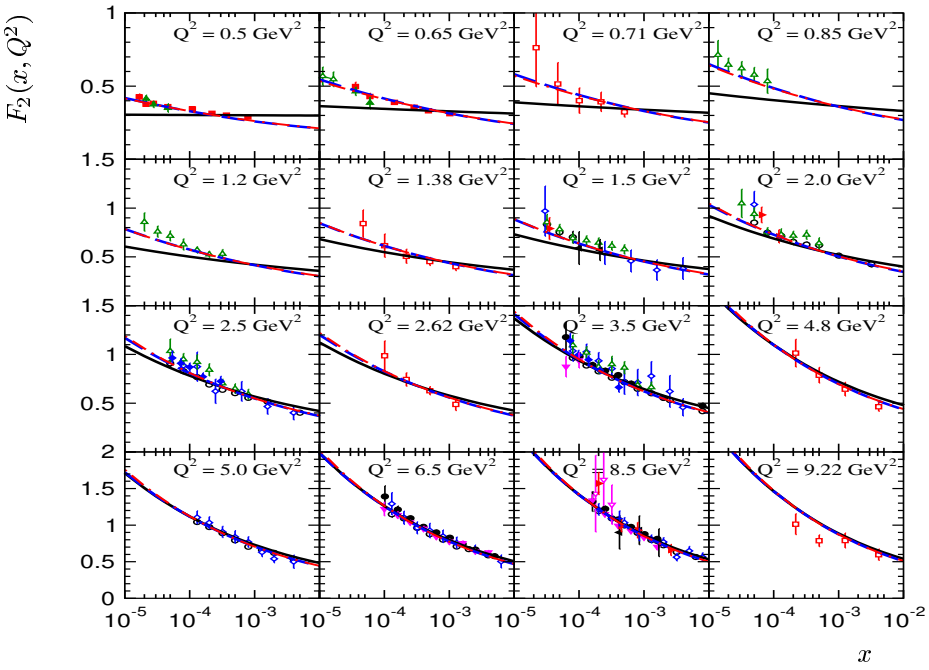
where $\mathrm{a}_a^{\tau 4}$ are the magnitudes which should be extracted from the fits of the experimental data. The variables $x_a = x\exp[p(\nu_a)]$, where $p(\nu_a) = [\Psi(1+\nu_a) - \Psi(\nu_a)]$ and ν_a are the powers of the $x \to 1$ asymptotics of the parton distributions, i. e. $f_a \sim (1-x)^{\nu_a}$ at $x \to 1$. From the quark counting rules we know that $\nu_q \approx 3$ and $\nu_G \approx 4$. Then, we get $p(\nu_q) \approx 11/6$ and $p(\nu_G) \approx 25/12$, and there derivatives $p'(\nu_q) \approx -49/36$ and $p'(\nu_G) \approx -205/144$ (see Ref. [7] for further details).

The functions $\widetilde{I}_\nu$ in Eqs. (7a, 7b) are related to the modified Bessel function I_ν and to the Bessel function J_ν by:

$$\widetilde{I}_\nu(\sigma) \;=\; \begin{cases} I_\nu(\bar{\sigma}),\ \text{if } \sigma^2 = \bar{\sigma}^2 \geq 0\;, \\ i^\nu J_\nu(\bar{\sigma}),\ \text{if } \sigma^2 = -\bar{\sigma}^2 < 0\;, \end{cases} \tag{8}$$

σ and ρ are the generalized Ball-Forte variables (see [7]). Note that the upper (down) line in the r.h.s. of Eq. (8) corresponds to the solution of the DGLAP equation for the "direct" ("backward") evolution in the DAS approximation.

3. The typical fits for the SF $F_2(x,Q^2)$ as a function of x for different Q^2 bins are presented on the figure below. The experimental points are from H1 [1] (open points) and ZEUS [2] (solid points). The solid line represents the NLO fit alone with $\chi^2/\mathrm{n.d.f.} = 1.31$. The dashed curve are obtained from the fit at the NLO, when the renormalon contributions of higher-twist terms have been incorporated. The corresponding $\chi^2/\mathrm{n.d.f.} = 0.86$. The dash-dotted curve (hardly distinguished from the dashed one) represents the fit at the LO together with the renormalon contributions of higher-twist terms. The corresponding $\chi^2/\mathrm{n.d.f.} = 0.84$. The results



demonstrate excellent agreement between theoretical predictions and experimental data for the region $Q^2 \geq 0.5$ GeV^2 as for SF $F_2(x,Q^2)$ as for the effective slope (see [7] and discussions therein).

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