Proceedings of the DIS'2004, Štrbské Pleso, Slovakia

UN-INTEGRATED UPDFS IN CCFM

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The un-integrated parton distribution functions (uPDFs) obtained from a CCFM evolution are studied in terms of the intrinsic transverse momentum distribution at low scales. The uPDFs are studied for variations of the renormalization and factorization scales.

1 Introduction

Un-integrated parton distributions are best suited to study the details of the perturbative as well as the non-perturbative QCD evolution. The general form of the integral equation for the parton evolution is:

$$x\mathcal{A}(x,k_t,\bar{q}) = x\mathcal{A}_0(x,k_t,\bar{q}) + \int \frac{dz}{z}\int \frac{dq^2}{q^2}\Theta(\text{order})\Delta_s\tilde{P}(z,q,k_t)x\mathcal{A}\left(\frac{x}{z},k_t',q\right) \quad (1)$$

with $\bar{q}$ being the evolution (factorization) scale. The ordering condition of the evolution is specified by $\Theta(\text{order})$ which in the case of CCFM is given by $\Theta(\text{order}) = \Theta(\bar{q}-zq)$. The first term of the rhs of eq.(1) gives the contribution of non-resolvable branchings between the starting scale Q_0 and the factorization scale $\bar{q}$ and is given by:

$$x\mathcal{A}_0(x,k_t,\bar{q}) = x\mathcal{A}_0(x,k_t,Q_0)\Delta_s(\bar{q},Q_0) \quad (2)$$

where the Sudakov form factor $\Delta_s(q_2,q_1)$ describes the probability of no radiation between Q_0 and $\bar{q}$. The second term of the rhs of eq.(1) describes the details of the QCD evolution, expressed by the convolution of the splitting function $\tilde{P}$ with the parton density and the Sudakov form factor Δ_s.

2 The starting distribution $x\mathcal{A}_0(x,k_t,Q_0)$

The non-resolvable branching contribution is directly related to the initial (starting) distribution $\mathcal{A}_0(x,k_t,Q_0)$, and especially to the intrinsic k_t distribution. Often a Gaussian type distribution with width k_0 is assumed and $\mathcal{A}_0$ can be parameterized as:

$$x\mathcal{A}_0(x,k_t,Q_0) = Nx^{p_0}(1-x)^{p_1}\cdot\exp\left(-k_t^2/k_0^2\right) \quad (3)$$

where N is a normalization constant, and p_0, p_1 being parameters to be determined experimentally.

In the following sections the dependence of the choice of p_0, p_1 and k_0 will be investigated. The CCFM evolution equation is applied to the starting distribution

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$x\mathcal{A}_0(x,k_t,Q_0)$ and then convoluted with the off-shell matrix elements. The simple gluon splitting function is applied [1,2],

$$P_{gg}(z,\bar{q},k_t) = \frac{\bar{\alpha}_s(q^2(1-z)^2)}{1-z} + \frac{\bar{\alpha}_s(k_t^2)}{z}\Delta_{ns}(z,q^2,k_t) \tag{4}$$

with $q = p_t/(1-z)$ and the non-Sudakov form factor Δ_{ns} is defined by:

$$\log\Delta_{ns}(z_i,q^2,k_t) = -\bar{\alpha}_s\int_z^1\frac{dz'}{z'}\int\frac{dq^2}{q^2}\cdot\Theta(k_t-q)\Theta(q-z'q) \tag{5}$$

The collinear cutoff $Q_g = 1.3$ GeV regulates the region of $z\to 1$, applied both to the real emissions as well as in the Sudakov form factor for the virtual corrections, which is equivalent to using the *plus* prescription of the splitting function P_+.

Due to the angular ordering in CCFM a kind of random walk in the propagator gluon k_t can be performed, and values of small k_t can be reached. A cut k_t^{cut} is introduced to avoid this region.

The CCFM evolution equations have been solved numerically [1] using a Monte Carlo method. In Tab. 1 the results of the fit for the un-integrated gluon densities are summarized. The input parameters are varied such that after convolution with the off-shell matrix elements the results agree best with the measured structure function $F_2(x,Q^2)$ in the range $x < 5\cdot 10^{-3}$ and $Q^2 > 4.5$ GeV^2 as measured at H1 [3,4] and ZEUS [5,6]. The starting distribution is given by eq.(3), where the parameter $p_1 = 4$ is kept fixed, but p_0 and the width of the Gaussian k_0 was adjusted to give the smallest $\chi^2/points$. The results are given in Tab. 1 and labeled as *set A0*.

Similarly, fits can be obtained using different values for the soft cut k_t^{cut} and a different value for the width of the intrinsic k_t distribution *set B0*. Motivated by the success of the saturation model [7], the parameterization of GBW can also serve as a starting distribution:

$$x\mathcal{A}_0(x,k_t,Q_0) = \frac{3\sigma_0}{4\pi^2\alpha_s}R_0^2(x)k_t^2\exp\left(-R_0^2(x)k_t^2\right) \tag{6}$$

with the parameters given in [7]. As seen from Tab. 1 a reasonable description of the data can be achieved *set GBW*.

The renormalization scale dependence of the final cross section can be estimated by changing the scale used in α_s in the off-shell matrix element. Since here we are using the LO α_s matrix elements, any scale variation will change the cross section. In order to obtain a reasonable result, the uPDF was fitted to describe F_2 by varying the scale μ_r. The *set A0-* correspond to a scale $\mu_r = 0.5p_t$ whereas *set A0+* correspond to a scale $\mu_r = 2p_t$. The resulting $\chi^2/points$ are also shown in Tab. 1.

The CCFM evolution is performed in an angular ordered region of phase space and the factorization scale is related to the maximum allowed angle for any emission. In CCFM the scale μ_f (coming from the maximum angle) can be written as:

$$\mu_f = \sqrt{\hat{s}+Q_\perp^2} \tag{7}$$

with $\hat{s}$ being the invariant mass of the $q\bar{q}$ subsystem, and $Q_\perp$ its transverse momentum. In this definition, the scale μ_f is related to the quark pair.

In order to investigate the uncertainties coming from the specific choice of the evolution scale given in eq.(7), another definition is applied, relating the factorization scale only to the quark (or anti-quark): $\mu_f = \frac{p_t}{1-z}$ with p_t being the transverse momentum of the quark(anti-quark) and $z = \frac{k_t \cdot \gamma}{y x_g s}$. This definition follows closely the definition of the rescaled transverse momentum $\bar{q}$ in CCFM. In Tab. 1 is shown, that also with this definition a reasonable description of the data can be achieved, but the parameter p_0 of the starting distribution is changed significantly, which is understandable, since the length of the evolution ladder is changed by μ_f. However changing the scale μ_r and μ_f simultaneously with the same factor results in a much larger χ^2.

set	μ_f	μ_r $= \sqrt{p_t^2 + m^2}$	p_0	k_0 (GeV)	k_t^{cut} (GeV)	$\chi^2/points$ $\chi^2/points$
$A0$	$\sqrt{\hat{s} + Q_\perp^2}$	$1 \cdot \mu_f$	0	1.33	1.33	$276/248 = 1.1$
$A0-$	$\sqrt{\hat{s} + Q_\perp^2}$	$0.5 \cdot \mu_f$	-0.01	1.33	1.33	$433/248 = 1.75$
$A0+$	$\sqrt{\hat{s} + Q_\perp^2}$	$2 \cdot \mu_f$	-0.01	1.33	1.33	$316/248 = 1.3$
$A1$	$\frac{p_t}{1-z}$		-0.1	0.8	1.33	$275/248 = 1.1$
$B0$	$\sqrt{\hat{s} + Q_\perp^2}$	$1 \cdot \mu_f$	0	0.8	0.25	$434/248 = 1.75$
GBW	$\sqrt{\hat{s} + Q_\perp^2}$	1	-	-	0.25	$349/248 = 1.4$

Table 1. The different settings of the CCFM un-integrated gluon densities. The initial distribution is parameterized as $x\mathcal{A}_0(x, k_t, Q_0) = N x^{p_0}(1-x)^{p_1} \cdot \exp\left(-k_t^2/k_0^2\right)$ with $p_1 = 4$ and $Q_g = 1.33$ GeV. In the last line the starting distribution from the parameterization of GBW [7] is used. In the last column, the $\chi^2/points$ to HERA F_2 data [3,4,5,6] is given (for $x < 5 \cdot 10^{-3}$ and $Q^2 > 4.5$ GeV^2).

3 k_t dependence of $x\mathcal{A}(x, k_t, \mu_f)$

In Fig. 1 the uPDFs (as described in Tab. 1) are shown as a function of k_t for different values of x. Also shown is the contribution of the first term of the rhs of eq.(1). It can be seen, that the influence of the intrinsic k_t distribution is concentrated at small values of k_t, whereas at values $k_t \gtrsim 1$ GeV the perturbative evolution is responsible. The specific choice of the k_t^{cut} results in different distributions at small values of k_t. It is also interesting to observe, that the *dip* in k_t visible in Fig. 1 (upper plot) is directly related to k_t^{cut}.

It is interesting to observe, that even with very different starting distributions, the uPDFs after perturbative evolution are similar (at larger k_t). Thus the small k_t region provides new information on the non-perturbative part of the parton density functions, which can only be investigated by using uPDFs. Especially the question of saturation effects at small k_t can be directly investigated with uPDFs. The difference of using a Gaussian intrinsic k_t distribution to a form motived by the saturation model of GBW is significant. It is also important to note, that these differences are washed out for inclusive distributions, as all uPDFs are able

to describe F_2 at a similar level.

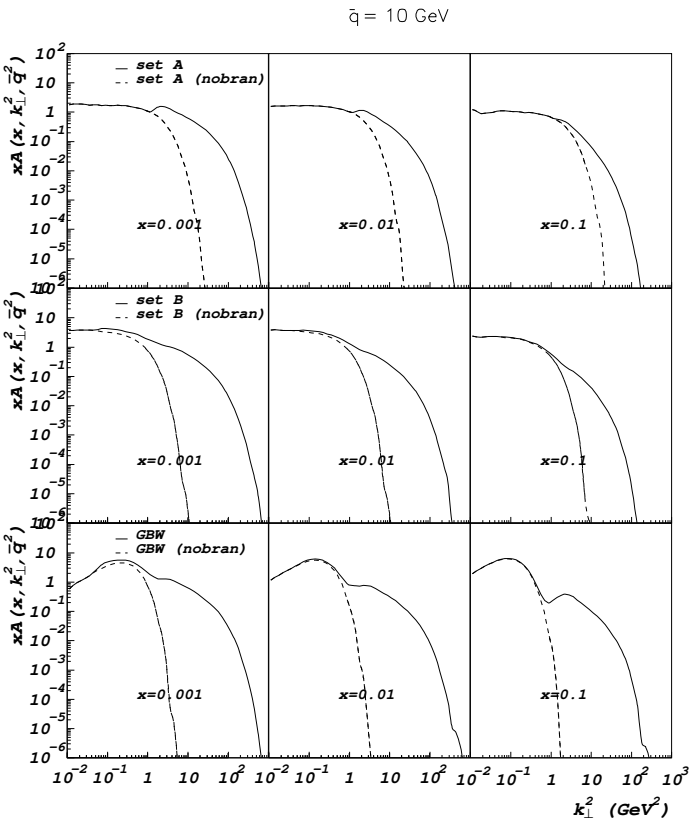

Figure 1. *Comparison of the different sets of un-integrated gluon densities obtained from the CCFM evolution as described in the text.*

Acknowledgments

I am grateful to the organizers of the DIS2004 workshop for the stimulating atmosphere and the good organization.

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