KWIECIŃSKI EVOLUTION OF UNINTEGRATED PARTON DISTRIBUTIONS

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The Kwieciński evolution of unintegrated parton distributions (UPDs) in the transverse-coordinate space is analyzed with the help of the Mellin transform. Numerical results are presented for the unintegrated pion distributions with a simple valence-like initial condition at the low scale, which follows from chiral large-N_c quark models. The effect of spreading of UPDs in the transverse momentum with the increasing scale is confirmed, with $\langle k_\perp^2 \rangle$ growing asymptotically as $Q^2\alpha(Q^2)$. Formal aspects of the equations, such as the limits of UPDs at $x \to 0$, $x \to 1$, and at low and large b, are straightforward to obtain with our method.

This talk is based on Refs. [1, 2] and focuses on numerical as weel as formal aspects of the QCD evolution of UPDs proposed by Kwieciński [3]. Practical applications are presented by Szczurek [4], while similar approaches are discussed by Jung [5] and Lonnblad [6] in these proceedings.

The definition of the leading-twist UPDs [7] may be read-off from Fig. 1(a). The transverse momentum in the partonic loop, $k_\perp$, is left *unintegrated* and is not constrained. Kwieciński's approach is based on the CCFM formalism with the following modifications:

- One-loop CCFM is used, with the angular ordering replaced with a stronger condition on the scaled gluon momenta of the form $q'_{\perp,i} > q'_{\perp,i-1}$, where $q'_i \equiv q_i/(1-z_i)$, $z_i \equiv x_i/x_{i-1}$, and x_i denotes the fraction of the longitu-

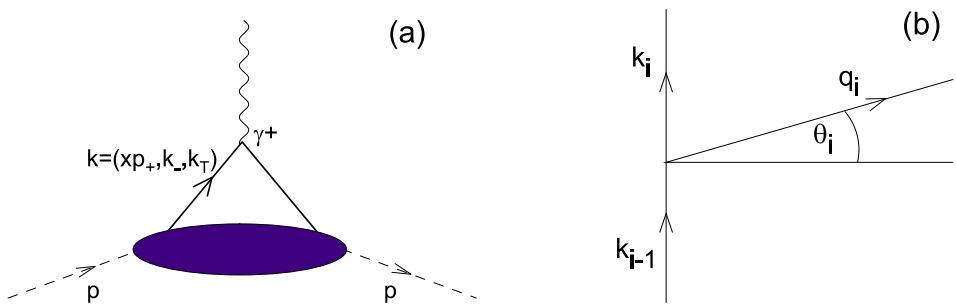

Figure 1. (a) Definition of the leading-twist UPD. (b) Kinematics inside the partonic cascade.

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dinal momentum of the hadron carried by the parton.

- Both singlet and non-singlet quarks are included.
- Non-Sudakov form factor is set to unity.
- The evolution is written in the transverse-coordinate space, b, Fourier-conjugated to the transverse momentum.

The Kwieciński equations are diagonal in the b-space and read [3]:

$$Q^2 \frac{\partial f_{\rm NS}(x,b,Q)}{\partial Q^2} = \tag{1}$$
$$\frac{\alpha_s(Q^2)}{2\pi} \int_0^1 dz\, P_{qq}(z) \left[\Theta(z-x)\, J_0((1-z)Qb)\, f_{\rm NS}\left(\frac{x}{z},b,Q\right) - f_{\rm NS}(x,b,Q) \right],$$

with similar coupled equations for the sea quarks and gluons. Note that for the integrated PDs (case of $b=0$) the equations assume the familiar DGLAP form.

In this talk we consider the uPDF of the pion. The initial conditions for the evolution are taken from the chiral quark models [2, 8, 9], which in the chiral limit provide a particularly simple, factorized form. Moreover, these conditions are consistent with the formal requirements, such as proper normalization, correct support, crossing symmetry, and the momentum sum rule. The working scale of the model, Q_0, is estimated using the fact that at Q_0 all the momementum is carried by the quarks, which are the only degrees of freedom at that scale (see [2] for details). The Spectral Quark Model [9] provides a factorized initial condition of the form

$$q(x,b,Q_0) = F(b)\theta(x)\theta(1-x), \tag{2}$$
$$F(b) = \left(1+\frac{bM_V}{2}\right)\exp\left(-\frac{M_V b}{2}\right)$$

for the valence quarks, with no gluons nor sea quarks at the scale Q_0. Note that the functional dependence in b in Eq. (2) factorizes in Eq. (1). Thus, it is convenient to denote $f(x,b,Q) = F(b) f^{\rm evol}(x,b,Q)$ and present the results for the the *evolution-generated* part $f^{\rm evol}$.

Equations (1) can be solved very efficiently using the Mellin transform [2]. In the moment space we find

$$Q^2 \frac{df_{\rm NS}(n,b,Q)}{dQ^2} = -\frac{\alpha(Q^2)}{8\pi}\gamma_{n,NS}(Qb) f_{\rm NS}(n,b,Q), \tag{3}$$

where $f_{\rm NS}(n,b,Q)$ denote the nth moment, and $\gamma_{n,NS}(b)$ are the b-dependent anomalous dimensions [2]. The formal solution of Eq. (3) is

$$\frac{f_{\rm NS}(n,b,Q)}{f_{\rm NS}(n,b,Q_0)} = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{dQ'^2 \alpha(Q'^2)}{8\pi Q'^2} \gamma_{\rm NS}(n,b,Q') \right], \tag{4}$$

and similarly in the singlet channel. In ordert to come back to the x-space, the inverse Mellin transform is carried out numerically. The numerical results are shown in Figs. 2 and 3. In Fig. 4 we plot the mean squared transverse momentum generated by the evolution. As Q is increased, $\langle k_\perp^2 \rangle$ grows as $\alpha_S(Q^2)Q^2$.

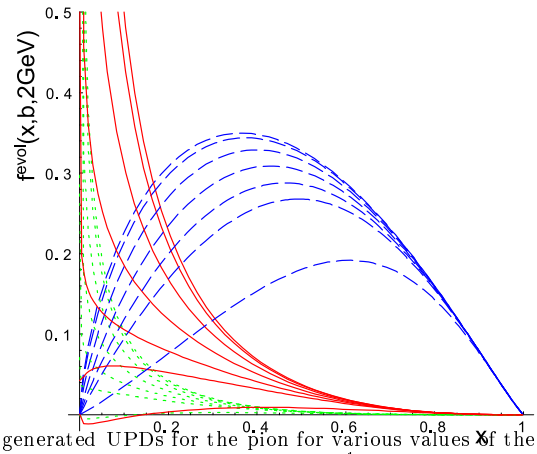

Figure 2. Evolution-generated UPDs for the pion for various values of the transverse coordinate (from top to bottom: $b = 0$, 1, 2, 3, 4, 5 and 10 GeV^{-1}), plotted as functions of the Bjorken x. The evolution is made with the initial condition (2) at $Q_0 = 313$ MeV up to $Q = 2$ GeV. Solid lines – gluons, dashed lines – valence quarks, dotted lines – sea quarks.

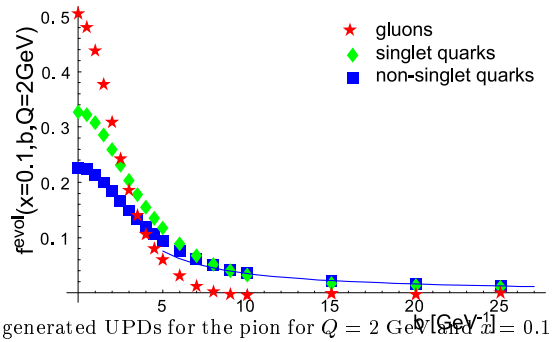

Figure 3. Evolution-generated UPDs for the pion for $Q = 2$ GeV and $x = 0.1$, plotted as functions of b. The evolution is made with the initial condition (2) at $Q_0 = 313$ MeV. The numerical results are represented by squares for the non-singlet quarks, diamonds for the singlet quarks, and stars for the gluons, while the solid line shows the asymptotic power-law formula for the case of non-singlet quarks. We note the much faster fall-of for the gluons than for the quarks. As Q is increased or x decreased, the distributions in b become narrower, which is equivalent to spreading in $k_\perp$.

To summarize, we have proposed an efficient numerical method to solve Eq. (1) via the Mellin transform. The method allows to study formal aspects of the equations, *e.g.* the asymptotic forms of the evolution-generated UPDs at large b, or at $x \to 0$ and $x \to 1$. At large b the evolution-generated b-dependent UPDs exhibit power-law fall-off, with the magnitude of the exponents growing with the probing scale [2]. The fall-off is steeper for the gluons than for the quarks. At $x \to 0$ we have found generalizations of the DLLA behavior. We have also shown that for large b the solution for the valence UPD of the pion grows linearly with x for not too large x, and the slope decreases with b as a power law. At $x \to 1$ the evolution-generated b-dependent UPDs approach the integrated distributions as $(1-x)^2$. We find the spreading of the $k_\perp$ distributions with the probing scale Q, with the effect strongest

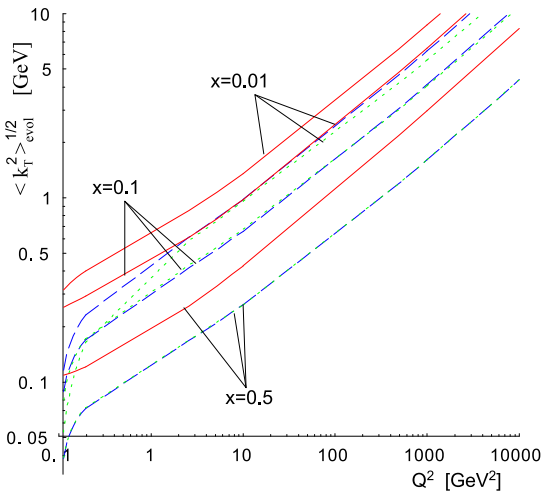

Figure 4. The rms transverse momenta of UPDs of the pion for $x = 0.01$, 0.1, and 0.5, plotted as functions of the renormalization scale Q^2. Solid lines – gluons, dashed lines – non-singlet quarks, dotted lines – singlet quarks.

for gluons and increasing with decreasing x. We have also shown that the widths $\langle k_\perp^2 \rangle_i^{\rm evol}$ in all channels i increase at large Q^2 as $Q^2\alpha(Q^2)$.

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