Ideal convergence in topological spaces

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Joint results with Szymon Głąb and Pratulananda Das

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Ideal convergence

Definition

Let $\mathcal{I} \subseteq 2^\omega$ be an ideal and let (X, d) be a metric space. We say that a sequence $(x_n)_{n\in\omega}$ is $\mathcal{I}$-convergent to $x \in X$ provided that

$$\{n \in \omega : d(x_n, x) \geq \varepsilon\} \in \mathcal{I}.$$

When $\mathcal{I} = Fin$ then $\mathcal{I}$-convergence coincides with the normal convergence.

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$\mathcal{F}$-convergent subsequence

$\mathcal{I}$-ideal on ω, $Fin \subseteq \mathcal{I}$, X-topological space

Definition

Let $(x_n)_{n\in\omega}$ be a sequence in X. Let $\mathcal{F} \subseteq [\omega]^\omega$, we say that $(x_n)_{n\in\omega}$ has convergent $\mathcal{F}$-subsequence provided that there exists $A \in \mathcal{F}$ such that $(x_n)_{n\in A}$ is convergent.

Remark

We will be mostly interested in the case when

- $\mathcal{F} = [\omega]^\omega := \{A \subseteq \omega : |A| = \omega\}$ (subsequence)
- $\mathcal{F} = \mathcal{I}^+ := \{A \subseteq \omega : A \notin \mathcal{I}\}$ (not small subsequence)
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The $\mathcal{I}(\mathcal{F})$ property of X

$\mathcal{I}$-ideal on ω, $Fin \subseteq \mathcal{I}$, X-topological space, $\mathcal{F} \subseteq [\omega]^\omega$.

Definition

We say that X has the $\mathcal{I}(\mathcal{F})$ property, provided that any sequence $(x_n)_{n\in\omega} \subseteq X$ that is $\mathcal{I}$-convergent has a convergent $\mathcal{F}$-subsequence.

Well-known, Kostyrko-Šalát-Wilczyński

Let X be a non-discrete metric space.

- X has $\mathcal{I}(\mathcal{I}^*)$ property iff $\mathcal{I}$ is a P-ideal.
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Subspaces

Proposition

Let $M \subseteq X$. If X has the $\mathcal{I}(\mathcal{F})$ property, then M has $\mathcal{I}(\mathcal{F})$ property also.

Definition

We denote

$$add^*(\mathcal{I}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \forall I \in \mathcal{I}\ \exists A \in \mathcal{A}\ \ A \not\subseteq^* I\}.$$

Equivalently

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Products

Theorem

Let $\kappa < add^*(\mathcal{I})$ and let $\{X_\alpha : \alpha < \kappa\}$ be a family of topological spaces such that every X_α has the $\mathcal{I}(\mathcal{I}^*)$ property. Then $X = \prod_{\alpha<\kappa} X_\alpha$ has the $\mathcal{I}(\mathcal{I}^*)$ property.

Theorem

The space $\{0,1\}^\kappa$ has the $\mathcal{I}(\mathcal{I}^*)$ property if and only if $\kappa < add^*(\mathcal{I})$.

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Sketch of the proof

- Let $\mathcal{A} = \{A_\alpha : \alpha < \kappa\} \subseteq \mathcal{I}^*$ be any family of cardinality κ
- Define $(x_n)_{n\in\omega} \subseteq \{0,1\}^\kappa$ by

$$x_n(\alpha) = \begin{cases} 1, \text{ when } n \in A_\alpha \\ 0, \text{ when } n \notin A_\alpha. \end{cases}$$

- $(x_n)_{n\in\omega}$ is $\mathcal{I}$-convergent to $x \in \{0,1\}^\kappa$, where $x(\alpha) = 1$.
- By $\mathcal{I}(\mathcal{I}^*)$ property, there exists a set $A \in \mathcal{I}^*$ such that $(x_n)_{n\in A}$ is convergent.
- Hence, $A \subseteq^* A_\alpha$ for every $\alpha < \kappa$ and $add^*(\mathcal{I}) > \kappa$.

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Main results

Theorem

Let X be a topological space with the character $\chi(X) < add^*(\mathcal{I})$. Then X has the $\mathcal{I}(\mathcal{I}^*)$ property.

Definition

An ideal $\mathcal{I}$ is called tall provided that any infinite subset of ω contains an infinite set from $\mathcal{I}$.

Theorem

If $\mathcal{I}$ is not tall then X has $\mathcal{I}([\omega]^\omega)$.

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Example

Let $X = \mathbb{N} \cup \{\infty\}$ and $\tau_{\mathcal{I}}$ be a topology in X such that:

- it is discrete on $\mathbb{N}$
- $U \subseteq \mathbb{N} \cup \{\infty\}$ is an open neighborhood of ∞ whenever $\mathbb{N} \setminus U \in \mathcal{I}$.

Then X does not have the $\mathcal{I}([\omega]^\omega)$ property if and only if $\mathcal{I}$ is tall.

Remark

Let X be a topological space without $\mathcal{I}([\omega]^\omega)$. Then X contains a copy of $(\mathbb{N} \cup \{\infty\}, \tau_{\mathcal{I}})$.

Remark

The continuous image of a space that has $\mathcal{I}(\mathcal{F})$ property does not need to have $\mathcal{I}(\mathcal{F})$ property.

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Thank you for your attention :)