Szymon Głąb

Algebrability and strange functions

(with Artur Bartoszewicz)

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algebrability

Let A be a linear algebra. We say that $X \subset A$ is (strongly) κ-algebrable if there is (free) κ-generated subalgebra A' of A with $A' \subset X \cup \{0\}$.

classes of surjective functions

$f : \mathbb{R} \to \mathbb{R}$
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What is known

$\mathcal{J} \subset \mathcal{PES} \subset \mathcal{SES} \subset \mathcal{ES}$;
Real Jones functions $\mathcal{J}$ is $2^{\mathfrak{c}}$-lineable (Gámez-Merino, 2011);
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Question

Is $\mathcal{J}(\mathbb{C})$ strongly $2^{\mathfrak{c}}$-algebrable?

For every $n \in \mathbb{N}$ let $\mathcal{H}^n$ be a set of surjective functions $h : \mathbb{C}^n \to \mathbb{C}$ such that $|\mathcal{H}^n| \leq \mathfrak{c}$.

Theorem

There is a family $\{f_\xi : \xi < 2^{\mathfrak{c}}\} \subset \mathbb{C}^{\mathbb{C}}$ such that for any $n \in \mathbb{N}$, any $h \in \mathcal{H}^n$ and distinct ordinals $\xi_1 < \xi_2 < \cdots < \xi_n < 2^{\mathfrak{c}}$ we have $h(f_{\xi_1}, \ldots, f_{\xi_n}) \in \mathcal{J}(\mathbb{C})$.In particular, the family of complex Jones functions is strongly $2^{\mathfrak{c}}$-algebrable.

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Characterize when there exists a **closed** infinite dimensional algebra of functions with a particular "strange" property?

topologies on $\mathbb{C}^X$ and $\mathbb{R}^X$

τ_p – pointwise topology
τ_u – uniform topology

Proposition

Let $\mathcal{A}$ be a subalgebra of $\mathbb{C}^X$ or $\mathbb{R}^X$. Then for any $f \in \mathcal{A}$ the characteristic function of $\{x \in X : f(x) \neq 0\}$ is in $\mathrm{cl}_{\tau_p}(\mathcal{A})$.

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algebra $\mathcal{A}(f)$

Let $f \in \mathbb{R}^X$ (or $f \in \mathbb{C}^X$). Fix the partition $\{B_\xi : \xi < \kappa\}$ of X.

$$\mathcal{A}(f) = \{\bigcup_{\xi<\kappa} f_\xi : f_\xi \in \mathcal{A}_\xi\}$$

where $\mathcal{A}_\xi$ is a subalgebra of $\mathbb{R}^{B_\xi}$ or $\mathbb{C}^{B_\xi}$ generated by $f \restriction B_\xi$.

Theorem

Assume that $f \restriction B_\xi$ is unbounded for every $\xi < \kappa$. Then $\mathcal{A}(f)$ is τ_u-closed.

Corollary

There exists τ_u-closed algebra $\mathcal{A}$ of cardinality $2^{\mathfrak{c}}$ such that $\mathcal{A} \setminus \{0\}$ consists of complex perfectly everywhere surjective functions.

Let $\{B_\xi : \xi < \mathfrak{c}\}$ be a decomposition of $\mathbb{C}$ into $\mathfrak{c}$ many Bernstein sets. For any $\xi < \mathfrak{c}$ let $f_\xi : B_\xi \to \mathbb{C}$ a free generator such that algebra generated by f_ξ consists of perfectly everywhere surjective functions. Put $f = \bigcup_{\xi<\mathfrak{c}} f_\xi : \mathbb{C} \to \mathbb{C}$. Then $\mathcal{A}(f)$ is a desired algebra.

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where $\mathcal{A}_\xi$ is a subalgebra of $\mathbb{R}^{B_\xi}$ or $\mathbb{C}^{B_\xi}$ generated by $f \restriction B_\xi$.

Theorem

Assume that $f \restriction B_\xi$ is unbounded for every $\xi < \kappa$. Then $\mathcal{A}(f)$ is τ_u-closed.

Corollary

There exists τ_u-closed algebra $\mathcal{A}$ of cardinality $2^{\mathfrak{c}}$ such that $\mathcal{A} \setminus \{0\}$ consists of complex perfectly everywhere surjective functions.

Let $\{B_\xi : \xi < \mathfrak{c}\}$ be a decomposition of $\mathbb{C}$ into $\mathfrak{c}$ many Bernstein sets.For any $\xi < \mathfrak{c}$ let $f_\xi : B_\xi \to \mathbb{C}$ a free generator such that algebra generated by f_ξ consists of perfectly everywhere surjective functions.Put $f = \bigcup_{\xi<\mathfrak{c}} f_\xi : \mathbb{C} \to \mathbb{C}$.Then $\mathcal{A}(f)$ is a desired algebra.

everywhere discontinuous functions with finite range

By $\mathcal{EDF}$ denote the family of all functions $f : \mathbb{R} \to \mathbb{R}$ which are everywhere discontinuous and $f(\mathbb{R})$ is finite. It was proved (Bartoszewicz, Bienias & G., 2012) that $\mathcal{EDF}$ is $2^{\mathfrak{c}}$-algebrable(but not strongly 1-algebrable).

Theorem

Let $\mathcal{A} \subset \mathcal{EDF} \cup \{0\}$ be an algebra.Then $\mathcal{A}$ is τ_p-closed if and only if $\mathcal{A}$ is finitely generated.

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There is a linear algebra $\mathcal{A} \subset \mathbb{R}^{\mathbb{R}}$ of $2^{\mathfrak{c}}$ generators such that for any function $f \in \mathrm{cl}_{\tau_p}(\mathcal{A}) \setminus \{0\}$ there is open set U such that $f^{-1}(U)$ is a Bernstein set.

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A family $\{A_\alpha : \alpha < \kappa\}$ of subset of Y is called σ-**independent**, if $\bigcap_{\alpha \in X} A_\alpha^{\varepsilon(\alpha)} \neq \emptyset$ for every countable set $X \subset \kappa$ and every $\varepsilon : X \to \{0,1\}$ where $A^0 = A$ and $A^1 = Y \setminus A$.By the Tarski theorem there exists a σ-independent family on $\mathfrak{c}$ of cardinality $2^{\mathfrak{c}}$.

Proof.:

Let $\{B_\alpha : \alpha < \mathfrak{c}\}$ be a partition of $\mathbb{R}$ into $\mathfrak{c}$ many Bernstein sets.Let $\{A_\xi : \xi < 2^{\mathfrak{c}}\}$ be a σ-independent family on $\mathfrak{c}$.For any $\xi < 2^{\mathfrak{c}}$ put $C_\xi = \bigcup\{B_\alpha : \alpha \in A_\xi\}$.Let $\mathcal{B}$ be a σ-algebra generated by $\{C_\xi : \xi < 2^{\mathfrak{c}}\}$. Let $\mathcal{A}$ be an algebra generated by $\{\chi_{C_\xi} : \xi < 2^{\mathfrak{c}}\}$.$f \in \mathcal{A}$ is a simple function $f = \sum_{k=1}^{2^n} c_k \chi_{D_k}$ where D_k are Boolean combinations of $C_{\xi_1}, \ldots, C_{\xi_n}$.If $f \in \mathrm{cl}_{\tau_p}(\mathcal{A}) \setminus \{0\}$, then there are $f_n \in \mathcal{A}$ which tend pointwisely to f.Let $X \subset 2^{\mathfrak{c}}$ be the smallest set such that each f_n is measurable with respect to σ-algebra $\mathcal{B}_X$ generated by $\{C_\xi : \xi \in X\}$.X is countable.There is $\alpha < \mathfrak{c}$ which does not belong to any A_ξ, $\xi \in X$. Then $B_\alpha \subset \bigcap_{\xi \in X} \mathbb{R} \setminus C_\xi$.Thus $f_n \restriction B_\alpha = 0$ and $f \restriction B_\alpha = 0$.Since $f(x) \neq 0$ for some $x \in \mathbb{R}$, there is $\delta > 0$ such that $f^{-1}(f(x) - \delta, f(x) + \delta)$ is disjoint with $f^{-1}(0)$.But f is $\mathcal{B}_X$-measurable, $f^{-1}(f(x) - \delta, f(x) + \delta)$ contains a Bernstein set of the form $\bigcap_{\xi \in X} C_\xi^{\varepsilon(\xi)}$ for some $\varepsilon : X \to \{0,1\}$.A set which contains a Bernstein set and is disjoint with some other Bernstein set is also a Bernstein set.

Thank you for your attention!