
Summary of Working Group A Structure Functions and Low x Theory part

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Overview

- proton structure function: global fits [R. Thorne, Wu-Ki Tung](#)
- photon structure function: parametrisation [A. Zembruski](#)
- nuclear parton distributions [S.Kumano](#)
- lattice QCD results [A.Shindler](#)
- higher order calculations (NNLO splitting function) [A.Vogt](#)
- unintegrated parton distributions [W.Broniowski, A.Szczurek, H.Jung, W.Broniowski](#)
- low x
 - resummation at low x [G.Salam](#)
 - BFKL Pomeron at nonzero temperature [L.Lipatov](#)
 - perturbative-nonperturbative interface [S.Bondarenko](#)
 - studies of NLL BFKL [A.Sabio-Vera, C.Royon](#)
 - γ^*g impact factor, $\gamma^*\gamma^*$ scattering [A. Kyrieleis, M. Lublinsky](#)
 - higher twists, renormalons [A.Kotikov](#)

Overview (continued)

- dipole picture and saturation at low x
 - nonlinear evolution (Balitsky-Kovchegov equation and beyond) L.Lipatov, R.Peschanski, K.Golec-Biernat, E.Iancu, M.Kozlov
 - saturation in impact parameter space E.Naftali
 - odderon in dipole model Y.Kovchegov
 - instantons (*WGB Summary*) A.Utermann
 - shadowing in diffraction and DVCS (*WGB summary*) S.Munier, T.Rogers, L.Favart
 - hadronic collisions C.Marquet
 - shadowing in nuclei W.Schafer

In total: 31 theory talks

Lattice

Structure functions on lattice

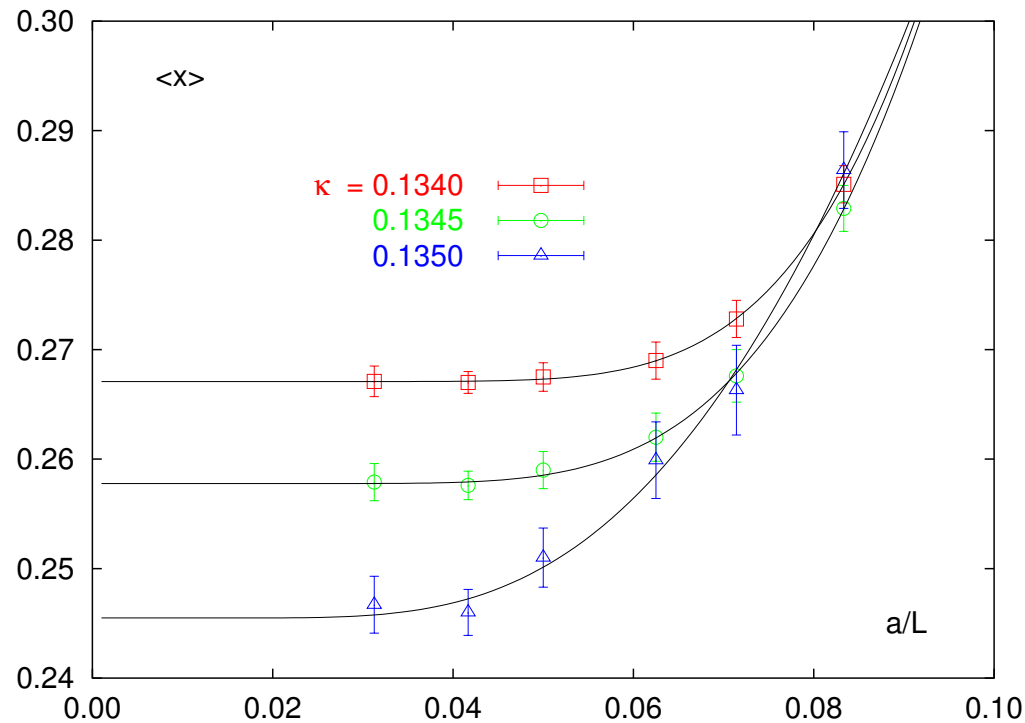
A.Shindler: Review of recent results on the moments of structure functions from lattice QCD.

Detailed study of systematic uncertainties:

- non-perturbative renormalisation
- continuum limit
- finite volume effects
- chiral limit (lattice formulation of the Dirac operator)
- quenching

Structure functions on lattice

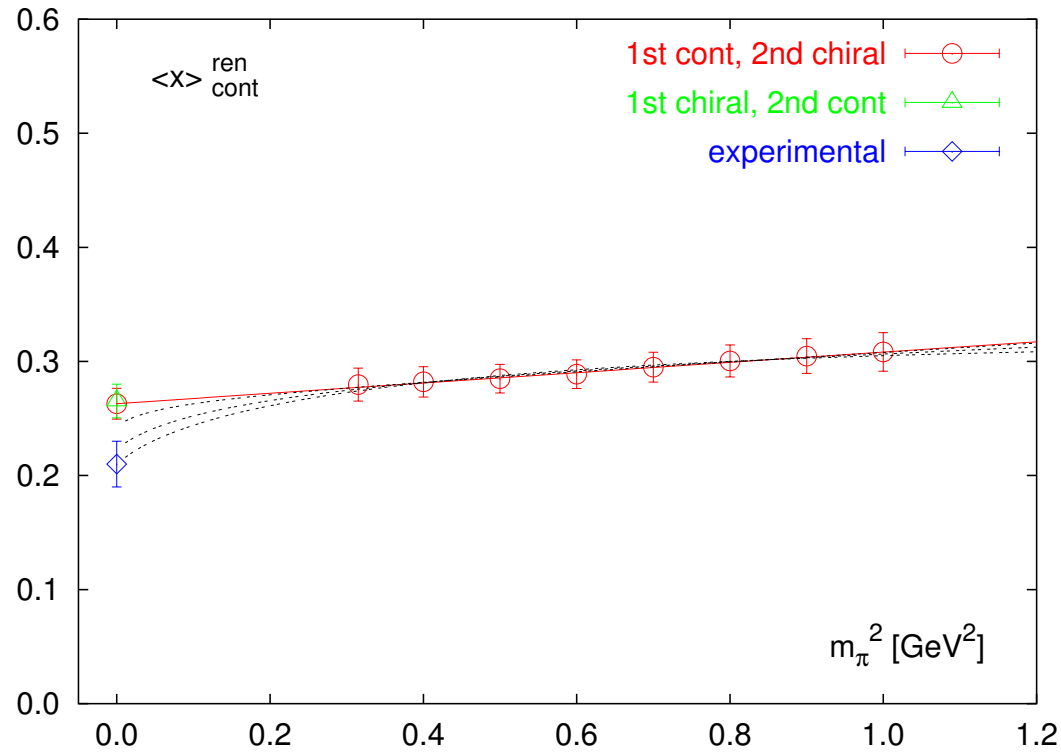
Pion matrix element of the twist 2 operator which corresponds to average momentum of the nonsinglet quark density: $\langle x \rangle_\pi$



Significant finite volume effects in this case. FVE are expected to be also very large in case of nucleon which might be a possible explanation for the current deviation of lattice calculations from experiment in case of $\langle x \rangle_N$.

Structure functions on lattice

Calculation of $\langle x \rangle_\pi$ using non-perturbative renormalisation



Chiral extrapolation is an important issue. Non-linear fit gives value close to experiment, however more studies are needed, especially for lower quark masses.

Splitting functions

Splitting function at NNLO

A.Vogt: Complete results for next-to-next-to-leading order (NNLO) non-singlet and singlet splitting functions.

Small x limit of nonsinglet functions:

$$P_{(x \rightarrow 0)}^i(x) = D_0^i \ln^4 x + D_1^i \ln^3 x + D_2^i \ln^2 x + D_3^i \ln x$$

where

$$D_0^+ \simeq 1.58, \quad D_1^+ \simeq 29.63 - 2.37n_f,$$
$$D_2^+ \simeq 295.04 - 32.20n_f + 0.59n_f^2, \quad D_3^+ \simeq 1261.11 - 152.60n_f + 4.35n_f^2$$

Strongly increasing coefficients of small x terms.

New, unpredicted small x term in P_{ns}^s :

$$\sim d^{abc} d_{abc} n_c \alpha_s^3 \ln^4 x$$

Splitting function at NNLO

Small x limit in singlet case:

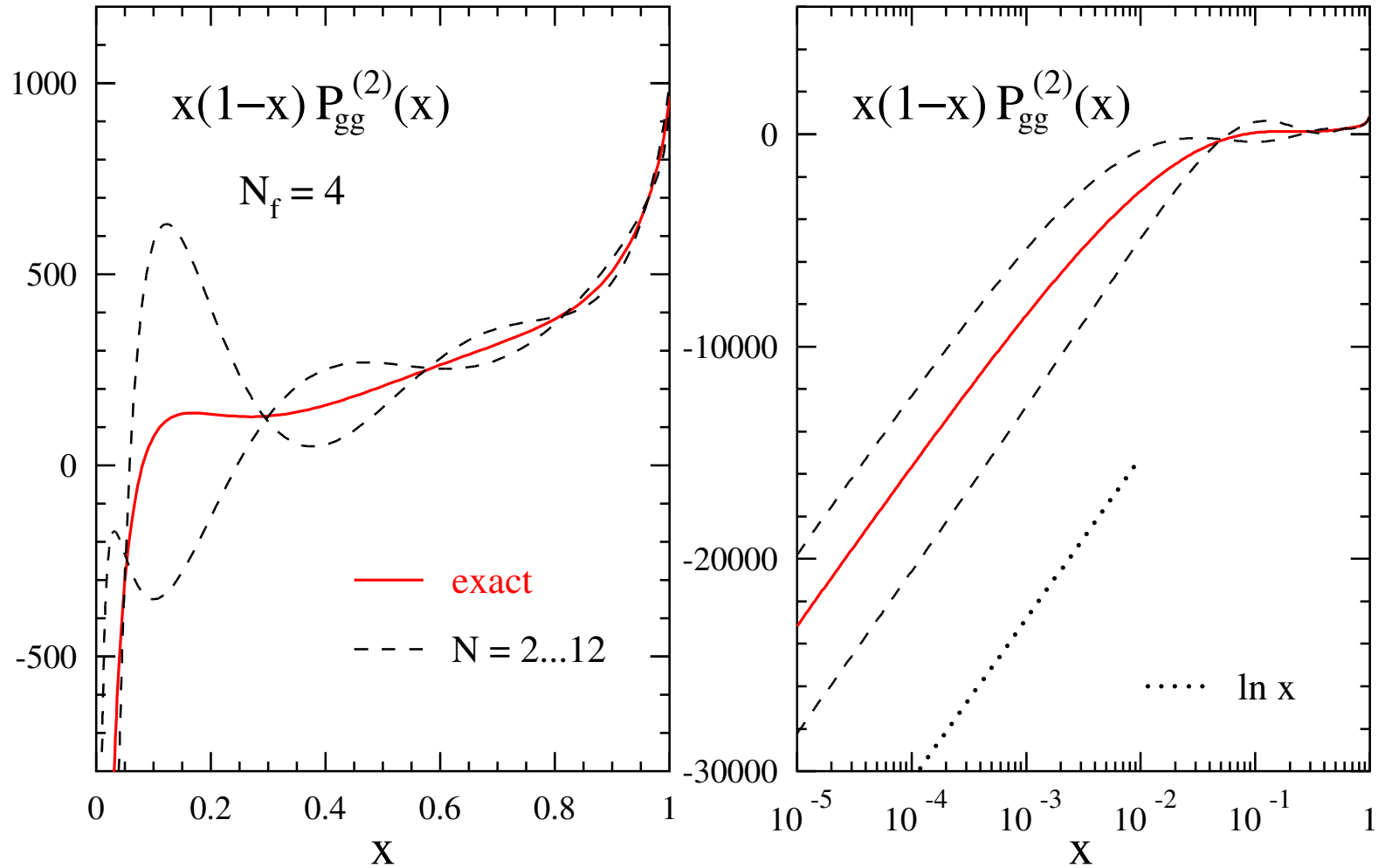
$$P_{ab}^{(2)} = E_1^{ab} \frac{\ln 1/x}{x} + E_2^{ab} \frac{1}{x} + \dots$$

Small x terms E_1^{gg}, E_1^{qg} consistent with previous calculations: NLLx BFKL and Catani-Hautmann.

Exact values of P_{ab} within the error bands of the previous estimates.

One should not expect big differences from previous NNLO MRST estimates.

Splitting function at NNLO



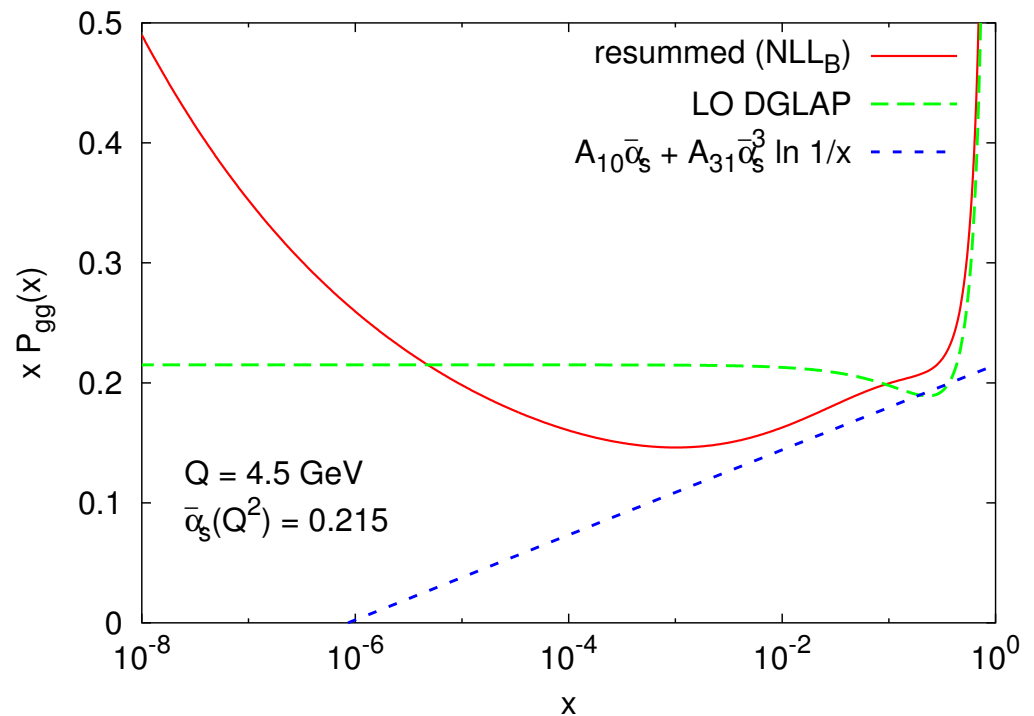
Note: P_{gg} is negative

Resummation at low x

G.Salam: Calculation of P_{gg} splitting function from the improved small x evolution equation with collinear logarithms.

Interesting feature: *DIP* present at $x \sim 10^{-3}$, instead of immediate increase at low x .

Observed in all low x resummation methods (see Altarelli et.al, Thorne).



Dip in P_{gg}

New classification of small x terms: $\bar{\alpha}_s \log^2 1/x \sim 1$.

	LLx	NLLx	NNLx	...
α_s	x	-	-	
α_s^2	0	n_f	-	
α_s^3	0	x	x	
α_s^4	x	x	x	$\ln^0 1/x$
α_s^5	0	x	x	$\ln 1/x$
\vdots				$\ln^2 1/x$ $\ln^3 1/x$

→ Leading for $\alpha_s \ln^2 1/x \sim 1$
→ Subleading for $\alpha_s \ln^2 1/x \sim 1$

$$xP_{gg}(x) \simeq -|A_{31}|\bar{\alpha}_s^3 \log \frac{1}{x} + |A_{43}|\bar{\alpha}_s^4 \log^3 \frac{1}{x} + \dots$$

Initial negative slope, starts at NNLO.

Poorly convergent expansion in $\sqrt{\bar{\alpha}_s}$, though NNLO quite stable, up to the position of the dip. Implications for HERA: **NNLO vs Resummation**.

Low x

BFKL Pomeron at $T > 0$

Collision of heavy nuclei \rightarrow formation of quark-gluon plasma

Parton-parton scattering leads to thermalisation of plasma

The confining potential between $q\bar{q}$ disappears, which leads to J/Ψ suppression.

A similar mechanism could occur for glueballs.

L. Lipatov: Study of influence of $T > 0$ on a Pomeron: cylinder topology

Results:

- resulting equation still has conformal symmetry, despite the presence of additional scale T
- energy dependence of the Pomeron the same as at $T = 0$
- extension to nonlinear equation
- different topologies \rightarrow connection with string dynamics

Saturation

Balitsky-Kovchegov non-linear equation

$$\frac{\partial N_Y(\underline{x}_0, \underline{x}_1)}{\partial Y} = \bar{\alpha}_s \int \frac{d^2 \underline{x}_2}{2\pi} \frac{(\underline{x}_0 - \underline{x}_1)^2}{(\underline{x}_0 - \underline{x}_2)^2 (\underline{x}_1 - \underline{x}_2)^2} [N_Y(\underline{x}_0, \underline{x}_2) + N_Y(\underline{x}_2, \underline{x}_1) - N_Y(\underline{x}_0, \underline{x}_1) - N_Y(\underline{x}_0, \underline{x}_2)N_Y(\underline{x}_2, \underline{x}_1)]$$

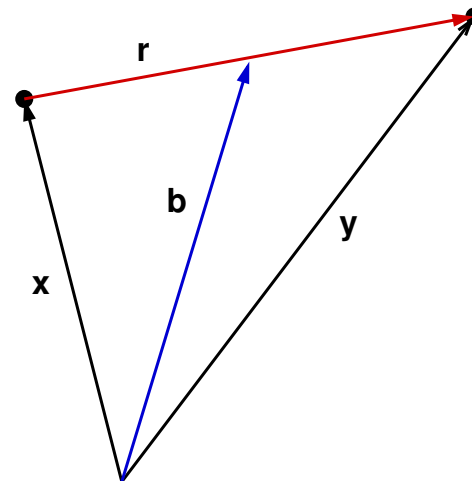
$N_Y(\underline{x}_0, \underline{x}_1)$ forward amplitude for scattering of the $q\bar{q}$ dipole on a nucleus target. **Linear** + **rescattering** term.

$$Y = \ln\left(\frac{1}{x}\right)$$

$$\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

$$\mathbf{r} = \underline{x}_0 - \underline{x}_1$$

$$\mathbf{b} = \frac{\underline{x}_0 + \underline{x}_1}{2}$$



E.Naftali: study of solution with impact parameter + confinement

BFKL formalism and CDP

L.Lipatov: Using **Möbius representation** one can derive:

- Original BFKL equation (in LLx) is completely equivalent to the Mueller-BFKL equation in the Color Dipole Picture.
- The equation with the Triple Pomeron vertex is equivalent to Balitsky-Kovchegov equation (in the large N_c)
- New equation for finite N_c

$$\frac{dN_{xy}}{dY} = \bar{\alpha}_s \int \frac{d^2 z}{2\pi} \frac{xy^2}{xz^2 yz^2} f_{xz,yz}$$

where

$$f_{xz,yz} = N_{xz} + N_{yz} - N_{xy} - G_{xz,yz}^{(4)} - \frac{1}{2(N_c^2 - 1)} (N_{xz} + N_{yz} - N_{xy})^2$$
$$G_{xz,yz}^{(4)} = N_{xz} N_{yz} + \Delta G^{(4)}$$

- powerful approach, possibly NLLx corrections can be implemented in the CDP

Travelling waves and Saturation scale

In the approximation of large nucleus $R \gg r$, impact parameter b in BK equation can be neglected leading to 1+1 dim. problem

$$\partial_Y N = \bar{\alpha}_s \chi(-\partial_L) N - \bar{\alpha}_s N^2$$

with χ BFKL eigenvalue.

R.Peschanski: BK equation falls into class of nonlinear equation with general properties:

- $N = 0$ is unstable fixed point w.r.t. linear evolution
- nonlinearity tames the growth when $N \sim \mathcal{O}(1)$
- the initial condition must be sufficiently steep at large k , ($N \sim 1/k^2$)

Relation with other fields in physics: **Disordered phenomena, spin glass phase transitions, polymer diffusion**

Travelling wave fronts

N is a travelling wave

$$N(\ln k^2 - \ln Q_s^2(Y))$$

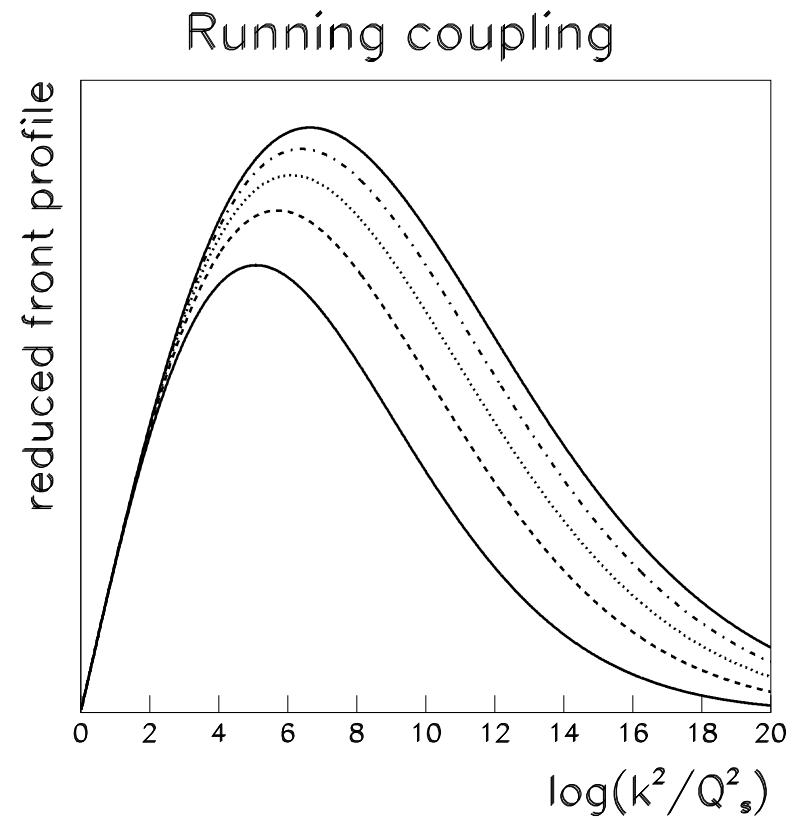
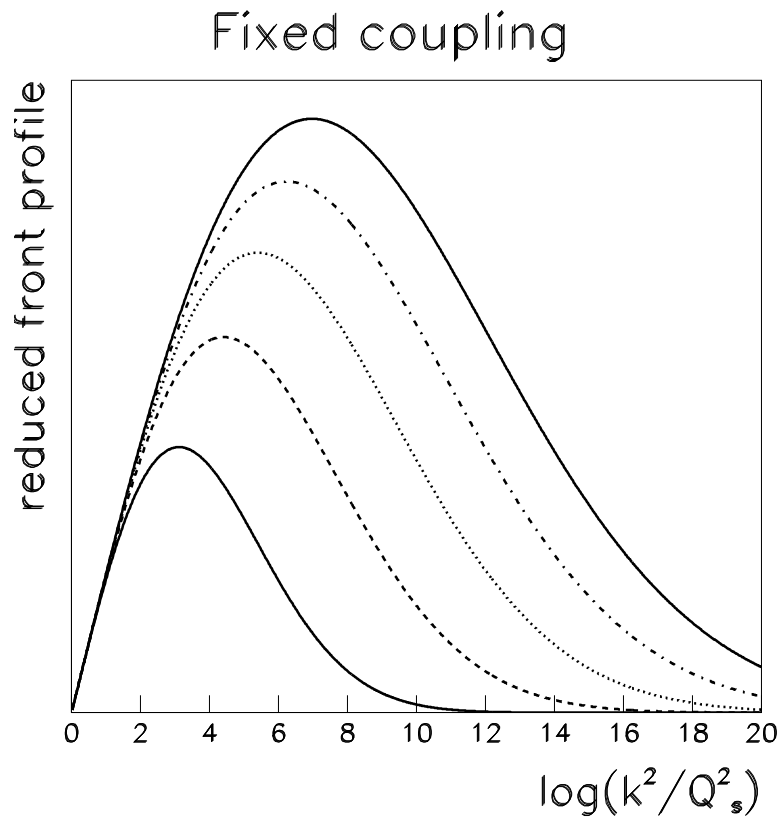
which corresponds to geometric scaling. A systematic expansion has been provided for $Q_s(Y)$

$$\ln Q_s^2(Y) = \bar{\alpha}_s \frac{\chi(\gamma_c)}{\gamma_c} Y - \frac{3}{2\gamma_c} \ln Y - \frac{3}{\gamma_c^2} \sqrt{\frac{2\pi}{\bar{\alpha}_s \chi''(\gamma_c)}} \frac{1}{\sqrt{Y}}$$

Solution N around the wave front

$$N(k, Y) \simeq \left(\frac{k^2}{Q_s^2(Y)} \right)^{-\gamma_c} e^{-\frac{\ln^2 k^2 / Q_s^2(Y)}{2\bar{\alpha}_s \chi''(\gamma_c) Y}} \times \{ \gamma_c \ln(k^2 / Q_s^2(Y)) + \dots \}$$

Travelling wave fronts

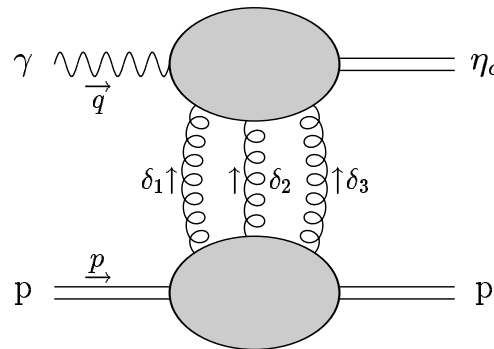


Rigorous results for $Q_s(Y)$ and N also in the running coupling case.

Numerical study: [K. Golec-Biernat](#)

Odderon: recap

- **Odderon**: partner of Pomeron but with $C = -1$
- It gives contribution to the difference between particle-particle and particle-antiparticle collisions cross sections.
- Test at HERA: an exclusive photo(electro)-production of η_c **pseudoscalar meson**.



- QCD evolution equation for the Odderon - **BKP equation**

Odderon in dipole picture

Y.Kovchegov: Odderon evolution equation in rapidity Y

$$\frac{\partial \mathcal{O}_Y(\underline{x}_0, \underline{x}_1)}{\partial Y} = \bar{\alpha}_s \int \frac{d^2 \underline{x}_2}{2\pi} \frac{(\underline{x}_0 - \underline{x}_1)^2}{(\underline{x}_0 - \underline{x}_1)^2 (\underline{x}_1 - \underline{x}_2)^2} \cdot [\mathcal{O}_Y(\underline{x}_0, \underline{x}_1) + \mathcal{O}_Y(\underline{x}_2, \underline{x}_1) - \mathcal{O}_Y(\underline{x}_0, \underline{x}_1)]$$

with **C-odd** initial conditions

$$\mathcal{O}_{Y=0}(\underline{x}_0, \underline{x}_1) = -\mathcal{O}_{Y=0}(\underline{x}_1, \underline{x}_0) = c_0 \alpha_s^3 \ln^3 \frac{x_0}{x_1}$$

$\underline{x}_0, \underline{x}_1$ are transverse coordinates of $q\bar{q}$ dipole pair. The evolution equation for Odderon is the same as for Pomeron, the difference being in **C-odd** or **C-even** initial conditions.

Odderon in dipole picture

The leading high energy intercept is then

$$\omega_{\text{Odd}} - 1 = \bar{\alpha}_s \chi(n=1, \nu=0) = 0$$

the same as Bartels-Lipatov and Vacca.

Including saturation: coming through the Pomeron amplitude N (which includes multiple scatterings)

$$\frac{\partial \mathcal{O}_Y(\underline{x}_0, \underline{x}_1)}{\partial Y} = K \otimes \mathcal{O}_Y(\underline{x}_0, \underline{x}_1) - \frac{\alpha_s N_c}{2\pi^2} \int d^2 \underline{x}_2 \frac{\underline{x}_{01}^2}{\underline{x}_{02}^2 \underline{x}_{12}^2} \cdot [\mathcal{O}_Y(\underline{x}_0, \underline{x}_2) N_Y(\underline{x}_2, \underline{x}_1) + N_Y(\underline{x}_0, \underline{x}_2) \mathcal{O}_Y(\underline{x}_2, \underline{x}_1)]$$

The solution not known, likely to be decreasing function on energy.

S - matrix in the high energy limit

Two different formalisms at high energy which include unitarity corrections:

Color Glass Condensate
(JIMWLK equation)

Color Dipole Picture

E.lancu: Two approaches are in fact equivalent.

This means that the full solution to **JIMWLK equation** should match the Monte Carlo simulations for the onium-onium scattering done by G.Salam.

Role of fluctuations

Monte Carlo simulations done by G.Salam show the crucial role of **fluctuations** in the onium wave function → **correlations between multiple scatterings**.

Full Monte Carlo:

$$S_Y \sim \exp(-\kappa \alpha_s^2 Y^2)$$

Incoherent scattering:

$$S_Y \sim \exp(-\kappa_0 \alpha_s^2 e^{Y\lambda})$$

E.lancu: S matrix is much larger when one takes the fluctuations into account.

Unitarity effects can be therefore significant and set in early, even when S is still quite large (or amplitude $N = 1 - S$ relatively small).

Saturation in hadronic collisions

C.Marquet: Extension of the GBW saturation model for study of Mueller-Navelet jets. Assuming k_T factorisation theorem:

$$\sigma = \int d^2r_1 d^2r_2 \phi^{(1)}(r_1, Q_1^2) \phi^{(2)}(r_1, Q_1^2) \sigma_{dd}(\Delta\eta, r_1, r_2)$$

with

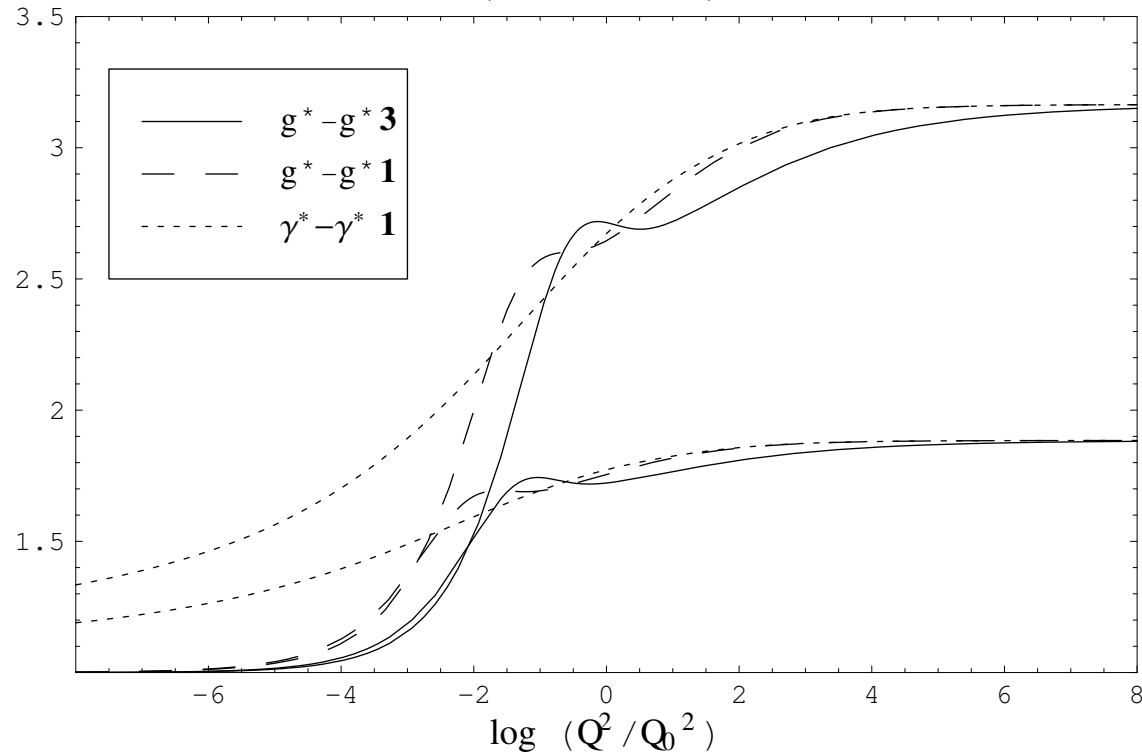
$$\sigma_{dd}(\Delta\eta, r_1, r_2) = \sigma_0 \left\{ 1 - \exp\left(-\frac{r_{eff}^2}{4R_0^2(\Delta\eta)}\right) \right\}$$

taken from GBW generalisation to $\gamma^*\gamma^*$ collisions (N.Timneau et.al). r_{eff}

is a combination of r_1 and r_2 .

Saturation in hadronic collisions

$R_{8/4}$ and $R_{4.6/2.4}$



Cross section ratios:

$$R_{i|j} = \frac{\sigma(Q_1, Q_2, \Delta\eta_i)}{\sigma(Q_1, Q_2, \Delta\eta_j)}$$

More abrupt transition to saturation than in the $\gamma^*\gamma^*$ case.

Some open questions: qqg state, validity of k_T factorisation

Saturation at HERA

BK equation \longrightarrow F_2 from dipole picture \longrightarrow HERA data

but

BK equation is not complete: running α_s ?, $NLLx$?, impact parameter dependence?, applicability at small Q^2 ?

Dipole picture: is it valid beyond LLx ?

Need to study $NLLx$ photon-gluon impact factor [A.Kyrieleis](#)

Usually description of HERA data with some model containing numbers of parameters which are not well under control.

F_2 data too inclusive, perhaps better to study more exclusive processes:

diffraction, DVCS ([S.Munier](#), [L.Favart](#), [T.Rogers](#))

Unintegrated parton distributions

Unintegrated parton distributions

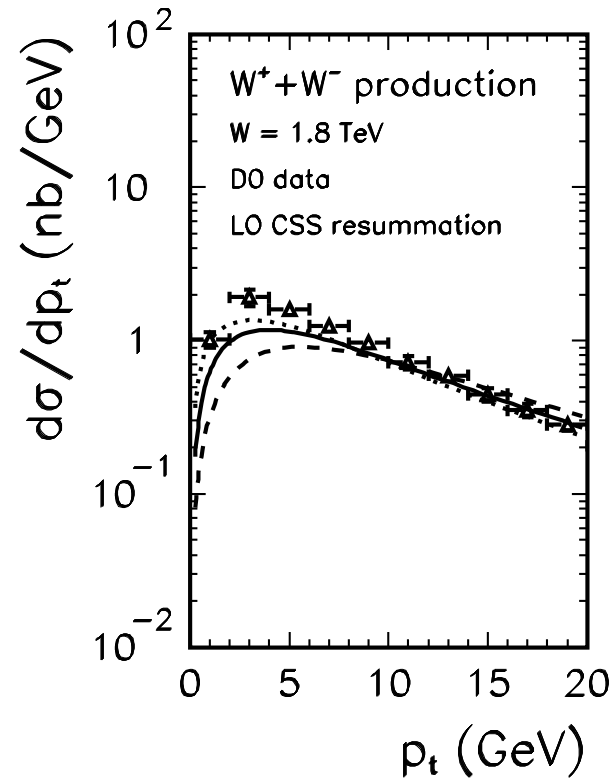
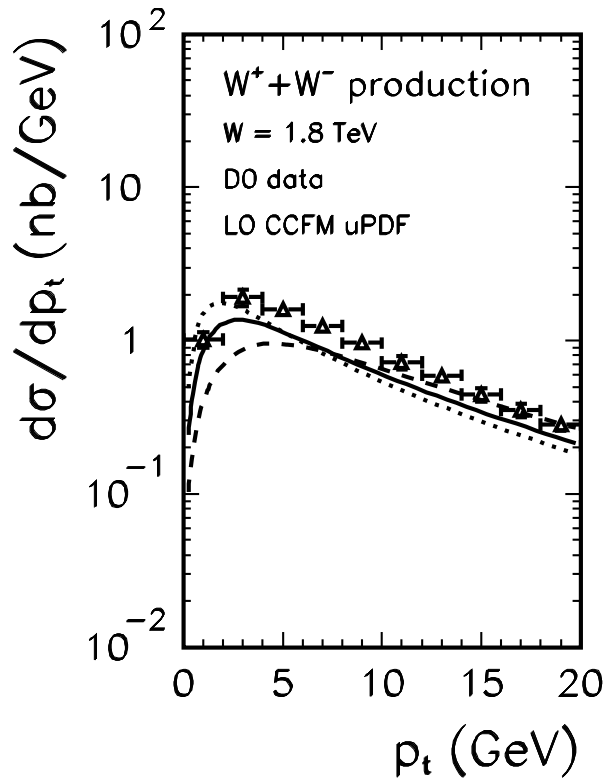
W. Broniowski, A. Szczurek, H. Jung, L. Lonnblad: unintegrated parton distributions from CCFM, one loop approximation to CCFM, Linked Dipole Chain model

Various applications at hadron colliders:

- heavy quark production
- p_T distribution of W^\pm, Z^0
- p_T distribution of the Higgs
- exclusive diffractive Higgs
- π^+, π^- spectra

p_T distribution of W at Tevatron

A.Szczurek



Soft resummation formulae (CSS) can be obtained from CCFM equation in the one-loop approximation (papers by J.Kwieciński et al)