Proton spin structure and intrinsic motion of constituents

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Introduction

- Presented results follow from QPM, in which (valence) quarks are considered as quasifree fermions on mass shell, with effective mass $x_0=m/M$. Momenta distributions describing intrinsic quark motion have spherical symmetry and constraint $J=1/2$ is applied. The model is constructed in consistently covariant way [for details see P.Z. *Phys.Rev.D65,054040(2002) and D67,014019(2003)*]. In this talk some properties of spin functions obtained in the model will be discussed:
- Sum rules for $g1$,$g2$
- $g1$,$g2$ from valence quarks, comparison with experimental data
- Discussion about Γ_1 and standard naïve QPM model
- Transversity

Model

Input:

set of $G_{k,\lambda}(p_0)$, which measure probability to find a fermion in state:

$$u(p,\lambda\mathbf{n}) = \frac{1}{\sqrt{N}}\begin{pmatrix} \phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}} \end{pmatrix}; \qquad \frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}}, \qquad \lambda = \pm\frac{1}{2},$$

where **n** coincides with the direction of target polarization **J**. Define:

$$H(p_0) = \sum_{k=1}^{3} e_k^2 \Delta G_k(p_0), \qquad \Delta G_k(p_0) \equiv G_{k,+1/2}(p_0) - G_{k,-1/2}(p_0).$$

Model

Output:

If one assume, $Q^2 \gg 4M^2x^2$ then:

$$g_1(x) = \frac{1}{2}\int H(p_0)\left(m + p_1 + \frac{p_1^2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}; \quad x = \frac{Q^2}{2M\nu},$$

$$g_2(x) = -\frac{1}{2}\int H(p_0)\left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0},$$

$$g_T(x) \equiv g_1(x) + g_2(x) = \frac{1}{2}\int H(p_0)\left(m + \frac{p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}.$$

Comments

- ...procedure complex, but unambiguous, task is well-defined.
- As a result there is a naïve QPM, improved not by QCD dynamic, but in **kinematics**: covariance + spheric symmetry constrained by *$J=1/2$*.
- We shall try to demonstrate, that it is also very important...

Sum rules

- **Basis:**

Obtained functions have general form

$$\int H(p_0) f(p_0, p_1, p_T) \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p,$$

which, due to spheric symmetry and δ –function, can be expressed as a combination of:

$$V_n(x) = \int H(p_0) \left(\frac{p_0}{M}\right)^n \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p.$$

One can prove, that these functions satisfy

$$\frac{V'_j(x)}{V'_k(x)} = \left(\frac{x}{2} + \frac{x_0^2}{2x}\right)^{j-k}; \qquad x_0 = \frac{m}{M},$$

Sum rules

...this gives possibility to obtain integral relations between $g_1(x)$ and $g_2(x)$:

$$g_2(x) = -\frac{x - x_0}{x} g_1(x) + \frac{x(x + 2x_0)}{(x + x_0)^2} \int_x^1 \frac{y^2 - x_0^2}{y^3} g_1(y) dy,$$

$$g_1(x) = -\frac{x}{x - x_0} g_2(x) - \frac{x + 2x_0}{x^2 - x_0^2} \int_x^1 g_2(y) dy.$$

In the limit $x_0 \to 0$:

$$g_2(x) = -g_1(x) + \int_x^1 \frac{g_1(y)}{y} dy,$$

$$g_1(x) = -g_2(x) - \frac{1}{x} \int_x^1 g_2(y) dy,$$

the first is the known expression for Wanzura - Wilczek twist-2 term for g_2 approximation.

Sum rules

$$V_n(x) = \int H(p_0)\left(\frac{p_0}{M}\right)^n \delta\left(\frac{p_0+p_1}{M} - x\right) d^3p.$$

Further, if we define

$$\langle x^\alpha \rangle = \int_0^1 x^\alpha V_0(x)dx,$$

then

$$\int_0^1 x^\alpha [g_1(x) + g_2(x)]dx = \langle x^\alpha \rangle \frac{\alpha+1}{(\alpha+2)(\alpha+3)},$$

$$\int_0^1 x^\alpha g_2(x)dx = -\langle x^\alpha \rangle \frac{\alpha(\alpha+1)}{(\alpha+2)(\alpha+3)}$$

for *any* α, for which the integrals exist. These relations imply

$$\int_0^1 x^\alpha \left[\frac{\alpha}{\alpha+1} g_1(x) + g_2(x)\right] dx = 0,$$

which for $\alpha = 2,4,6,\ldots$ corresponds to the Wanzura - Wilczek sum rules. Other special cases correspond to the Burkhardt - Cottingham ($\alpha = 0$) and the Efremov - Leader - Teryaev (ELT, $\alpha = 1$) sum rules.

Comment

- ... all these rules were here obtained from covariant kinematics and rotational symmetry, *$J=1/2$*.

spin

Valence quarks

We assume:

1) Spin contribution from the sea of quark-antiquark pairs and gluons can be neglected, so proton spin is generated only by the valence quarks.

2) *SU(6)* approach:

$$s_u = 4/3, \qquad s_d = -1/3.$$

Quark distributions are normalized as

$$\frac{1}{2}\int h_u(p_0)d^3p = \int h_d(p_0)d^3p = 1$$

and generic distribution reads

$$H(p_0) = \sum e_j^2 \Delta h_j(p_0) = \left(\frac{2}{3}\right)^2 \frac{2}{3} h_u(p_0) - \left(\frac{1}{3}\right)^2 \frac{1}{3} h_d(p_0).$$

Valence quarks

For unpolarized structure functions we have (P.Z. Phys.Rev. D **55**, 4290 (1997)):

$$F_2(x) = x^2 \int G(p_0)\frac{M}{p_0}\delta\left(\frac{p_0+p_1}{M}-x\right)d^3p; \qquad G(p_0) = \sum_q e_q^2 h_q(p_0),$$

If one use standard notation

$$F_{2val}(x) = \frac{4}{9}xu_V(x) + \frac{1}{9}xd_V(x),$$

then

$$q_V(x) = x\int h_q(p_0)\frac{M}{p_0}\delta\left(\frac{p_0+p_1}{M}-x\right)d^3p; \qquad q = u,d.$$

Valence quarks

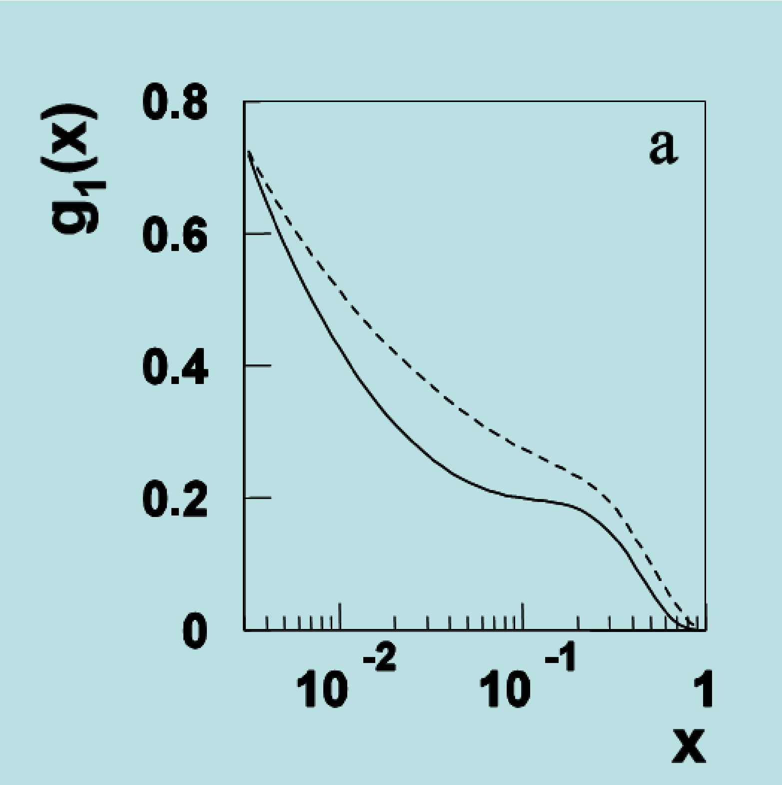
...from which, using technique of integral transforms, the functions g_1^q, g_2^q can be obtained:

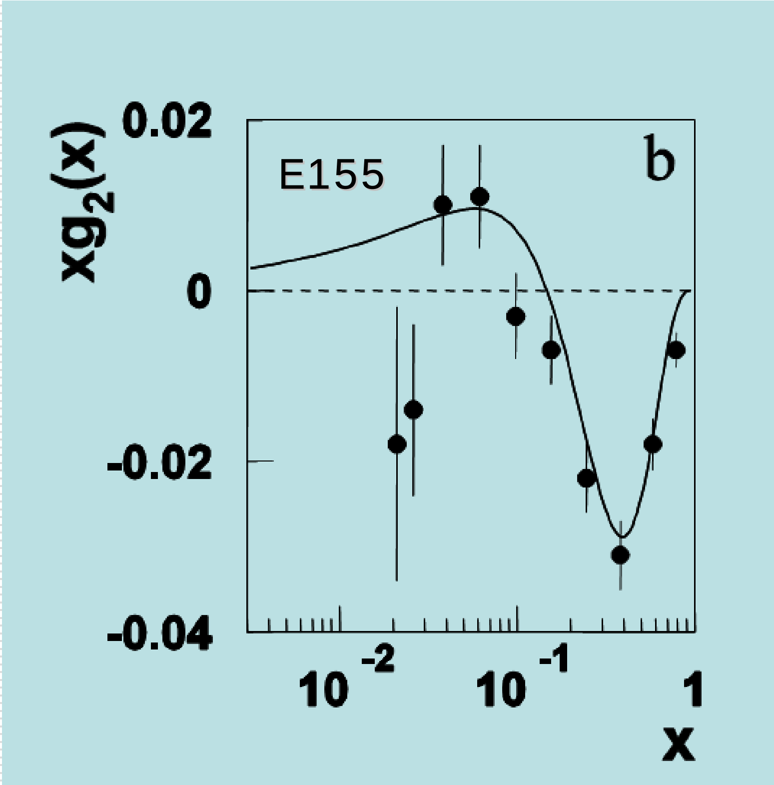
$$g_1^q(x) = \frac{1}{2}\left[q_V(x) - 2x^2\int_x^1 \frac{q_V(y)}{y^3}dy\right],$$

$$g_2^q(x) = \frac{1}{2}\left[-q_V(x) + 3x^2\int_x^1 \frac{q_V(y)}{y^3}dy\right],$$

Now, taking quark charges and corresponding $SU(6)$ factors, one can directly calculate g_1, g_2.

Valence quarks





Calculation - solid line, data - dashed line (left) and circles (right)

g_1 - analysis

- Integrating g_1 gives:

$$\Gamma_1 = \int g_1(x)dx = \frac{1}{2}\int H(p_0)\left(\frac{1}{3} + \frac{2m}{3p_0}\right)d^3p,$$

then, in $SU(6)$ as used above, one gets

$$\frac{5}{18} \geq \Gamma_1 \geq \frac{5}{54}$$

static quarks: $p_0 \to m$

massless quarks: $m \to 0$

- ...so, it seems: more motion=less spin?

How to understand it?

Lesson of QM

- Forget structure functions for a while and calculate another task.
- Remember, that angular momentum consists of ***j=l+s***.
- In relativistic case ***l,s*** are not conserved separately, only ***j*** is conserved. So, we can have pure states of ***j (j²,j_z)*** only, which are represented by relativistic spherical waves:

$$\psi_{jlj_z}(\mathbf{p}) = \frac{1}{\sqrt{2p_0}}\begin{pmatrix} i^{-l}\sqrt{p_0+m}\,\Omega_{jlj_z}(\frac{\mathbf{p}}{p}) \\ i^{-l'}\sqrt{p_0-m}\,\Omega_{jl'j_z}(\frac{\mathbf{p}}{p}) \end{pmatrix}; \qquad j = l \pm \frac{1}{2}, \qquad l' = 2j - l,$$

$$\Omega_{l+1/2,l,j_z}\left(\frac{\mathbf{p}}{p}\right) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}\,Y_{l,j_z-1/2} \\ \sqrt{\frac{j-j_z}{2j}}\,Y_{l,j_z+1/2} \end{pmatrix},$$

$$\Omega_{l'-1/2,l',j_z}\left(\frac{\mathbf{p}}{p}\right) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}\,Y_{l',j_z-1/2} \\ \sqrt{\frac{j+j_z+1}{2j+2}}\,Y_{l',j_z+1/2} \end{pmatrix}$$

Lesson of QM

$$j = j_z = \frac{1}{2}, \qquad l = 0 \qquad \Rightarrow \qquad l' = 1$$

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = i\sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{11} = -i\sqrt{\frac{3}{8\pi}}\sin\theta\exp(i\varphi)$$

$$\psi_{jlm}(\mathbf{p}) = \frac{1}{\sqrt{8\pi p_0}}\begin{pmatrix} \sqrt{p_0+m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -\sqrt{p_0-m}\begin{pmatrix} \cos\theta \\ \sin\theta\exp(i\varphi) \end{pmatrix} \end{pmatrix}$$

Lesson of QM

$$\Sigma_3 = \frac{1}{2}\begin{pmatrix} \sigma_3 & \cdot \\ \cdot & \sigma_3 \end{pmatrix} \Rightarrow$$

$$\psi^\dagger_{jlm}(\mathbf{p})\Sigma_3\psi_{jlm}(\mathbf{p}) = \frac{1}{16\pi p_0}[(p_0+m)+(p_0-m)(\cos^2\theta-\sin^2\theta)]$$

If a_p is the probability amplitude of the state ψ_{jlm}, then

$$\langle\Sigma_3\rangle = \int a_p^\star a_p \psi^\dagger_{jlm}(\mathbf{p})\Sigma_3\psi_{jlm}(\mathbf{p})d^3p = \frac{1}{2}\int a_p^\star a_p\left(\frac{1}{3}+\frac{2m}{3p_0}\right)p^2dp$$

$$\Gamma_1 = \frac{1}{2}\int H(p_0)\left(\frac{1}{3}+\frac{2m}{3p_0}\right)d^3p.$$

Spin and intrinsic motion

j=1/2

$m=p_0$

j=l+s

1≥‹s›/j≥1/3

QM:

For $p_0>m$ there
must be *some*
orbital momentum!

j=1/2

$m \approx 0$

Comparison with standard approach

Why the two approaches differ regarding the prediction Γ_1?

Usual QPM is closely connected with the preferred reference frame - IMF. The basic relations like

$$g_1(x) = \frac{1}{2}\sum e_j^2 \Delta q_j(x), \qquad F_2(x) = x\sum e_i^2 q_i(x)$$

are derived with the use of approximation

$$p_\alpha = xP_\alpha.$$

In the covariant formulation this relation is equivalent to the assumption, that the quarks are static. In present approach we do not use this approximation $\Rightarrow$ if $p_\alpha \neq xP_\alpha$ we obtain different relations between the distribution and structure functions.

Comment

Results suggest, that proton structure functions g_1 and g_2 can have a simple and natural interpretation even in terms of a naive QPM, provided that the model is based on a consistently covariant formulation, which takes into account intrinsic motion and spheric symmetry connected with the constraint $J=1/2$. This is not satisfied for standard formulation of QPM, which is based on simplified one-dimensional kinematics related only to the preferred reference system (infinite momentum frame). As a result, there is e.g. the known fact, that function g_2 has no well-defined meaning in the standard naive QPM. In this case it is just result of simplified kinematics and not because of absence of dynamics.

Summary

We studied spin functions in system of quasifree fermions having fixed effective mass $x_0 = m/M$ and total spin $J = 1/2$ - representing a covariant version of naive QPM.

Results:

1) Spin functions g_1 and g_2 depend on intrinsic motion. In particular, the momenta Γ_1 corresponding to the static (massive) fermions and massless fermions, can differ significantly:
$\Gamma_1(m \ll p_0)/\Gamma_1(p_0 \approx m) = 1/3$. It is due to splitting of angular momentum into spin and orbital part, as soon as intrinsic motion is present.

2) g_1 and g_2 are connected by a simple transformation, which is for $m \to 0$ identical to Wanzura - Wilczek relation for twist-2 term of the g_2 approximation. Relations for the $n-th$ momenta of the structure functions have been obtained, their particular cases are identical to known sum rules: Wanzura - Wilczek ($n = 2, 4, 6 \ldots$), Efremov - Leader - Teryaev ($n = 1$) and Burkhardt - Cottingham ($n = 0$).

3) Model has been applied to the proton spin structure, assuming proton spin is generated only by spins and orbital momenta of the valence quarks with *SU(6)* symmetry and for quark effective mass $m \to 0$. As an input we used known parameterization of the valence terms, then without any other free parameter, the functions $g_1,\ g_2$ were obtained. Comparison with the proton data demonstrates a good agreement.

4) Comparison with the corresponding relations for the structure functions following from naive QPM was done. Both the approaches are equivalent for the static quarks. Differences for quarks with internal motion inside the proton are result of the conflict with the assumption $p_\alpha = xP_\alpha$, which is crucial for derivation of the relations in the standard QPM.

Transversity

(preliminary, P.Z.+A.Efremov, O.Teryaev)

- First, remind our procedure for g_1, g_2 :

$$t_{\alpha\beta} = m\varepsilon_{\alpha\beta\lambda\sigma} q^\lambda w^\sigma, \qquad H(p_0) = G_+(p_0) - G_-(p_0)$$

$$\Rightarrow T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \frac{m}{2Pq} \int H\left(\frac{pP}{M}\right) w^\sigma \delta\left(\frac{p_0 + p_1}{M} - x\right) \frac{d^3 p}{p_0}$$

$$\Rightarrow g_1(x), g_2(x)$$

- Transversity may be related to auxiliary polarized process described by interference of axial vector and scalar currents. (*see G.R. Goldstein, R.L. Jaffe and X.D. Ji, Phys. Rev. D 52, 5006 (1995); B.L. Ioffe and A. Khodjamirian, Phys. Rev. D 51, 3373 (1995)*). We try to use simplest form of such vector, giving:

$$\tau_\alpha = \varepsilon_{\alpha\beta\lambda\sigma} p^\beta q^\lambda w^\sigma$$

$$\Rightarrow T_\alpha = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \frac{1}{2Pq} \int H\left(\frac{pP}{M}\right) p^\beta w^\sigma \delta\left(\frac{pq}{Pq} - x\right) \frac{d^3p}{p_0}$$

$$\Rightarrow \delta q(x) = \int H(p_0) \left(Mx - \frac{p_T^2/2}{p_0 + m}\right) \delta\left(\frac{p_0 + p_1}{M} - x\right) \frac{d^3p}{p_0}$$

Using technique of integral transforms gives:

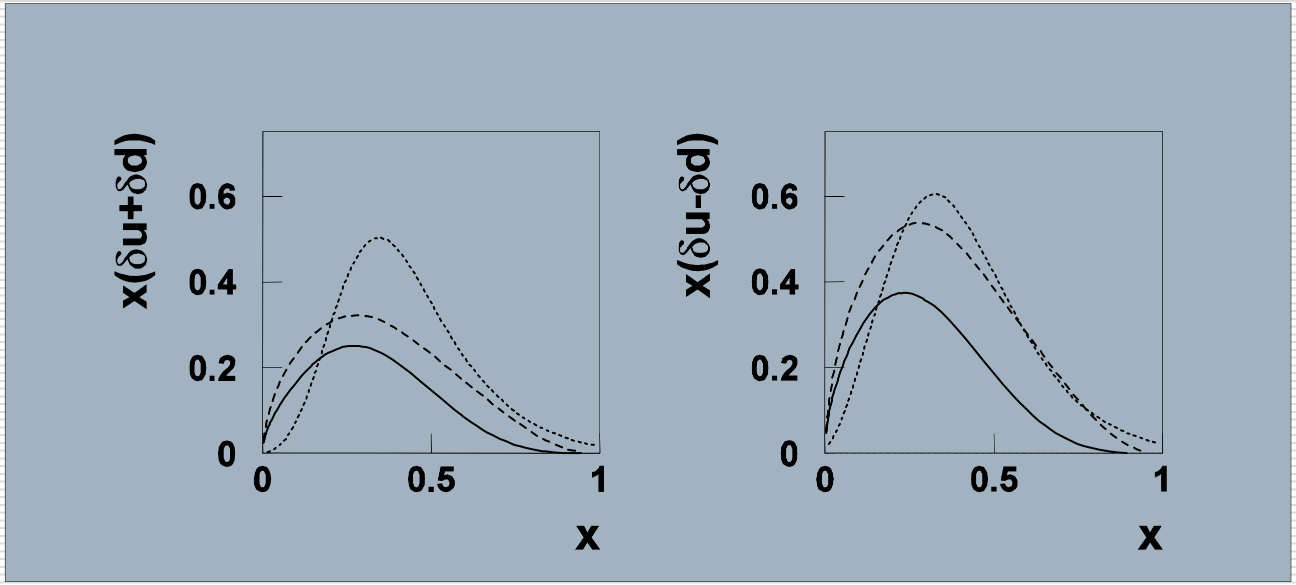
$$\delta q(x) = 2(g_1(x) + g_T(x))$$

$$\delta q(x) = 2\left(g_1(x) + \int_x^1 \frac{g_1(y)}{y} dy\right)$$

$$q_V(x) \to g_1(x) \to \delta q(x)$$

Calculation

- Dashed line – from g_1
- Full line – from q_v
- Dotted – calculation by *P.Schweitzer, D.Urbano, M.V.Polyakov, C.Weiss, P.V.Pobylitsa and K.Goeke, Phys.Rev. D 64, 034013 (2001).*



- But generally, obtained functions (in particular d-quarks) may not satisfy Soffer inequality. Why? One should consistently take into account interference nature of transversity...

$$f_1 \propto \sum_X \left(a^*_{++}(X)a_{++}(X) + a^*_{+-}(X)a_{+-}(X)\right)$$

$$g_1 \propto \sum_X \left(a^*_{++}(X)a_{++}(X) - a^*_{+-}(X)a_{+-}(X)\right)$$

$$\delta q \propto \sum_X \left(a^*_{++}(X)a_{--}(X) + a^*_{--}(X)a_{++}(X)\right)$$

$$\sum_X a^*_{++}(X)a_{++}(X) = G_+(p_0), \qquad \sum_X a^*_{+-}(X)a_{+-}(X) = G_-(p_0)$$

Transversity based on the expression...

$$G_T(p_0) \equiv \sum_X (a^*_{++}(X)a_{--}(X) + a^*_{--}(X)a_{++}(X))$$

$$|\delta q(x)| \leq \delta q_{\max}(x) \leq \frac{1}{2}(q(x) + \Delta q(x))$$

$$\delta q_{\max}(x) = \int G_+(p_0)\left(Mx - \frac{p_T^2/2}{p_0 + m}\right)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0}$$

$$q_V(x) \to \delta q_{\max}(x), g_1(x)$$

satisfies Soffer bound, in fact it satisfies a new, more strict limit...

Calculation

- Dashed line – Soffer bound
- Full line – δq_{max}
- Both limits are equivalent either for static quarks or for pure states with polarization +.