A Simple Model for Generalised Parton Distributions of the Pion

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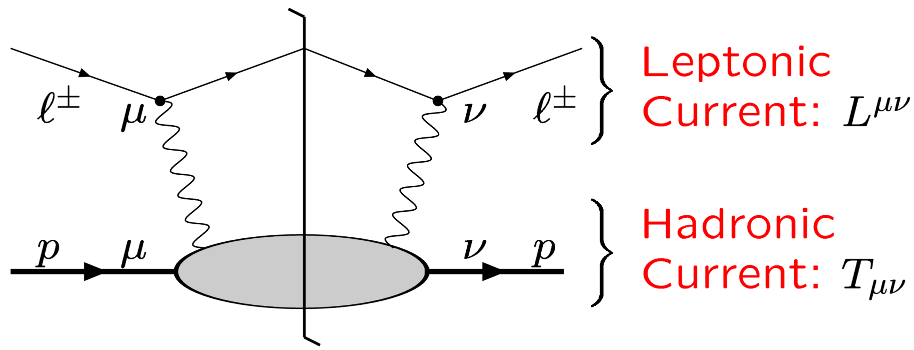
The Structure Functions

- For deeply inelastic electron-hadron scattering, we have in general

$$\mathbf{d}\boldsymbol{\sigma} \propto \boldsymbol{L^{\mu\nu} T_{\mu\nu}}, \tag{1}$$

with $L^{\mu\nu}$ and $T_{\mu\nu}$ the leptonic and hadronic current tensor.



- One can decompose $T_{\mu\nu}$, introducing the **Structure Functions** F_1 and F_2:

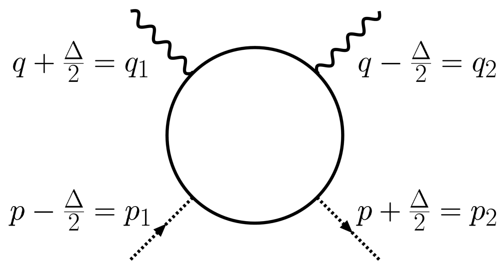
$$\boldsymbol{T_{\mu\nu}(p,q) = (-g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2}) \frac{F_1(Q^2,\nu)}{M} + [p_\mu + (\frac{p\cdot q}{Q^2}) q_\mu][p_\nu + (\frac{p\cdot q}{Q^2}) q_\nu] \frac{F_2(Q^2,\nu)}{M\nu}}, \tag{2}$$

where q ($q^2 = -Q^2$) is the momentum of the two photons.

- They give information about the Momentum carried by the quarks

The Off-Forward Case...

- If the momentum of the 2 photons is different, the decomposition involves more than 2 functions F_i: **the Off-Forward Structure Functions**
- They give information about the Correlations of the quarks
- They thus appear in the description of photon-hadron scatterings,



DIS Limit: -$\Delta = 0$
-only $\Im m T$

  - if $q_2^2 = 0$ (real photon), we have (Deeply) Virtual Compton Scattering.
  - Studied @ HERA (H1,Zeus,Hermes), @ JLab

- They also appear in $\gamma^\star\gamma^{(\star)} \to \pi\pi$ processes.

➪ Compared with the diagonal case, the amplitude is decomposed in five independent and fully general tensorial structures ($\rightarrow$ 5 F_i's) we have:

$$
\begin{aligned}
T_{\mu\nu}(q,p,\Delta) = -\mathcal{P}_{\mu\sigma} g^{\sigma\tau} \mathcal{P}_{\tau\nu} F_1 + \frac{\mathcal{P}_{\mu\sigma} p^\sigma p^\tau \mathcal{P}_{\tau\nu}}{p\cdot q} F_2 \\
+ \frac{\mathcal{P}_{\mu\sigma}(p^\sigma(\Delta^\tau - 2\xi p^\tau) + (\Delta^\sigma - 2\xi p^\sigma) p^\tau)\mathcal{P}_{\tau\nu}}{2p\cdot q} F_3 \\
+ \frac{\mathcal{P}_{\mu\sigma}(p^\sigma(\Delta^\tau - 2\xi p^\tau) - (\Delta^\sigma - 2\xi p^\sigma) p^\tau)\mathcal{P}_{\tau\nu}}{2p\cdot q} F_4 \\
+ \mathcal{P}_{\mu\sigma}(\Delta^\sigma - 2\xi p^\sigma)(\Delta^\tau - 2\xi p^\tau)\mathcal{P}_{\tau\nu} F_5 .
\end{aligned}
\tag{3}
$$

with

- the projector $\mathcal{P}_{\mu\nu} = g_{\mu\nu} - \frac{q_{2\mu} q_{1\nu}}{q_1\cdot q_2}$ that ensures current conservation ($\mathcal{P}_{\mu\nu} q_1^\mu = \mathcal{P}_{\mu\nu} q_2^\nu = 0$)
- $p = \frac{p_1+p_2}{2}$
- $q = \frac{q_1+q_2}{2}$
- $\Delta = q_1 - q_2$; $t = -\Delta^2$
- $\xi = \frac{\Delta\cdot q}{2p\cdot q}$

➪ **Twist analysis** ($\simeq \frac{1}{Q^n}$ expansion): Only 3 structures and thus 3 form factors needed : $\mathcal{H}$,$\mathcal{H}^3$ & $\tilde{\mathcal{H}}^3$.

$$T_{\mu\nu}(q,p,\Delta) = -\mathcal{P}_{\sigma\mu} g^{\sigma\tau} \mathcal{P}_{\nu\tau} \frac{q \cdot F_1^V}{2p\cdot q} + (\mathcal{P}_{\sigma\mu} p^\sigma \mathcal{P}_{\nu\rho} + \mathcal{P}_{\rho\mu} p^\sigma \mathcal{P}_{\nu\sigma}) \frac{F_2^{V\rho}}{p\cdot q} - \mathcal{P}_{\sigma\mu} i\epsilon^{\sigma\tau q\rho} \mathcal{P}_{\nu\tau} \frac{F_{1\,\rho}^A}{2p\cdot q}. \quad (4)$$

where the F_i^V's and F_1^A read

$$F_{1\,\rho}^V = 2p_\rho \mathcal{H} + (\Delta_\rho - 2\xi p_\rho)\mathcal{H}^3 + \text{tw4}, F_{1\,\rho}^A = \frac{i\epsilon_{\rho\Delta pq}}{p\cdot q}\tilde{\mathcal{H}}^3, F_{2\,\rho}^V = xF_{1\,\rho}^V - \frac{x}{2}\frac{p_\rho}{p\cdot q} q\cdot F_1^V + \frac{i}{4}\frac{\epsilon_{\rho\sigma\Delta q}}{p\cdot q} F_1^{A\sigma} + \text{tw4}.$$

➪ A straightforward identification of the two decompositions of $T_{\mu\nu}$ leads to simple relations between the five structure functions F_i's and the $\mathcal{H}$'s:

$$\begin{aligned}
F_1 &= \mathcal{H}, \\
F_2 &= 2x\mathcal{H} + \mathcal{O}(\frac{1}{Q^2}), \\
F_3 &= \frac{2x}{x^2-\xi^2}\left(x\mathcal{H}^3 x^2 + \tilde{\mathcal{H}}^3 \xi x - \mathcal{H}\xi\right) + \mathcal{O}(\frac{1}{Q^2}), \\
F_4 &= \frac{2x}{x^2-\xi^2}\left(\mathcal{H}^3 \xi x + \tilde{\mathcal{H}}^3 x^2 - \mathcal{H}x\right) + \mathcal{O}(\frac{1}{Q^2}), \\
F_5 &= \mathcal{O}(\frac{1}{Q^2}).
\end{aligned} \quad (5)$$

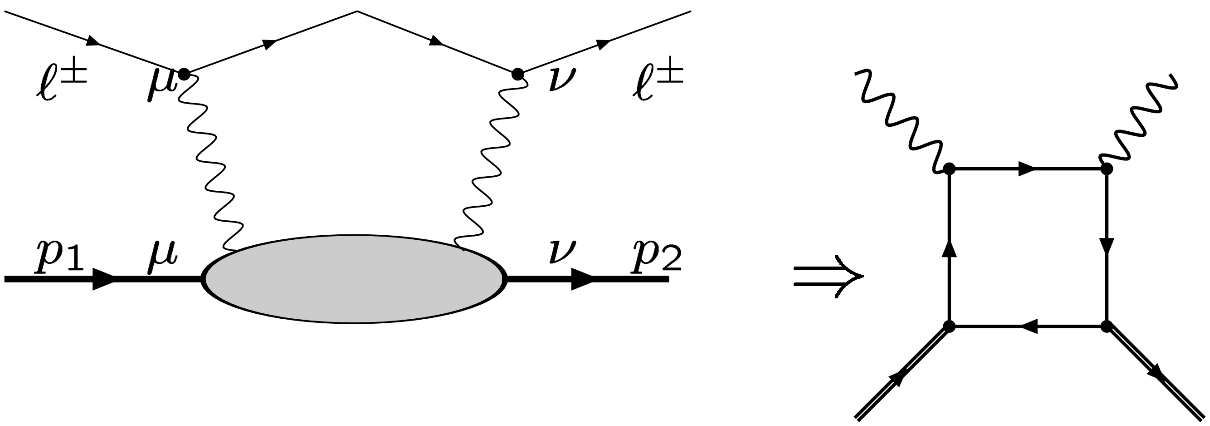
The simplest model for the pionic current: $T^{\mu\nu}$

[details in hep-ph/0207107, Phys.Lett.B547:210-218,2002]

- Pion-photon interaction analyzed at the quark level
- Using classical perturbative method (similar to NJL formalism):
  - Pion-quark coupling constant fixed to get the charge sum rule:

$$\int F_1(x)dx = \frac{5}{18} \tag{6}$$

- No gluons
- Schematically, we model the pionic current as follows



Link with GPD's

- One can introduce the **Generalised Parton Distributions H's** thanks to the $\mathcal{H}$'s:

$$\left\{ \begin{array}{c} \mathcal{H} \\ \widetilde{\mathcal{H}} \end{array} \right\}(x,\xi,t) = \sum_{i=u,d,\ldots} \int dy \left\{ \begin{array}{c} \frac{Q_i^2}{x(1-y/x-i\epsilon)} - (y \to -y) \\ \frac{Q_i^2}{x(1-y/x-i\epsilon)} + (y \to -y) \end{array} \right\}(x,y) \left\{ \begin{array}{c} H_i \\ \widetilde{H}_i \end{array} \right\}(y,\xi,t), \quad (7)$$

- As $\mathcal{H}(x,\xi) = \mathcal{H}(-x,\xi)$ for the π^0, it is easy to show that

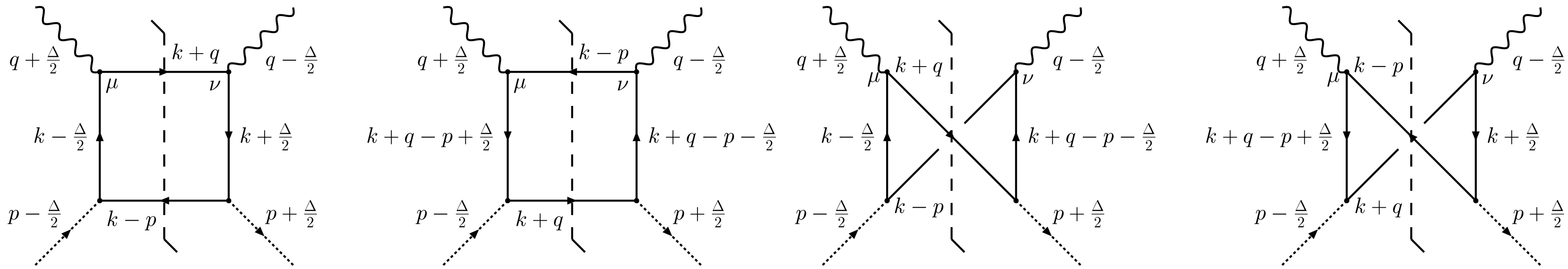
$$\Im m \mathcal{H} = H. \quad (8)$$

- Our calculation of the **imaginary part of** $T_{\mu\nu}$ gives $\Im m \mathcal{H}$**'s**

- Our calculation of $\Im m T_{\mu\nu}$ is sufficient to determine the GPD's (in the entire (x,ξ) plane).

Coming back to our model. . .

- At leading order, we have 4 diagrams contributing to the imaginary part of $T^{\mu\nu}$:



- The amplitude for the first **box diagram** is

$$T_a^{\mu\nu} = 3g^2(e_u^2+e_d^2)\int \mathrm{d}^4k \,\mathrm{Tr}\Big(\gamma^5((\not k-\not p)+m_q)\gamma^5\frac{\not k+\frac{\not\Delta}{2}+m_q}{(k+\frac{\Delta}{2})^2-m_q^2}\gamma^\nu((\not k+\not q)+m_q)\gamma^\mu\frac{\not k-\frac{\not\Delta}{2}+m_q}{(k-\frac{\Delta}{2})^2-m_q^2}\Big).$$

- g: Pion-quark coupling constant (fixed to satisfy the charge Sum Rule Eq. (6))

- Pseudo-scalar property of the pion $\Rightarrow \gamma^5$

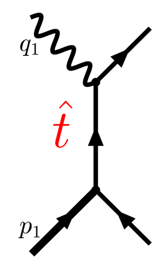
The pion size and the cut-off

- Finite size of the pion

  $\Rightarrow$ relative momentum p_{rel} of the quarks is **limited**.

- Simplest way $\longrightarrow$ cut-off

What about Gauge invariance ?



- $\boldsymbol{p^2_{rel} \propto \hat{t}}$, where $\hat{t}$ is the momentum transfer of $\gamma^* \pi \longrightarrow q\bar{q}$

  $\Rightarrow$ a cut-off on $\hat{t} \equiv$ a cut-off on p_{rel}.

- $\hat{t}$, $\mathcal{M}(\hat{t})$ and $\int_{\Omega(\Lambda)} \mathcal{M}(\hat{t}) d\hat{t}$ are potentially observable

  (since we consider only the imaginary part)

  $\Rightarrow$ these are all three gauge-invariant quantities.

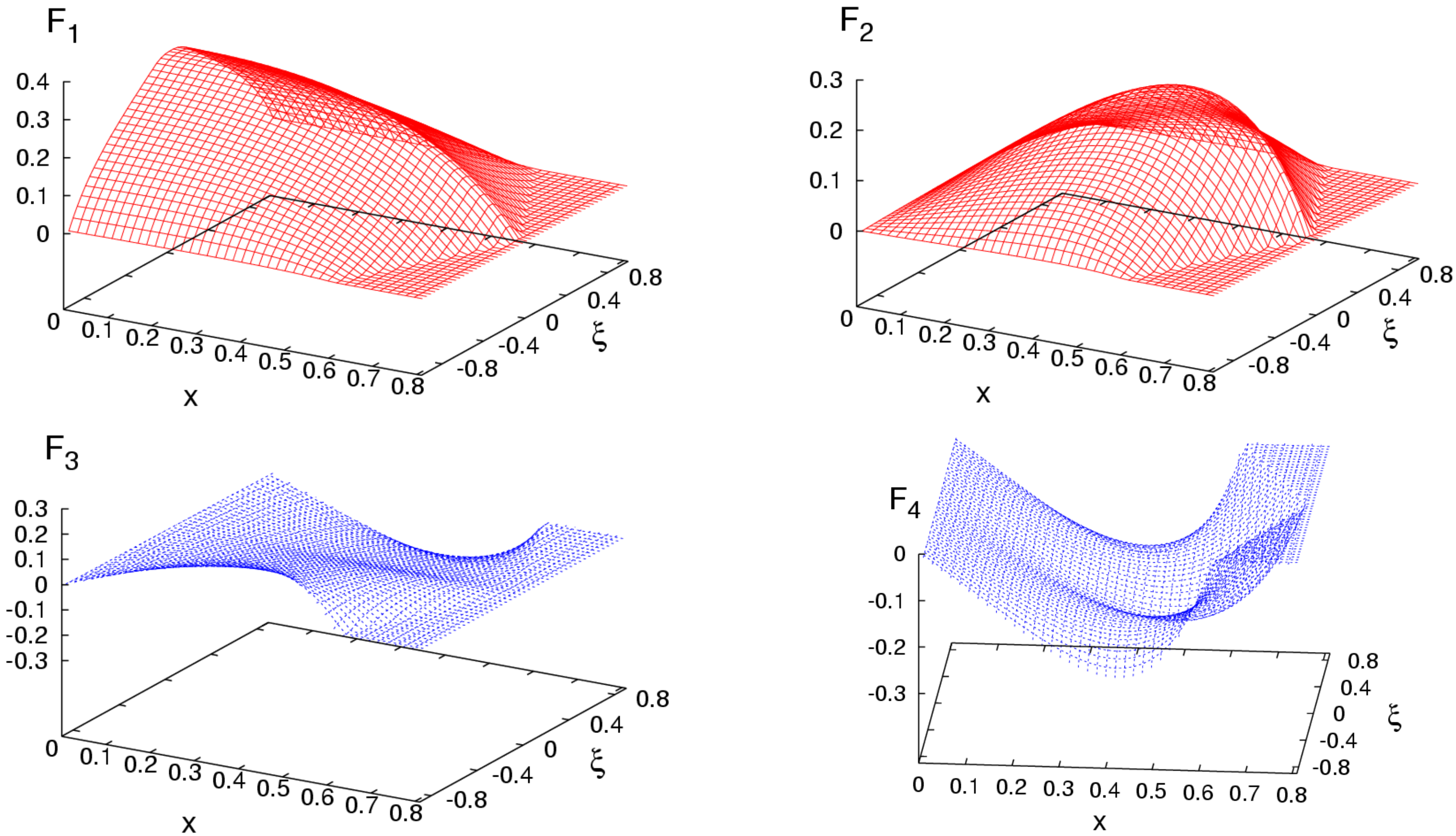
- In other words, **the cut-off procedure is gauge invariant !**

Results

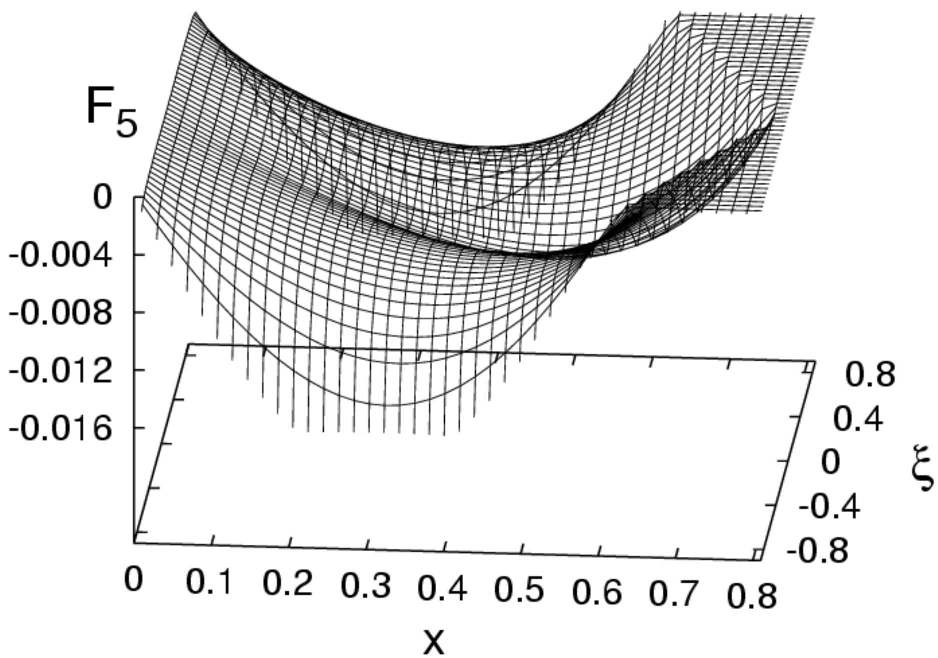
[details in hep-ph/0310184, to appear in Phys.Lett.B]

Plots

$\Lambda = 0.75$ GeV; $Q^2 = 10$ GeV^2; $m_\pi = 0$; $m_q = 300$ MeV



Plots (Cont.)



High-Q^2 behaviour

- Diagonal case: $2\langle x_B \rangle = 0.6$: Violation of the momentum sum rule

- Expanding the ratios of $\frac{F_2}{F_1}$,$\frac{F_3}{F_1}$, $\frac{F_4}{F_1}$,$\frac{F_5}{F_1}$, we get

$$F_2 = 2xF_1 + \mathcal{O}(1/Q^2) \quad F_3 = \frac{2x\xi}{\xi^2-1}F_1 + \mathcal{O}(1/Q^2)$$
$$F_4 = \frac{2x}{\xi^2-1}F_1 + \mathcal{O}(1/Q^2) \quad F_5 = \mathcal{O}(1/Q^2)$$

  - First relation: similar to the Callan-Gross relation for the diagonal structure functions $(x \leftrightarrow x_B)$ (spin one-half constituents in general).
  - This shows that F_2, F_3 and F_4 are simply related to F_1 at leading order.
  - Confirmation of the symmetries of the F_i's, since we have

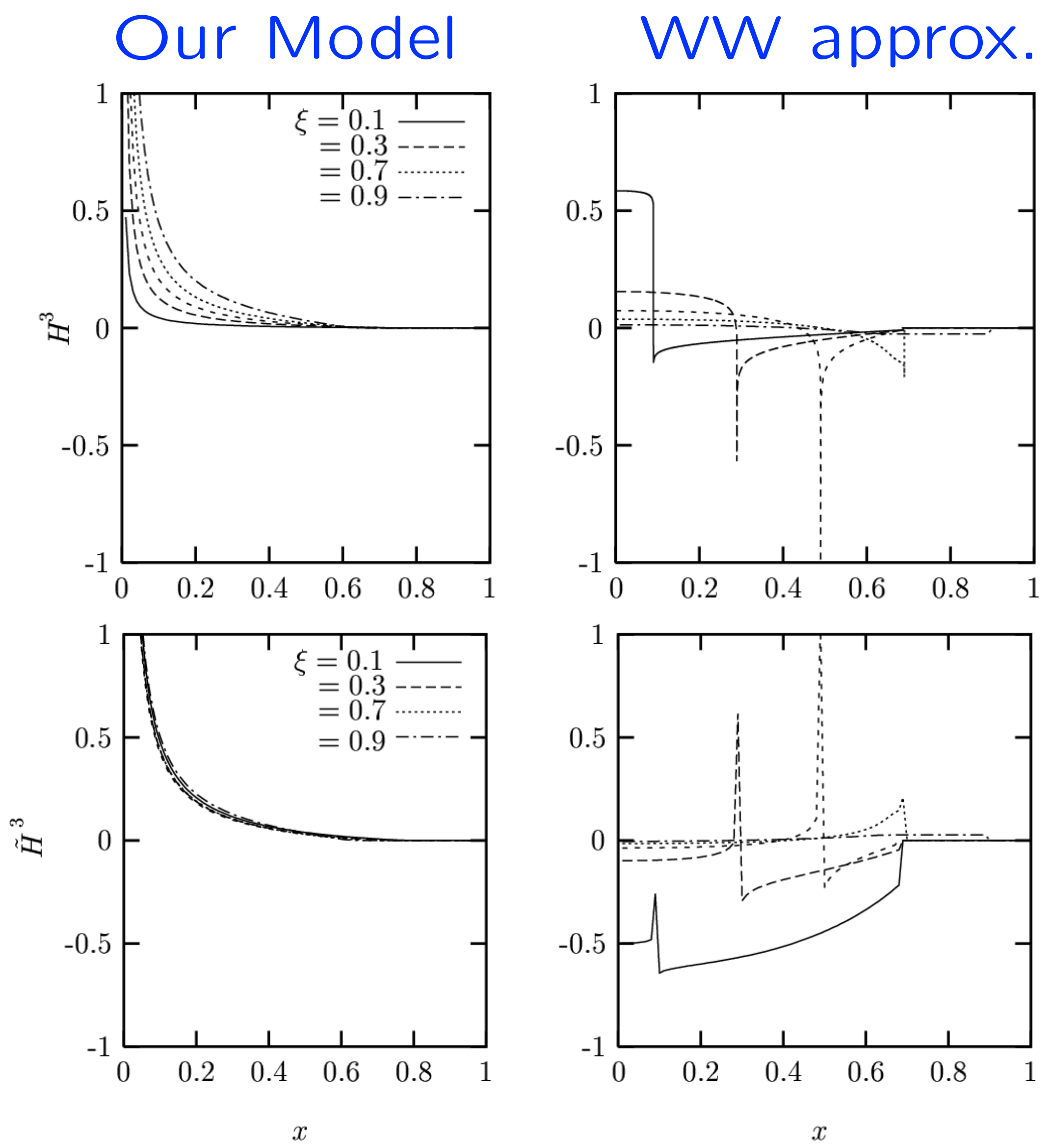
$$F_3 = \xi F_4.$$

➪ In terms of the H's, we get

$$\tilde{H}^3 = \frac{H(x-1)}{x(\xi^2-1)} + \mathcal{O}(\frac{1}{Q^2})$$
$$H^3 = \frac{H(x-1)\xi}{x(\xi^2-1)} + \mathcal{O}(\frac{1}{Q^2}) = \xi\widetilde{H}^3 + \mathcal{O}(\frac{1}{Q^2}).$$

Relation to the

Wandzura-Wilczek approximation ?


Our Model
WW approx.
$\xi = 0.1$
$= 0.3$
$= 0.7$
$= 0.9$
H^3
$\tilde{H}^3$
x
x

Conclusion

- Simplest model for **Off-forward structure functions**

- **Included physics:**
  - Relativistic kinematics
  - Pion size (via cut-off)
  - Gauge invariance

- **Results:**
  - F_2, F_3 and F_4 are simply related to F_1 at leading order, as well as H^3 and $\widetilde{H}^3$ to H
  - New relations between twist-2 and twist-3 contributions are inferred !