AAC analysis of polarized parton distributions with uncertainties

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Refs. **(1) Y. Goto et. al., Phys. Rev. D62 (2000) 034017.**
(2) M. Hirai, SK, N. Saito, Phys. Rev. D69 (2004) 054021.

April 15, 2004

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• proton-spin issue

polarized e/μ-proton scattering

→ **measurement of g_1** $\quad g_1^{LO} = \frac{1}{2}\sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i)$

proton, deuteron, ^{3}He g_1 data

with isospin symmetry

→ **valence and sea polarization** $\quad \Delta u_v, \Delta d_v, \Delta \bar{q}$

quark spin content $\quad \Delta\Sigma = \Delta u_v + \Delta d_v + 6 \cdot \Delta \bar{q}$

experimentally $\quad \int_0^1 dx \Delta\Sigma(x) \approx 0.1 - 0.3$

rest of the spin ???

Papers on the polarized PDFs

D. de Florian, L. N. Epele, H. Fanchiotti, C. A. Garcia Canal, and R. Sassot,
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C. Bourrely, F. Buccella, O. Pisanti, P. Santorelli, and J. Soffer,
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AAC (Y.Goto et al., M. Hirai et al.), Phys. Rev. D62 (2000) 034017; D69 (2004) 054021.

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J. Blümlein and H. Böttcher, Nucl. Phys. B636 (2002) 225.

Initial distributions

$$\Delta f_i(x,Q_0^2) = A_i\, x^{\alpha_i} (1 + \gamma_i\, x^{\lambda_i})\, f_i(x,Q_0^2)$$

$i = u_v, d_v, \bar{q}, g$ $\quad A_i\,,\ \alpha_i\,,\ \gamma_i\,,\ \lambda_i$: parameters

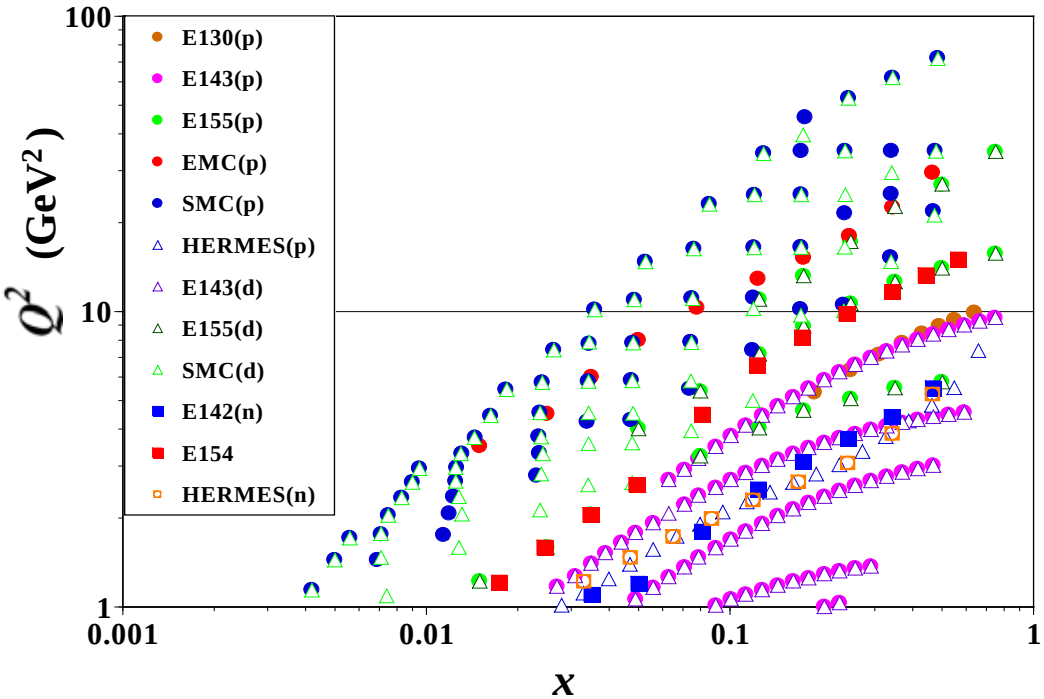
χ^2 fit to the data [p, n (^{3}He), d] $\quad \chi^2 = \sum_i \dfrac{(A_{1\,i}^{data} - A_{1\,i}^{calc})^2}{(\sigma_{A_{1\,i}}^{data})^2}$

$$A_1 \simeq \frac{g_1}{F_1} = g_1 \frac{2x(1+R)}{F_2} \qquad R = \frac{F_L}{2xF_1} = \frac{F_2 - 2xF_1}{2xF_1}$$

We analyzed with the following conditions.

- **unpolarized PDF** **GRV98**
- **initial Q^2** $\mathbf{Q_0^2 = 1\ GeV^2}$
- **number of flavor** $\mathbf{N_f = 3}$
- **positivity** $\mathbf{|\Delta f(x)| \le f(x)}$ **(to be precise,** $\mathbf{|\Delta\sigma| \le \sigma}$**)**
- **antiquark flavor:** $\mathbf{\Delta\bar{u} = \Delta\bar{d} = \Delta\bar{s}}$

Experimental data on spin asymmetry $A_1(x,Q^2)$



Total data 399 points
(Q^2>1GeV^2)

- **A_1 data**
  - Proton : E130, E143, EMC, SMC, HERMES, E155
  - Deuteron: E143, E155, SMC
  - Neutron : E142, E154, HERMES, JLab

Error estimation

Hessian method

$\chi^2(\xi)$ is expanded around its minimum ξ_0 (ξ =parameter)

$$\chi^2(\xi_0+\delta\xi)=\chi^2(\xi_0)+\sum_i\frac{\partial\chi^2(\xi_0)}{\partial\xi_i}\delta\xi_i+\frac{1}{2}\sum_{i,j}\frac{\partial^2\chi^2(\xi_0)}{\partial\xi_i\partial\xi_j}\delta\xi_i\delta\xi_j+\cdots$$

where the Hessian matrix is defined by $H_{ij}=\frac{1}{2}\frac{\partial^2\chi^2(a_0)}{\partial\xi_i\partial\xi_j}$

In the χ^2 analysis, 1σ standard error is

$$\Delta\chi^2=\chi^2(\xi_0+\delta\xi)-\chi^2(\xi_0)=\sum_{i,j}\delta\xi_i H_{ij}\delta\xi_j$$

$P(s)_N$: $\chi^2(=s)$ distribution with N degrees of freedom

$$\int_0^{\Delta\chi^2} ds\, P(s)_{N=11}=0.6826\rightarrow\Delta\chi^2=12.64 \quad (N=1 \text{ case},\ \Delta\chi^2=1)$$

The error of a distribution $F(x)$ is given by

$$\left[\delta F(x)\right]^2=\Delta\chi^2\sum_{i,j}\frac{\partial F(x)}{\partial\xi_i}H_{ij}^{-1}\frac{\partial F(x)}{\partial\xi_j}$$

Results

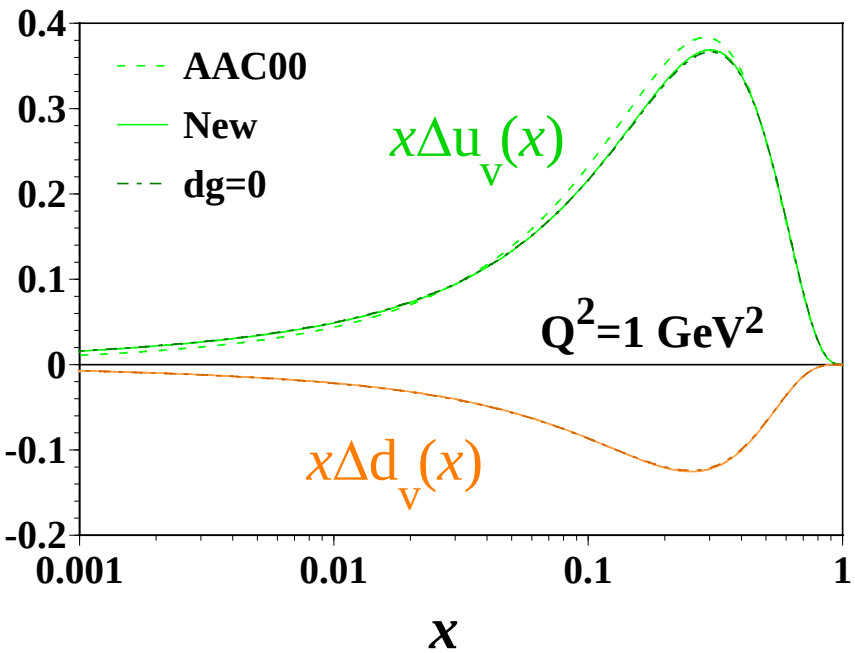
- Total χ^2

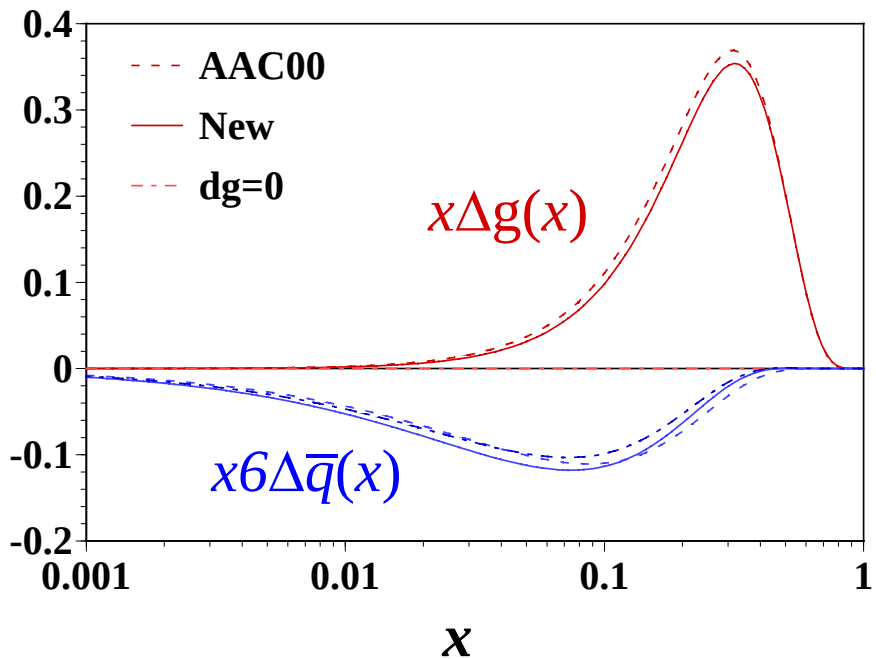
  χ^2/d.o.f. = 0.893

Comparison of AAC03 with AAC00

AAC00: Y.Goto et al., Phys. Rev. D62 (2000) 034017

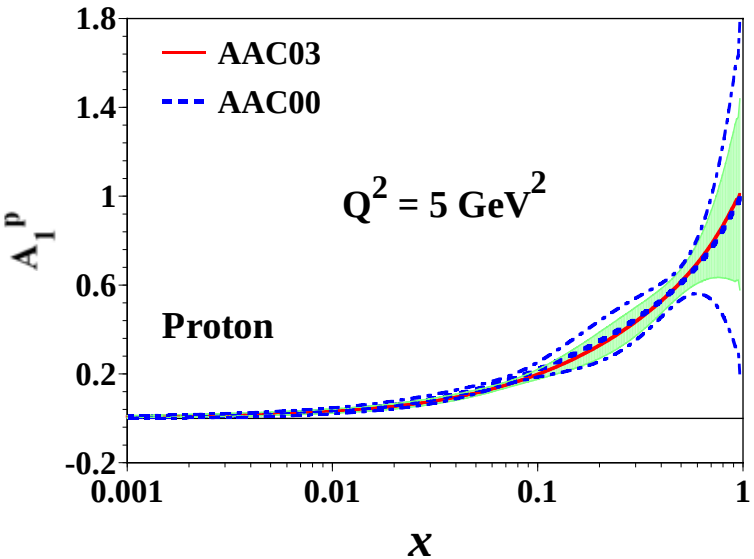
AAC03: M.Hirai et al., Phys. Rev. D69 (2004) 054021

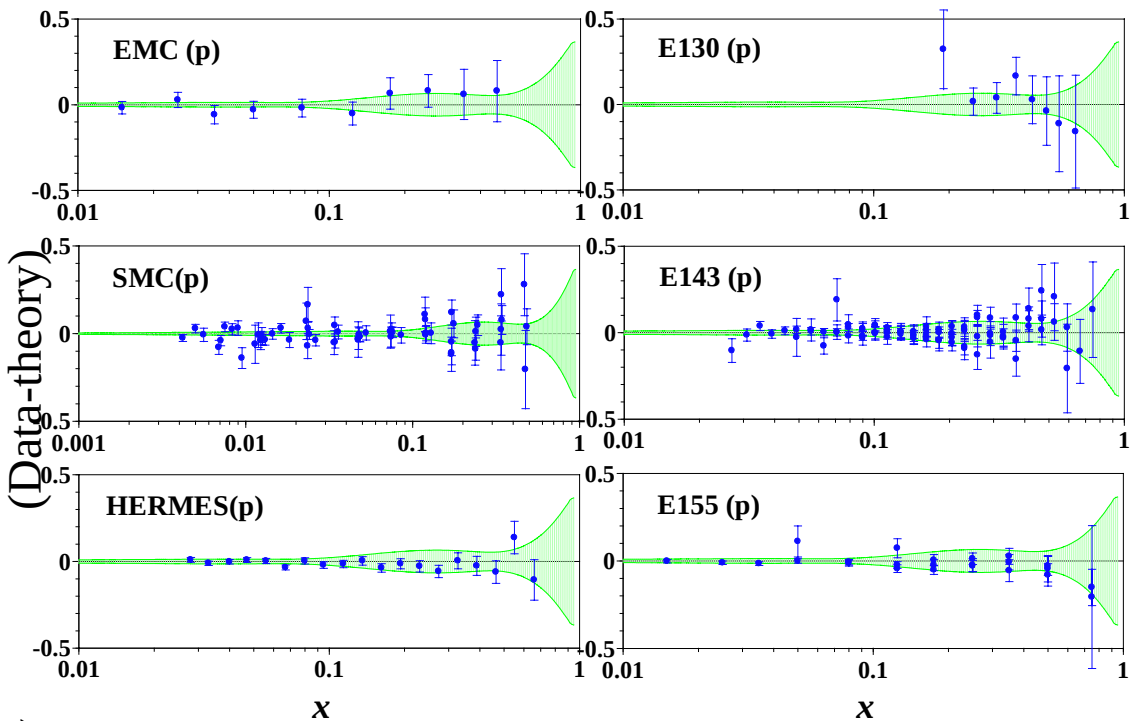




Spin asymmetry A_1^p

- Precise data (E155-proton) reduce asymmetry uncertainties
- Agreement with polarized DIS data
  - Quark and antiquark distributions are constrained
  - Gluon distribution need other constraint





AAC03 uncertainties

AAC00 uncertainties

AAC00 ↔ AAC03 (with E155-p)

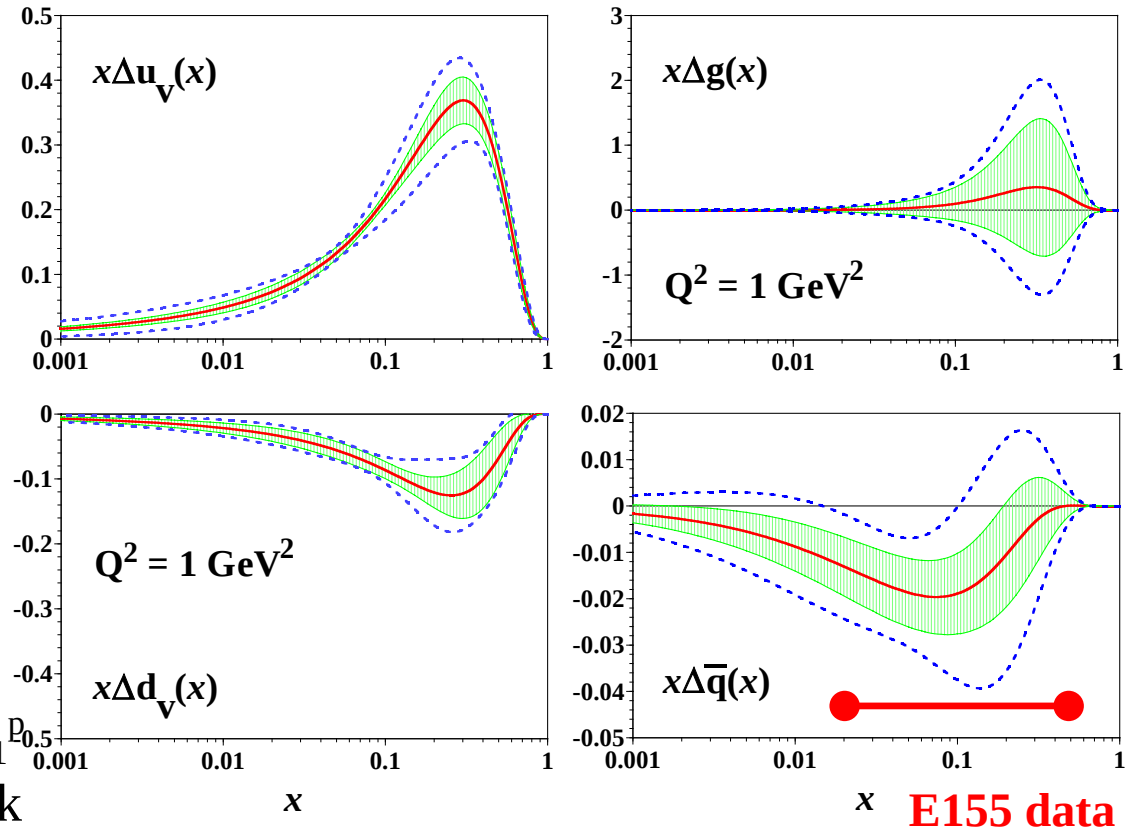
Polarized PDFs (AAC03)

- PDF uncertainties reduced by including precise (E155-p) data
- Valence-quark distributions are well determined
  - Small uncertainty of Δu_v, Δd_v
- Antiquark uncertainty is significantly reduced
  - $g_1^p \propto 4\Delta u_v + \Delta d_v + 12\Delta\bar{q}$
- $\Delta g(x)$ is not determined
  - Large uncertainty
  - Indirect contribution to g_1^p
  - Correlation with antiquark

AAC03 uncertainties

AAC00 uncertainties

AAC00 ↔ AAC03 (with E155-p)



E155 data

Correlation between $\Delta\bar{q}(x)$ and $\Delta g(x)$

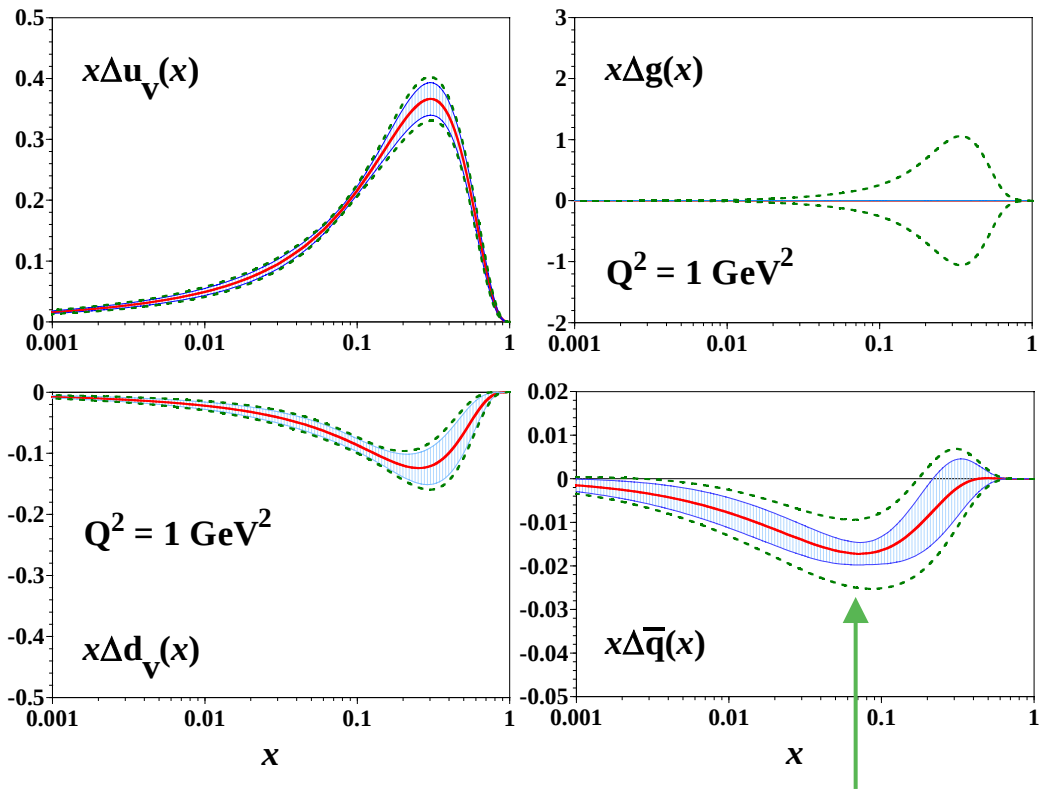
- **analysis with $\Delta g(x)=0$ at $Q^2=1$ GeV^2**
  - χ^2/d.o.f. = 0.915

- $\Delta q_v(x)$ uncertainties are not affected

- antiquark uncertainties are reduced
  - strong correlation with $\Delta g(x)$
  - correlation with $\Delta g(x)$ is almost terminated in the $\Delta g=0$ analysis

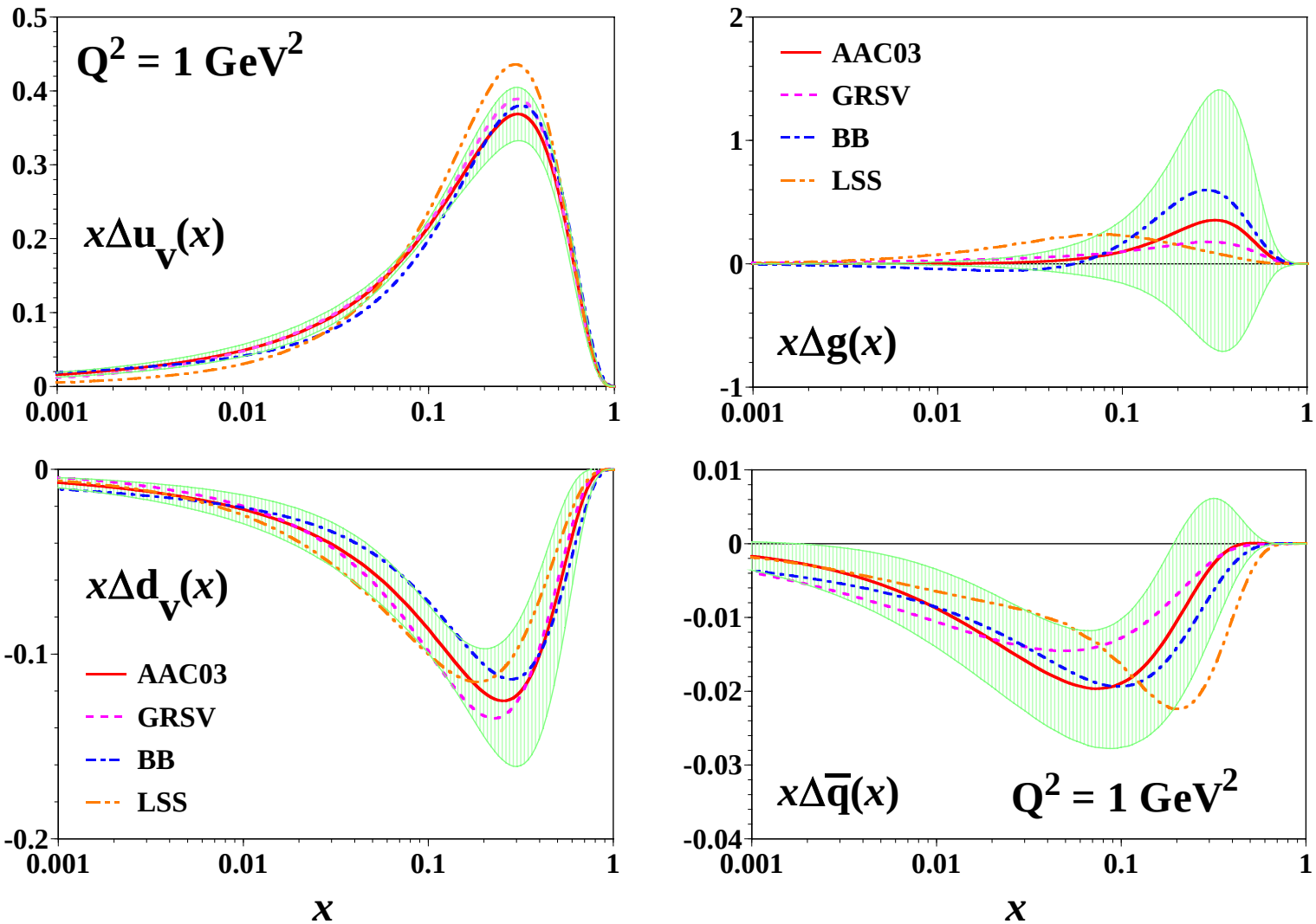
$\Delta g=0$ uncertainties

AAC03 uncertainties



The error band shrinks due to the correlation with $\Delta g(x)$.

Comparison with other parameterizations



1st moments $Q^2 = 1\,\mathrm{GeV}^2$

$\left(\Delta\Sigma = \Delta u_v + \Delta d_v + 6\Delta\bar{q}\right)$

	Δg	$\Delta\Sigma$
AAC03	0.499 ± 1.268	0.213 ± 0.138
GRSV01	0.420	0.204
LSS	0.680	0.210
BB	1.026	0.138

- GRSV01 [Phys. Rev. D63 (2001) 094005]
- LSS01 [Eur.Phys.J. C23 (2002) 479]
- BB02 [Nucl. Phys. B636 (2002) 225]

Polarized neutrino-proton scattering (CC)

$$W_{\mu\nu} = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\, F_1 + \frac{\hat{p}_\mu \hat{p}_\nu}{p\cdot q}\, F_2 - i\,\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda p^\sigma}{2p\cdot q}\, F_3 \qquad \text{where } \hat{p}_\mu = p_\mu - \frac{p\cdot q}{q^2} q_\mu$$

$$+ i\,\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda s^\sigma}{p\cdot q}\, g_1 + i\,\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda (p\cdot q\, s^\sigma - s\cdot q\, p^\sigma)}{(p\cdot q)^2}\, g_2$$

$$+ \left[\frac{\hat{p}_\mu\, \hat{s}_\nu + \hat{s}_\mu\, \hat{p}_\nu}{2p\cdot q} - \frac{s\cdot q\; \hat{p}_\mu\, \hat{p}_\nu}{(p\cdot q)^2}\right] g_3 + \frac{s\cdot q\; \hat{p}_\mu\, \hat{p}_\nu}{(p\cdot q)^2}\, g_4 + (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\frac{s\cdot q}{p\cdot q}\, g_5$$

new structure functions g_3, g_4, g_5

be careful about "various" definitions of g_3, g_4, g_5 !

$$\frac{d(\sigma^{CC}_{\lambda_p=-1} - \sigma^{CC}_{\lambda_p=+1})}{dx\,dy} = \frac{G_F^2 Q^2}{\pi(1+Q^2/M_W^2)^2 xy}\Big\{ \left[-\lambda_\ell y(2-y)x g_1^{CC} - (1-y) g_4^{CC} - y^2 x g_5^{CC}\right]$$

$$+ 2xy\frac{M^2}{Q^2}\left[\lambda_\ell x^2 y^2 g_1^{CC} + \lambda_\ell 2x^2 y\, g_2^{CC} + \left(1 - y - x^2y^2\frac{M^2}{Q^2}\right) x\, g_3^{CC}\right.$$

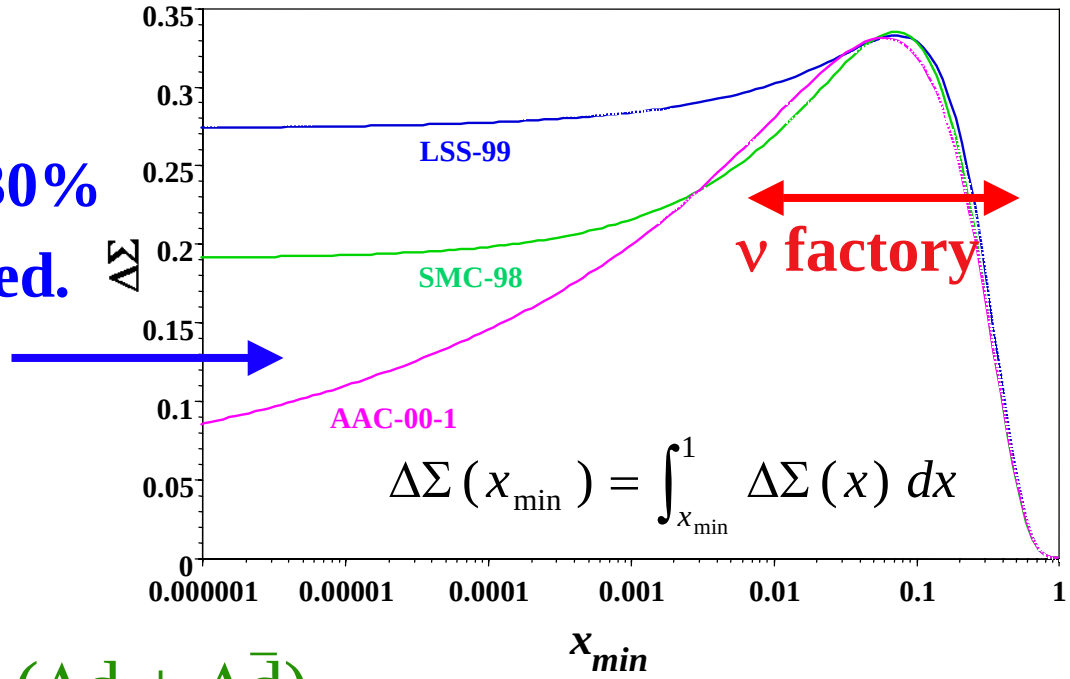
$$\left. - x\left(1 - \frac{3}{2}y - x^2y^2\frac{M^2}{Q^2}\right) g_4^{CC} - x^2y^2 g_5^{CC}\right]\Big\}$$

0 at $Q^2 >> M^2$

Quark spin content

e/μ scattering → ΔΣ = 0 ~30%

It is not uniquely determined.



ν scattering

$$g_1^{\nu p} + g_1^{\bar{\nu} p} = (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) + (\Delta s + \Delta \bar{s}) + (\Delta c + \Delta \bar{c})$$

on ν factory
see e.g. hep-ph/0310166

in LO $\int dx\,(g_1^{\nu p} + g_1^{\bar{\nu} p}) = \Delta\Sigma$

independent determination of quark spin content ΔΣ !

Summary

- Global analysis for polarized PDFs
  - $\Delta u_v(x)$, $\Delta d_v(x)$ are determined well
  - $\Delta\Sigma = 0.213 \pm 0.138$ ($Q^2 = 1$ GeV^2)
  - $\Delta g(x)$ could not be constrained

- **Uncertainties of polarized**
- **Effects of E155-proton data**
- **Global analysis also with $\Delta g=0$**
- **Error correlation between Δg and Δq**

AAC03-polarized-pdfs code could be obtained from
http://spin.riken.bnl.gov/aac/
http://hs.phys.saga-u.ac.jp/aac.html (mirror site)