# Charge asymmetry in $2\pi$ and $3\pi$ electroproduction on proton at high energies

A.I.Ahmedov[1,2], V.V.Bytev[1] and E.A.Kuraev[1], S. Bakmaev[1]

[1] Joint Institute for Nuclear Research, Dubna, Russia,
[2] Institute of Physics, Azerbaijan National Academy of Sciences, Baku

**Abstract**
The charge asymmetry induced by interference of amplitudes with $\sigma$ and $\rho$ mesons decaying in the $\pi^+\pi^-$ pair created in the fragmentation region of proton is suggested to be a test of the degeneration hypothesis of $\rho$ and $\sigma$ mesons. Some numerical estimations are given.

Charge asymmetry in $3\pi$ production provides the measurement of $2\gamma \to 3\pi$ anomaly of Wess-Zumino-Witten effective lagrangian.

# 1 Introduction

Theoretical reasons for the existence of scalar neutral meson ($\sigma$-meson) were formulated in the late 60s [1]. It was recognized that the chiral and scaling symmetry of strong interactions were violated, which can be realized within an effective lagrangian by including a scalar field $\sigma(x)$. In the frameworks of QCD it was shown that breaking of scale invariance was related to the trace of the energy-momentum tensor [2]. In the papers by Schechter, Ellis, and Lanik [3] the effective QCD lagrangian with broken scale and chiral symmetries was constructed where a scalar gluonic current was related with $\sigma(x)$: $G^2_{\mu\nu} \sim m^4_\sigma \sigma(x)^4/G_0, G_0 =< 0|(\alpha_s/\pi G^2_{\mu\nu}|0 >= 0.017 GeV^4$- is the gluonic condensate. Besides the widths of 2 pion and 2 gamma decay channels it was obtained

$$\Gamma(\sigma \to \pi^+\pi^-) = \frac{m^5_\sigma}{48\pi G_0}, \quad \Gamma(\sigma \to \gamma\gamma) = \frac{3}{4}\left(\frac{R\,\alpha}{8\pi^2}\right)^2 \Gamma(\sigma \to \pi^+\pi^-), \qquad (1)$$

with $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$. Experimental evidence of possible existence of $\sigma(750)$ was obtained at CERN experiments for the process $\pi^- p \to \pi^-\pi^+ n$ [4, 5] with a polarized target. In S. Weinberg's paper [6] itwas shown that the relation $m_\rho = m_\sigma$ is a consequence of broken chiral symmetry. A similar statement follows from the analysis of superconvergent sum rules for helicity amplitudes, as was shown in the paper by Gillman and Harari [7]. Really, taking this value for the $\sigma$ meson we obtain for its total (mainly 2 pion) width $\Gamma(\sigma \to 2\pi) = 150 MeV = \Gamma_\rho$.

To obtain independent evidence of $\sigma$ and the validity of the degeneracy $m_\rho = m_\sigma$, $\Gamma_\rho = \Gamma_\sigma$, we suggest measuring the charge asymmetry of two pion production at electron-proton collisions

$$e(p_1) + p(p) \to e'(p'_1) + p'(p') + \pi^+(q_+) + \pi^-(q_-), \tag{2}$$

which is defined as follows:

$$\begin{aligned} A_c &= \frac{d\sigma(q_1, q_2) - d\sigma(q_2, q_1)}{d\sigma(q_1, q_2) + d\sigma(q_2, q_1)} \\ &= \frac{N(\pi^+(q_1), \pi^-(q_2)) - N(\pi^+(q_2), \pi^-(q_1))}{N(\pi^+(q_1), \pi^-(q_2)) + N(\pi^+(q_2), \pi^-(q_1))}, \end{aligned} \tag{3}$$

where $d\sigma(q_1, q_2)$ means the inclusive cross section with $\pi^-$ meson with momentum $q_1$ and $\pi^+$ with momentum $q_2$, and $N(\pi^+(q_1), \pi^-(q_2))$ is the number of corresponding events.

Asymmetry in the case of electroproduction does not depend on the total center of mass energy ($\sqrt{s}$) due to a dominant contribution of amplitudes with photon exchange in the scattering channel.

In the case of pion pair photoproduction on proton the asymmetry is decreasing function of the center-of-mass energy, which is contrary to the case of electroproduction.

Interference of 2 photon mechanism and bremsstrahlung mechanism of electroproduction contains the anomalous vertex $\gamma\gamma \to 3\pi$ of effective Wess-Zumino-Witten lagrangian. It can be measured as a charge-odd asymmetry of $3\pi$ electroproduction.

## 2 Calculation of asymmetry

Charge asymmetry can be more pronounced at invariant mass of pions close to the $\rho$ meson mass where due to the Breit-Wigner enhancement of the cross section the counting rate is expected to be large.

The matrix element in this region can be put in the form $\mathcal{M} = \mathcal{M}_\rho + \mathcal{M}_\sigma$ (see fig.1). Then the charge asymmetry will have the form

$$A_c = \frac{2\mathcal{M}_\rho(\mathcal{M}_\sigma)^*}{|\mathcal{M}_\sigma|^2 + |\mathcal{M}_\rho|^2}. \tag{4}$$

To obtain the realistic estimation magnitude of the effect we take into account only Feynman amplitudes containing the $\sigma, \rho$ intermediate states.

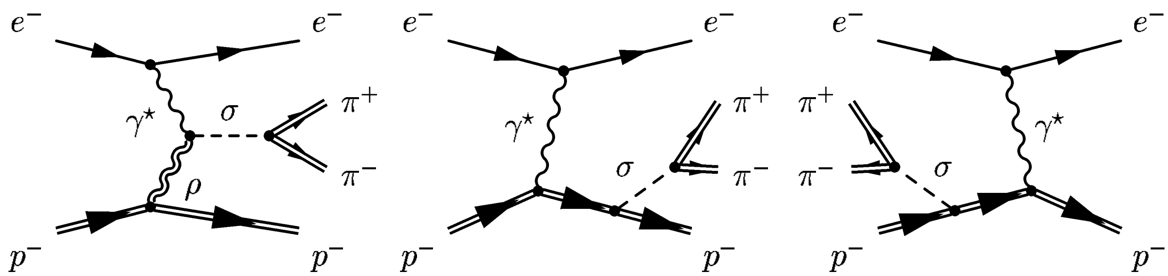


Figure 1: Feynman Diagrams (FD) for the matrix element $\mathcal{M}_\sigma$. By exchange $\sigma \to \rho, \rho \to \sigma$ one can obtain FD for $\mathcal{M}_\rho$

In Appendix we give the cross section and the asymmetry in kinematics of fragmentation of proton in both exclusive and inclusive set-up.

The matrix elements have the form

$$\mathcal{M}_\sigma = J_\mu \bar{u}(p')T^\mu u(p) R_\sigma g_{\sigma\pi\pi} g_{\sigma nn}, \tag{5}$$
$$\mathcal{M}_\rho = J_\mu \bar{u}(p')T^{\mu\nu} u(p)(q_- - q_+)^\nu R_\rho g_{\rho\pi\pi} g_{\rho nn},$$

with (for kinematics see (2))

$$J_\mu = \frac{4\pi\alpha}{Q^2}\bar{u}(p_1')\gamma_\mu u(p_1), \quad R_{\sigma,\rho} = \frac{1}{s_1 - m^2_{\sigma,\rho} + i(m\Gamma)_{\sigma,\rho}}, \tag{6}$$
$$Q^2 = -q^2, \quad s_1 = q_2^2, \quad q_2 = q_+ + q_-, \quad q = p_1 - p_1'.$$

Besides, $g_{\rho\pi\pi} = 4\sqrt{3\pi/\beta_\rho^3}\sqrt{\Gamma_\rho/m_\rho}$ and $g_{\sigma\pi\pi} = 4\sqrt{\pi/\beta_\sigma}\sqrt{m_\sigma\Gamma_\sigma}$ are the coupling constants, $\beta_{\rho,\sigma} = \sqrt{1-(4m_\pi^2/m^2_{\rho,\sigma})}$ is the velocity of pions in the rest frame of decaying $\sigma$ or $\rho$ mesons.

The hadronic currents obey the current conservation condition

$$\bar{u}(p')T_\mu u(p)q^\mu = 0, \quad \bar{u}(p')T^{\mu\nu}u(p)q_\mu = 0, \quad \bar{u}(p')T^{\mu\nu}u(p)q_{2\nu} = 0. \tag{7}$$

The expressions for $T$ we have used are:

$$T^{\mu} = \Lambda \frac{1}{q_1^2 - m_\rho^2} R_1^{\mu\nu}\gamma_\nu + \frac{\hat{p} + \hat{q} + m_p}{(p+q)^2 - m_p^2}\gamma^\mu + \gamma_\mu \frac{\hat{p}' - \hat{q} + m_p}{(p'-q)^2 - m_p^2}, \tag{8}$$

and

$$T^{\mu\nu} = \frac{1}{\Lambda}\frac{1}{q_1^2 - m_\sigma^2} R_2^{\mu\nu} + \gamma_\nu \frac{\hat{p} + \hat{q} + m_p}{(p+q)^2 - m_p^2}\gamma_\mu + \gamma_\mu \frac{\hat{p}' - \hat{q} + m_p}{(p'-q)^2 - m_p^2}\gamma_\nu, \tag{9}$$

with $\Lambda = g_{\rho nn}/g_{\sigma nn}$, $q_1 = p - p'$. The vertex $\gamma^* \sigma \rho$ we parameterize as

$$R_{1,2}^{\mu\nu} = \frac{g\, m}{m^2 + Q^2}(q^\nu q_{1,2}^\mu - g^{\mu\nu} q q_{1,2}), \tag{10}$$

which is an anzatz inspired by low-order triangle Feynman diagram calculation, and $m = 300 MeV$ is the constituent quark mass. The coupling $g$ is chosen in order to reproduce the $(g_{\rho\pi\pi}/e)^2 \Gamma(\sigma \to \gamma\gamma)$. The factor $(g_{\rho\pi\pi}/e)^2$ is introduced to take into account the replacement of one of the photons by the vector meson. Our estimate gives $g \approx 2$.

By calculating the matrix element which is given above we obtain charge asymmetry in the form

$$A_c = \frac{2[(s_1 - m_\rho^2)(s_1 - m_\sigma^2) + m_\rho m_\sigma \Gamma_\rho \Gamma_\sigma] Q^{\mu\lambda}(q_- - q_+)^\nu Z_{\mu\nu\lambda} \Lambda g_{\rho\pi\pi} g_{\sigma\pi\pi}}{Q^{\eta\sigma}[|R_\rho|^{-2} Z_{\eta\sigma} g_{\sigma\pi\pi}^2 + \Lambda^2 |R_\sigma|^{-2} Z_{\eta\mu_1\sigma\nu_1} g_{\rho\pi\pi}^2 (q_- - q_+)_{\mu_1}(q_- - q_+)_{\nu_1}]} \tag{11}$$

with

$$\begin{aligned} Q^{\mu\nu} &= 2p_1^\mu p_1^\nu + (Q^2/2) g^{\mu\nu}, \\ Z_{\mu\nu\lambda} &= Tr(\hat{p}' + m_p) T_\mu (\hat{p} + m_p) \tilde{T}_{\lambda\nu}, \\ Z_{\eta\sigma} &= Tr(\hat{p}' + m_p) T_\eta (\hat{p} + m_p) \tilde{T}_\sigma, \\ Z_{\eta\mu_1\sigma\nu_1} &= Tr(\hat{p}' + m_p) T_{\eta\mu_1} (\hat{p} + m_p) \tilde{T}_{\sigma\nu_1}. \end{aligned} \tag{12}$$

For inclusive set-up (pions as well as the scattered electron are detected) the numerator and the denominator must be integrated by phase volume of the scattered proton.

# 3 Discussion

We neglect above the final-state pion and pion-nucleon interaction which causes the pion-pion scattering phases [8]. It can be justified within 5% accuracy for the case of rather large effective mass of pions in the final state.

In Tables 1-4 we present the $x_+, x_-$ distribution (which means energy fraction of $\pi^+$ and $\pi^-$ in the case of the energies of experiment H1) for the equal masses $m_\sigma = m_\rho = 769$ MeV and different masses.

The integration of asymmetry was made over variables $\vec{q}_\pm$ in the region $0.01 - 0.9$ GeV for each $x, y$ component of the vector $\vec{q}_\pm$:

$$Ac_{int}(x_+, x_-, Q) = \frac{\int d^2q_+ \int d^2q_- 2\mathcal{M}_\rho(\mathcal{M}_\sigma)^*}{\int d^2q_+ \int d^2q_-(|\mathcal{M}_\sigma|^2 + |\mathcal{M}_\rho|^2)} \qquad (13)$$

We can see that the case $m_\sigma = m_\rho$ can be unambiguously separated from the $m_\sigma \neq m_\rho$ case within 5% accuracy of experimental data.

Asymmetry has a magnitude of order 1 ($|A_c| \sim 1$) in the fragmentation region of the initial proton for small values of momentum transferred $Q \sim 0.5$ GeV $\ll \sqrt{s} \sim 100$ GeV (DESY, H1). At higher $Q$ it decreases rapidly: for $Q \sim 3$ GeV $|A_c| < 0.1$.

In Tables 5-7 we give asymmetry as a function of angles $\theta_+, \theta_-$ for the HERMES energies, $\theta_\pm = \widehat{p_e q_\pm}$, integrated over $\varepsilon_+, \varepsilon_-$ (energies of $\pi_+, \pi_-$) in the region $0.5 - 20$ GeV, and over the angles $\phi_+, \phi_-$ ($\phi_\pm = \widehat{q_\perp q_{\pm\perp}}$) in the region $0 - 2\pi$, $Q = 0.2$ GeV

$$Ac_{int}(\theta_+, \theta_-, Q = 0.2) = \frac{\int_0^{2\pi}\int_0^{2\pi} d\phi_+ d\phi_- \int\int d\varepsilon_+ d\varepsilon_- 2\mathcal{M}_\rho(\mathcal{M}_\sigma)^*}{\int_0^{2\pi}\int_0^{2\pi} d\phi_+ d\phi_- \int\int d\varepsilon_+ d\varepsilon_-(|\mathcal{M}_\sigma|^2 + |\mathcal{M}_\rho|^2)} \qquad (14)$$

Note that asymmetry in Tables 5-7 turns to zero not only when $\theta_+ = \theta_-$, but also at the points $\theta_+ = -\theta_-$.

In all numerical estimates given in Tables we used the following values:

$$\Lambda = \frac{g_{\rho nn}}{g_{\sigma nn}} = 1, \quad m = 0.300\ GeV, \quad m_\rho = 0.769\ GeV, \qquad (15)$$
$$\Gamma_\rho = \Gamma_\sigma = 150\ GeV, \quad M_p = 0.98\ GeV, \quad m_\pi = 0.139\ GeV.$$

# 4 Acknowledgements

One of us (V.V.B) is grateful to the HERMES Collaboration for valuable discussions and warm hospitality during May 2003. We acknowledge the support of RFBR, grant 03-02-17077, and two of us E.K. and V.B. the support INTAS, grant 00-00-366.
E.A.K. is grateful to L.Shimanovski and O.Teryaev for critical discussions.

# 5 Appendix

In the proton fragmentation region (invariant mass of the scattered proton jet) $(p' + q_+ + q_-)^2 \ll s = 2pp_1$ we can use the Sudakov decomposition of 4-momenta (2):

$$q = \beta\tilde{p}_1 + \alpha\tilde{p} + q_\perp, \quad q_\pm = \beta_\pm\tilde{p}_1 + x_\pm\tilde{p} + q_{\pm\perp}, \quad p' = \beta'\tilde{p}_1 + x\tilde{p} + p_\perp, \quad (16)$$
$$\tilde{p}_1 = p_1 - p\frac{m_1^2}{s}, \quad \tilde{p} = p - p_1\frac{m_p^2}{s},$$

with the conservation law and on mass shell conditions

$$\frac{m_p^2}{s} + \beta = \beta_+ + \beta_- + \beta', \quad \vec{q} = \vec{q}_+ + \vec{q}_- + \vec{p}, \quad x + x_- + x_+ = 1, \quad (17)$$
$$s\beta_\pm = A_\pm = \frac{1}{x_\pm}[m_\pi^2 + \vec{q}_\pm^{\,2}], \quad s\beta' = A = \frac{1}{x}[m_p^2 + \vec{p}^{\,2}].$$

Above we put designations for transversal components of 4-vectors:

$$\vec{q}_\pm^{\,2} = -q_{\pm\perp}^2, \quad \vec{p}^{\,2} = -p_\perp^2, \quad (18)$$

and $m_p$ ($m_\pi$) are the proton (pion) masses.

Phase space volume

$$d\Gamma = (2\pi)^{-8}\frac{d^3p_1'}{2\epsilon_1'}\frac{d^3p'}{2\epsilon'}\frac{d^3q_+}{2\epsilon_+}\frac{d^3q_-}{2\epsilon_-}\delta^4(p_1 + p - p' - p_1' - q_+ - q_-)$$

can be transformed in the form

$$d\Gamma = (2\pi)^{-8}\frac{d^2\vec{q}d^2\vec{q}_+d^2\vec{q}_-}{8s}\frac{dx_+dx_-}{x_+x_-(1 - x_+ - x_-)}. \quad (19)$$

By using Gribov's representation for the metric tensor

$$g^{\mu\nu} = g_\perp^{\mu\nu} + \frac{2}{s}[\tilde{p}_1^\mu\tilde{p}^\nu + \tilde{p}_1^\nu\tilde{p}^\mu]$$

the matrix element can be written in the form

$$\mathcal{M} = \frac{8\pi\alpha is}{q^2}N(\Phi_\rho g_{\rho\pi\pi}g_{\rho nn} + \Phi_\sigma g_{\sigma\pi\pi}g_{\sigma nn}), \quad (20)$$
$$N = \frac{1}{s}\bar{u}(p_1')\tilde{p}u(p_1), \quad \Sigma|N|^2 = 2;$$

and

$$\Phi_{\sigma,\rho} = \frac{R_{\sigma,\rho}}{s}\bar{u}(p')O_{\sigma,\rho}u(p), \tag{21}$$

with

$$O_\sigma = as\hat{q} + b\hat{p}_1 + \frac{1}{d}(\hat{p}+\hat{q}+M)\hat{p}_1 + \frac{1}{d'}\hat{p}_1(\hat{p}'-\hat{q}+M), \tag{22}$$

$$O_\rho = cs + \frac{1}{d}(\hat{q}_- - \hat{q}_+)(\hat{p}+\hat{q}+M)\hat{p}_1 + \frac{1}{d'}\hat{p}_1(\hat{p}'-\hat{q}+M)(\hat{q}_- - \hat{q}_+). \tag{23}$$

Asymmetry (11) in the exclusive set-up (the scattered electron as well as all components of pion momenta are registered) has the form

$$A_c = \frac{2\Sigma Re(\Phi_\rho\Phi_\sigma^*)}{\Sigma|\Phi_\rho|^2\Lambda^2 g_{\rho\pi\pi}^2 + \Sigma|\Phi_\sigma|^2 g_{\sigma\pi\pi}^2}\Lambda g_{\rho\pi\pi}g_{\sigma\pi\pi}, \tag{24}$$

where the sum over the fermion spin states is implied. The asymmetry as well as the cross section do not depend on $s$ in the large $s$ limit. The notation is:

$$a = \frac{\Lambda\, g\, m\,(1-x)}{2(m^2+Q^2)(q_1^2-m_\rho^2)}, \quad b = -\frac{\Lambda\, g\, m\, qq_1}{2(m^2+Q^2)(q_1^2-m_\rho^2)}, \tag{25}$$

$$c = \frac{g\, m}{2\Lambda(m^2+Q^2)(q_1^2-m_\sigma^2)}[(qq_- - qq_+)(1-x) - (x_- - x_+)(qq_+ + qq_-)],$$

$$d = (p+q)^2 - m_p^2 = -Q^2 + A_- + A_+ + A - m_p^2,$$

$$d' = (p'-q)^2 - m_p^2 = -Q^2 + 2\vec{p}\vec{q} - x(A_- + A_+ + A - m_p^2),$$

Here we put the relevant scalar products which appear in (25) by using the

Gribov representation (16-18):

$$p_1^{'2}=p_1^2=0,\quad 2p_1q=-2p_1^{'}q=-Q^2,\quad p^2=(p')^2=m_p^2\,,$$
$$q^2=-Q^2=(p_1-p_1')^2,\quad p_1q_\pm=\frac{1}{2}sx_\pm,\quad p_1p=\frac{s}{2},\quad p_1p'=\frac{sx}{2},$$
$$q_1^2=(p-p')^2=-\frac{1}{x}[m_p^2(1-x)^2+\vec{p}^2],$$
$$s_1=q_2^2=(q_++q_-)^2=-\frac{1}{x_-x_+}[m_\pi^2(x_-+x_+)^2+(x_+\vec{q}_--x_-\vec{q}_+)^2],$$
$$qq_1=\frac{1}{2}[s_1+Q^2-q_1^2],\quad qq_\pm=-\vec{q}\vec{q}_\pm+\frac{x_\pm}{2}(A_-+A_++A-m_p^2),$$
$$pq_\pm=\frac{1}{2}[m_\pi^2+\vec{q}_\pm^2+m_p^2x_\pm^2]\frac{1}{x_\pm},\quad p'q_\pm=\frac{1}{2x_\pm x}[m_p^2x_\pm^2+x^2m_\pi^2+(x_\pm\vec{p}-x\vec{q}_\pm)^2],$$
$$qp=\frac{1}{2}(A_-+A_++A-m_p^2),\quad qp'=-\vec{q}\vec{p}+\frac{x}{2}(A_-+A_++A-m_p^2),$$
$$pp'=\frac{1}{2x}[m_p^2(1+x^2)+\vec{p}^2],\quad q_+q_-=\frac{1}{2x_+x_-}[m_\pi^2(x_+^2+x_-^2)+(x_+\vec{q}_--x_-\vec{q}_+)^2].$$

| $x_-\backslash x_+$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|---|---|---|---|---|---|---|---|---|
| 0.1 | 0 | -0.32 | -0.51 | -0.46 | -0.37 | -0.26 | -0.15 | -0.07 |
| 0.2 | 0.32 | 0 | 0.032 | -0.037 | -0.071 | -0.067 | -0.058 | |
| 0.3 | 0.51 | -0.03 | 0 | 0.001 | -0.039 | -0.055 | | |
| 0.4 | 0.46 | 0.037 | -0.001 | 0 | -0.019 | | | |
| 0.5 | 0.367 | 0.071 | 0.0398 | 0.0197 | | | | |
| 0.6 | 0.257 | 0.067 | 0.055 | | | | | |
| 0.7 | 0.148 | 0.058 | | | | | | |
| 0.8 | 0.07 | | | | | | | |

Table 1: Integrated Asymmetry for equal masses of $\sigma$ and $\rho$ mesons, $Q = 1.2$ GeV (for other numerical parameters see Section 3).

| $x_-\backslash x_+$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|---|---|---|---|---|---|---|---|---|
| 0.1 | 0 | -0.09 | -0.39 | -0.65 | -0.81 | -0.87 | -0.84 | -0.71 |
| 0.2 | 0.09 | 0 | -0.06 | -0.09 | -0.32 | -0.55 | -0.59 | |
| 0.3 | 0.39 | 0.06 | 0 | -0.039 | -0.031 | -0.065 | | |
| 0.4 | 0.653 | 0.093 | 0.039 | 0 | -0.021 | | | |
| 0.5 | 0.806 | 0.323 | 0.031 | 0.021 | | | | |
| 0.6 | 0.869 | 0.547 | 0.065 | | | | | |
| 0.7 | 0.845 | 0.595 | | | | | | |
| 0.8 | 0.71 | | | | | | | |

Table 2: Integrated Asymmetry for mass of $\sigma$ equals 1.2 Gev, $Q = 0.7$ GeV.

| $x_-\backslash x_+$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|---|---|---|---|---|---|---|---|---|
| 0.1 | 0 | -0.83 | -0.88 | -0.85 | -0.79 | -0.71 | -0.62 | -0.48 |
| 0.2 | 0.83 | 0 | -0.45 | -0.56 | -0.54 | -0.47 | -0.38 | |
| 0.3 | 0.88 | 0.44 | 0 | -0.22 | -0.29 | -0.28 | | |
| 0.4 | 0.85 | 0.55 | 0.22 | 0 | -0.11 | | | |
| 0.5 | 0.79 | 0.54 | 0.29 | 0.11 | | | | |
| 0.6 | 0.72 | 0.47 | 0.28 | | | | | |
| 0.7 | 0.62 | 0.38 | | | | | | |
| 0.8 | 0.48 | | | | | | | |

Table 3: Integrated Asymmetry for equal masses of $\sigma$ and $\rho$ mesons, $Q = 0.7$ GeV.

| $x_-\backslash x_+$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|---|---|---|---|---|---|---|---|---|
| 0.1 | 0 | -0.788 | -0.938 | -0.947 | -0.911 | -0.841 | -0.735 | -0.576 |
| 0.2 | 0.788 | 0 | -0.545 | -0.777 | -0.743 | -0.642 | -0.510 | |
| 0.3 | 0.938 | 0.545 | 0 | -0.299 | -0.454 | -0.442 | | |
| 0.4 | 0.947 | 0.777 | 0.299 | 0 | -0.168 | | | |
| 0.5 | 0.911 | 0.743 | 0.454 | 0.168 | | | | |
| 0.6 | 0.84 | 0.64 | 0.44 | | | | | |
| 0.7 | 0.73 | 0.51 | | | | | | |
| 0.8 | 0.57 | | | | | | | |

Table 4: Integrated Asymmetry for mass of $\sigma$ equals 0.469 Gev, $Q = 0.7$ GeV.

| $\theta_+\backslash\theta_-$ | 0.22 | 0.18 | 0.13 | 0.09 | 0.04 | 0 | -0.04 | -0.09 | -0.13 | -0.18 | -0.22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.22 | 0 | 0.02 | 0.06 | 0.16 | 0.12 | -0.53 | 0.12 | 0.16 | 0.06 | 0.02 | 0 |
| 0.18 | -0.02 | 0 | 0.07 | 0.23 | 0.29 | -0.34 | 0.29 | 0.23 | 0.07 | 0 | -0.02 |
| 0.13 | -0.06 | -0.07 | 0 | 0.22 | 0.45 | -0.21 | 0.45 | 0.22 | 0 | -0.07 | -0.06 |
| 0.09 | -0.16 | -0.23 | -0.22 | 0 | 0.57 | -0.12 | 0.57 | 0 | -0.22 | -0.23 | -0.16 |
| 0.04 | -0.12 | -0.29 | -0.45 | -0.57 | 0 | 0 | 0 | -0.57 | -0.45 | -0.29 | -0.12 |
| 0 | 0.53 | 0.34 | 0.21 | 0.12 | 0 | 0 | 0 | 0.12 | 0.21 | 0.34 | 0.53 |
| -0.04 | -0.12 | -0.28 | -0.45 | -0.57 | 0 | 0 | 0 | -0.57 | -0.45 | -0.29 | -0.12 |
| -0.09 | -0.16 | -0.23 | -0.22 | 0 | 0.57 | -0.12 | 0.57 | 0 | -0.22 | -0.23 | -0.16 |
| -0.13 | -0.06 | -0.07 | 0 | 0.22 | 0.45 | -0.21 | 0.45 | 0.22 | 0 | -0.07 | -0.06 |
| -0.18 | -0.02 | 0 | 0.07 | 0.23 | 0.29 | 0.34 | 0.29 | 0.23 | 0.07 | 0 | -0.02 |
| -0.22 | 0 | 0.02 | 0.06 | 0.16 | 0.12 | -0.53 | 0.12 | 0.16 | 0.06 | 0.02 | 0 |

Table 5: Integrated Asymmetry for mass of $\sigma$ equals 0.469 GeV.

| $\theta_+ \backslash \theta_-$ | 0.22 | 0.18 | 0.13 | 0.09 | 0.04 | 0 | -0.04 | -0.09 | -0.13 | -0.18 | -0.22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.22 | 0 | 0.03 | 0.10 | 0.22 | 0.09 | -0.44 | 0.09 | 0.22 | 0.10 | 0.03 | 0 |
| 0.18 | -0.03 | 0 | 0.13 | 0.37 | 0.23 | -0.27 | 0.23 | 0.37 | 0.13 | 0 | -0.03 |
| 0.13 | -0.10 | -0.13 | 0 | 0.43 | 0.34 | -0.16 | 0.34 | 0.43 | 0 | -0.13 | -0.10 |
| 0.09 | -0.22 | -0.37 | -0.43 | 0 | 0.36 | -0.08 | 0.36 | 0 | -0.43 | -0.37 | -0.22 |
| 0.04 | -0.09 | -0.23 | -0.34 | -0.36 | 0 | -0.03 | 0 | -0.36 | -0.34 | -0.23 | -0.09 |
| 0 | 0.43 | 0.27 | 0.16 | 0.08 | 0.03 | 0 | 0.03 | 0.08 | 0.16 | 0.27 | 0.44 |
| -0.04 | -0.09 | -0.23 | -0.34 | -0.36 | 0 | -0.03 | 0 | -0.36 | -0.34 | -0.23 | -0.09 |
| -0.09 | -0.22 | -0.37 | -0.43 | 0 | 0.36 | -0.08 | 0.36 | 0 | -0.43 | -0.37 | -0.22 |
| -0.13 | -0.10 | -0.13 | 0 | 0.43 | 0.34 | -0.16 | 0.34 | 0.43 | 0 | -0.13 | -0.10 |
| -0.18 | -0.02 | 0 | 0.13 | 0.37 | 0.23 | -0.27 | 0.23 | 0.37 | 0.13 | 0 | -0.03 |
| -0.22 | 0 | 0.03 | 0.10 | 0.22 | 0.09 | -0.44 | 0.09 | 0.22 | 0.10 | 0.03 | 0 |

Table 6: Integrated Asymmetry for mass of $\sigma$ equals 0.769 GeV.

| $\theta_+ \backslash \theta_-$ | 0.22 | 0.18 | 0.13 | 0.09 | 0.04 | 0 | -0.04 | -0.09 | -0.13 | -0.18 | -0.22 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.22 | 0 | 0.04 | 0.15 | 0.27 | 0.05 | -0.33 | 0.05 | 0.27 | 0.15 | 0.04 | 0 |
| 0.18 | -0.04 | 0 | 0.22 | 0.44 | 0.14 | -0.19 | 0.14 | 0.44 | 0.22 | 0 | -0.04 |
| 0.13 | -0.15 | -0.22 | 0 | 0.47 | 0.17 | -0.10 | 0.17 | 0.47 | 0 | -0.22 | -0.15 |
| 0.09 | -0.27 | -0.44 | 0.47 | 0 | 0.13 | -0.03 | 0.13 | 0 | -0.47 | -0.44 | -0.27 |
| 0.04 | -0.05 | -0.14 | -0.17 | -0.13 | 0 | -0.08 | 0 | -0.13 | -0.17 | -0.14 | -0.05 |
| 0 | 0.33 | 0.19 | 0.10 | 0.03 | 0.08 | 0 | 0.08 | 0.03 | 0.10 | 0.19 | 0.33 |
| -0.04 | -0.05 | -0.14 | -0.17 | -0.12 | 0 | -0.08 | 0 | -0.12 | -0.17 | -0.14 | -0.05 |
| -0.08 | -0.27 | -0.44 | -0.47 | 0 | 0.12 | -0.03 | 0.12 | 0 | -0.47 | -0.44 | -0.27 |
| -0.13 | -0.15 | -0.22 | 0 | 0.47 | 0.17 | -0.10 | 0.17 | 0.47 | 0 | -0.22 | -0.15 |
| -0.18 | -0.41 | 0 | 0.22 | 0.44 | 0.14 | -0.19 | 0.14 | 0.44 | 0.22 | 0 | -0.04 |
| -0.22 | 0 | 0.04 | 0.15 | 0.27 | 0.05 | -0.33 | 0.05 | 0.27 | 0.15 | 0.04 | 0 |

Table 7: Integrated Asymmetry for mass of $\sigma$ equals 1.200 GeV.

# 6 measuring the $\gamma\gamma \to 3\pi$ anomaly

Measurement of the C-odd correlation in 3 pion production process at electron-proton collisions

$$e(p_1) + P(p_2) \to e(p_1') + P(p_2') + \pi_+(q_+) + \pi_-(q_-) + \pi_0(q_0), \qquad (26)$$

provides the possibility to measure the "two gamma 3 pions anomaly" of Wess-Zumino-Witten effective Lagrangian [9]. Really, measuring the charge asymmetry defined as

$$A_c = \frac{d\sigma(\pi_+(q_+)\pi_-(q_-)\pi_0(q_0)) - d\sigma(\pi_+(q_-)\pi_-(q_+)\pi_0(q_0))}{d\sigma(\pi_+(q_+)\pi_-(q_-)\pi_0(q_0)) + d\sigma(\pi_+(q_-)\pi_-(q_+)\pi_0(q_0))}, \qquad (27)$$

one will measure the interference of amplitudes of two-photon and bremsstrahlung mechanisms of $3\pi$ system creation:

$$A_c = \frac{2Re\sum M_1 M_2^*}{\sum(|M_1|^2 + |M_2|^2)}, \qquad (28)$$

with the simbol $\sum$ implied the summation on the spin states of proton and electron. One of them, $M_1$ corresponds to contribution of two-photon mechanism:

$$M_1 = \frac{4\pi\alpha}{q^2}\bar{u}(p_1')\gamma_\nu u(p_1)\bar{u}(p_2')\gamma_\sigma u(p_2)D(q_2^2)M^{\nu\sigma}, \qquad (29)$$

with $D(q_2^2) = (1/q_2^2)(m_\rho^2/(q_2^2 - m_\rho^2))$, $q_2 = p_2 - p_2'$; $q = p_1 - p_1'$ and the amplitude of transition of two (virtual) photons to the set of three pions have a form:

$$M^{\nu\sigma} = \frac{\alpha}{\pi f_\pi^2}[\rho(\nu\sigma q q_2) + (\nu\sigma(q - q_2)q_0) - \frac{q_+^\sigma}{q_2 q_+}(\nu q q_- q_0) - \frac{q_-^\sigma}{q_2 q_-}(\nu q q_+ q_0) - \frac{q_+^\nu}{q q_+}(\sigma q q_- q_0) - \frac{q_-^\nu}{q q_-}(\sigma q q_+ q_0)],$$

$$(abcd) = \epsilon^{\alpha\beta\gamma\delta}a_\alpha b_\beta c_\gamma d_\delta, \qquad (30)$$

$f_\pi = 93 MeV$ and

$$\rho = \frac{2}{3} - \frac{2(q_+ + q_-)^2 - (q_+ + q_0)^2 - (q_- + q_0)^2}{(q_+ + q_- + q_0)^2 - m_\pi^2}. \qquad (31)$$

The bremsstrahlung mechanism describes the creation of the $\pi_+\pi_-\pi_0$ system through $\omega$ meson intermediate state with $\omega$ meson emitted by proton:

$$M_2 = \frac{4\pi\alpha}{q_2}\bar{u}(p_1')u(p_1)\bar{u}(p_2')O_{\rho\eta}u(p_2)\frac{g_{\omega NN}M^\eta}{q_1^2 - m_\omega^2 + im_\omega\Gamma_\omega}, \qquad (32)$$

with $q_1 = q_+ + q_- + q_0$, $g_{\omega NN} = g$-is the coupling constant of $\omega$ meson with nucleons; and (see [9],KRS paper)

$$M^\eta = (\eta q_1 q_- q_+)F,$$
$$F = \frac{3g}{4\sqrt{2}f_\pi^3\pi^2}[1 - \frac{3}{4}x + \frac{3}{32}x^2 - \frac{1}{4}x[(\frac{(q_+ + q_-)^2}{M^2} - 1 + i\frac{\Gamma_\rho}{M})^{-1} \quad (33)$$
$$+(\frac{(q_+ + q_0)^2}{M^2} - 1 + i\frac{\Gamma_\rho}{M})^{-1} + (\frac{(q_- + q_0)^2}{M^2} - 1 + i\frac{\Gamma_\rho}{M})^{-1}]],$$

with $x = 2g^2 f_\pi^2/M^2 \approx 2, M = M_\rho$. In the model used in [10] the term $3x^2/32$ in expression for $F$ did not appears. The matrix tensor $O_{\rho\eta}$ have a form:

$$O_{\rho\eta} = \gamma_\eta \frac{\hat{p}_2 + \hat{q} + M_p}{(q + p_2)^2 - M_p^2}\gamma_\rho + \gamma_\rho \frac{\hat{p}_2 - \hat{q}_1 + M_p}{(p_2 - q_1)^2 - M_p^2}\gamma_\eta. \quad (34)$$

Note that Proton current tensor satisfy gauge conditions

$$\bar{u}(p_2')O_{\rho\eta}u(p_2)q_1^\eta = 0; \quad \bar{u}(p_2')O_{\rho\eta}u(p_2)q^\rho = 0. \quad (35)$$

At high energy limit $s = (p_1 + p_2)^2 >> M_p^2$ both matrix elements are proportional $s$ (this fact provide the independence of the total cross section of process considered. Virtual photon emitted by electron Green function will have a form

$$G_{\nu\rho}(q) \approx \frac{1}{q^2}\frac{2}{s}p_{2\nu}p_{1\rho}. \quad (36)$$

The resulting expression for asymmetry will have a form:

$$A_c = 8\pi\alpha Re \frac{\int p_1^\lambda p_1^\rho M_{\lambda\sigma} M^\eta D(q_2^2)(q_1^2 - M_\omega^2 + iM_\omega\Gamma_\omega)^{-1} Sp_{\sigma\rho\eta} d\Phi}{\int d\Phi[(4\pi\alpha)^2 D(q_2^2)^2 p_1^\lambda p_1^\sigma M_{\lambda\sigma} M^*_{\lambda_1\sigma_1} Sp^{\sigma\sigma_1} + g^2 BW p_1^\rho p_1^{\rho_1} M^\eta M^{\eta *} Sp_{\rho\eta\rho_1\eta_1}]}, \quad (37)$$

with

$$BW = ((q_1^2 - M_\omega^2)^2 + M_\omega^2\Gamma_\omega^2))^{-1}, \quad (38)$$
$$Sp_{\sigma\rho\eta} = Sp(\hat{p}_2 + M_p)\gamma_\sigma(\hat{p}_2' + M_p)O_{\rho\eta};$$
$$Sp_{\sigma\sigma_1} = Sp(\hat{p}_2 + M_p)\gamma_\sigma(\hat{p}_2' + M_p)\gamma_{\sigma_1};$$
$$Sp_{\rho\eta\rho_1\eta_1} = Sp(\hat{p}_2 + M_p)O_{\rho\eta}(\hat{p}_2' + M_p)\tilde{O}_{\rho_1\eta_1}. \quad (39)$$

In this expression we imply the partial integration on the phase space of the final particles:

$$d\Phi = \frac{d^3p_1'}{2\epsilon_1'}\frac{d^3p_2'}{2\epsilon_2'}\frac{d^3q_+}{2\epsilon_+}\frac{d^3q_-}{2\epsilon_-}\frac{d^3q_0}{2\epsilon_0}\delta^4(p_1+p_2-p_1'-p_2'-q_+-q_--q_0). \quad (40)$$

Using Sudakov decomposition of 4-vectors

$$q_i = x_i\tilde{p}_2 + \beta_i p_1 + q_{i\perp}, p_2' = x\tilde{p}_2 + \beta p_1 + p_\perp, \quad (41)$$

we can present the phase volume in the form:

$$d\Phi = \frac{1}{32s}\frac{dx_+dx_-dx\theta(1-x_+-x_--x)}{x_+x_-x(1-x-x_+-x_-)}d^2qd^2pd^2q_+d^2q_-. \quad (42)$$

After performing the extra variables integration in the nominator and denominator we obtain the asymmetry $A_c$ as a function of charged pions energy fractions $x_\pm$. This value is presented in the table. One can see that the absolute value of $A_c$ is of the same order as the asymmetry in the process of two charged pions production at the high energy electron-proton diffractive scattering.

# References


[1] A. Salam, J. Strathedee Phys. Rev. **184**, 1760 (1969)

[2] S. Adler et al., Phys.Rev. **D15**, 1712 (1977)

[3] Ellis J, Lanik J, 1985 Phys.Lett. **150B**, 289
J. Schechter, Phys. Rev. **D21** 3393 (1980)

[4] Svec M, Phys.Rev. **D55**, 5727 (1997)
Svec M, 2002 hep-ph/0209323
Svec M, 2002 hep-ph/0210249

[5] Alexeev I et al., Nucl.Phys. **B541**, 3 (1999)

[6] Weinberg S, Phys.Rev.Lett. **65**, 1177 (1990)

[7] G. J. Gilman, H. Harari Phys.Rev. **165**, 1803 (1968)

[8] P.Hager, B.Pire, L.Shimanovski, O.Teryaev, Proc. of ICHEP 2002, p.437;
M.Diehl, T.Gousset, B.Pire and O.Teryaev, Phys.Rev.Lett. 81 (1998) 1782.

[9] Witten E. Nucl. Phys. 1983 B223, 422; Wess J., Zumino B. Phys. Lett. 1971, 37B, 95; Kaymakcalan O. Radjeev S. and Schechter J. Phys. Rev. 1984, D30, 594.

[10] Kuraev E. A. and Silagadze Z. K. Yad. Fiz. 58,1995, 1687.