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The Ins and Outs of $g-2$

Thomas Teubner (The University of Liverpool)

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VI. Outlook: Future prospects for Theory + Experiment

I. $g-2$: Testing QFT at the highest precision

- Magnetic moment $\vec{\mu}$: a fundamental observable

$$\vec{\mu} = g \frac{e}{2m} \vec{s}, \qquad \vec{s} \text{ the particle's spin.}$$

- Dirac equation $\rightsquigarrow$ $g = 2$, but quantum corrections lead to the anomalous magnetic moment: $a \equiv (g-2)/2$

- In an uniform magnetic field $\vec{B}$ there is circular motion with cyclotron frequency $\vec{\omega}_c = e\vec{B}/m$ and spin precession with $\vec{\omega}_s = ge\vec{B}/2m$.

  One measures the anomalous precession frequency

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = \frac{e}{m} a_\mu \vec{B}$$

  via the time spectrum of the electrons from the μ decays (and the $\vec{B}$ field via NMR magnetometers).

- Theory prediction in QED at one loop:

$$a \;=\; \alpha/(2\pi) \;\simeq\; 11\,614\,097 \cdot 10^{-10} \qquad \text{(Schwinger)}$$

- Experimentally: one of the most precisely measured quantities

$$a_\mu^{\rm exp} \;=\; 11\,659\,208\;(6) \cdot 10^{-10} \quad [0.5\,\text{ppm}]$$
$$a_e^{\rm exp} \;=\; 11\,596\,521.869\;(0.041) \cdot 10^{-10} \quad [3.5\,\text{ppb}]$$

- $a_e \sim$ 140 times more precise than a_μ (and serves to measure α), but:

  Sensitivity to New Physics: $a^{\rm NP} \;\propto\; m^2/\Lambda_{\rm NP}^2$

  $\longrightarrow$ typically enhanced for muon by $m_\mu^2/m_e^2 \;\sim\; 43000\,!$

- With this naive dimensional argument a discrepancy

$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} \sim 25 \cdot 10^{-10}$$

  would imply *New Physics* at the scale $M_{\rm NP} \sim 2$ TeV

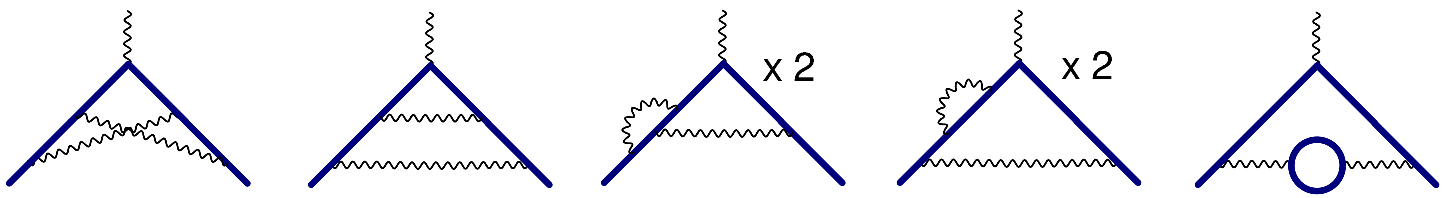
  $\hookrightarrow$ strong potential to establish or constrain NP!

II. The Standard Model prediction of $g-2$

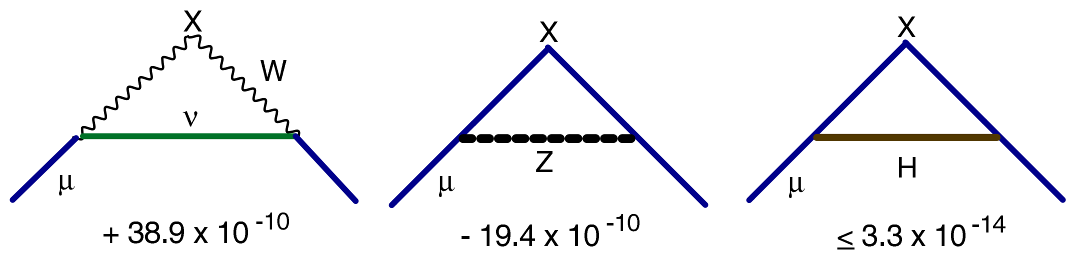
The different contributions in the SM: $a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$

Some typical Feynman graphs for illustration:

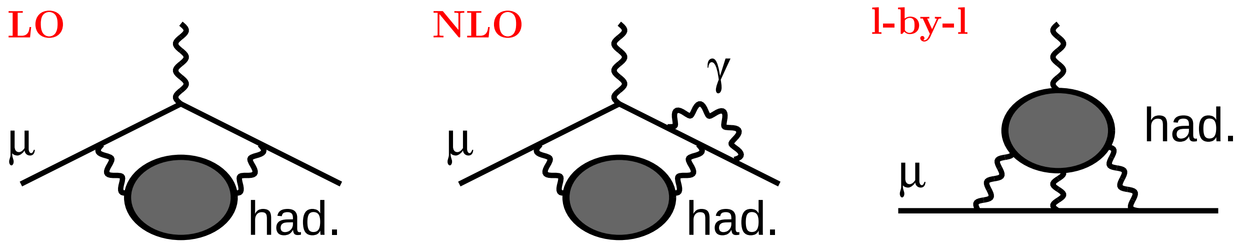
QED 2–loop:



ElectroWeak 1–loop:



Hadronic: $a_\mu^{\rm had} \quad = \quad a_\mu^{\rm had,LO} \quad + \quad a_\mu^{\rm had,NLO} \quad + \quad a_\mu^{\rm had,Light-by-Light}$



→ For the ‘hadronic blob’, data input needed; e.g. from the very important experiment CMD-2 at Novosibirsk:

(They provide the most precise $e^+e^- \to \pi^+\pi^-$ data with only 0.6% sys. error)

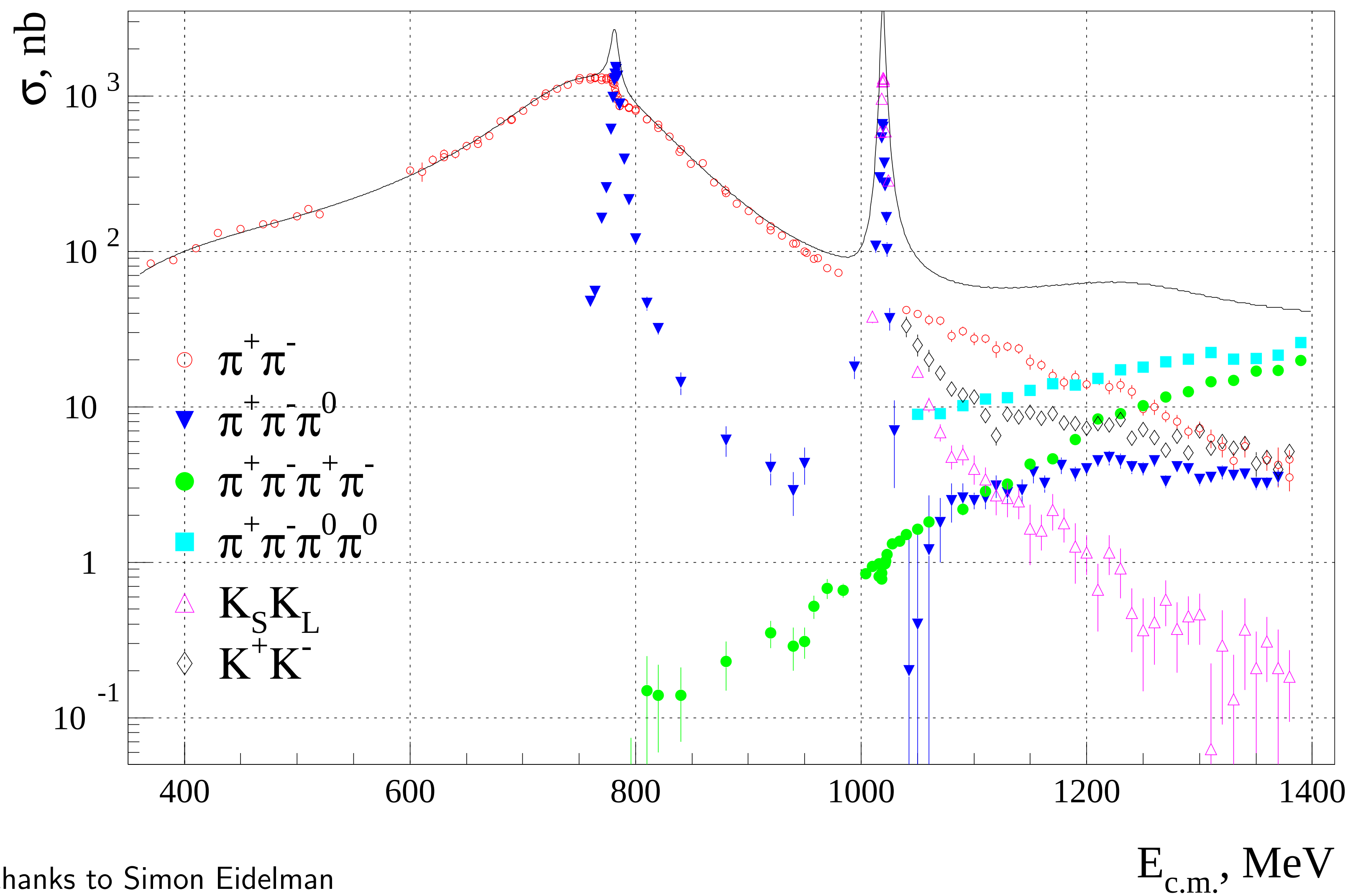


Figure thanks to Simon Eidelman

The different contributions numerically (up to date!):

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	$116\,584\,719.35 \pm 1.43$	up to 5-loop! (Laporta+Remiddi, Kinoshita et al.)
	(was $116\,584\,705.7 \pm 1.8$)	▶ incl. most recent corr. from Kinoshita+Nio
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		▶ agrees very well with Knecht+Peris+Perrottet+deRafael
LO hadr.	$7110 \pm 50 \pm 8 \pm 28$	Davier+Eidelman+Hoecker+Zhang '03b (τ)
(CMD-2 $\pi\pi$	$6963 \pm 62 \pm 36$	Davier+Eidelman+Hoecker+Zhang '03b (e^+e^-)
data re-anal.)	$6924 \pm 59 \pm 24$	Hagiwara+Martin+Nomura+T, hep−ph/0312250
NLO hadr.	-98 ± 1	HMNT, in agreem. with Krause '97, Alemany+D+H '98
L-by-L	136 ± 25	new: Melnikov+Vainshtein, hep−ph/0312226
$<$ Dec. 2003:	80 ± 40	compilation from Nyffeler, hep−ph/0203243
$<$ Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \to 1.6\,\sigma$
$\sum$	$(11659183.5 \pm 6.7)10^{-10}$	with HMNT (e^+e^-)

III. Squeezing the error: hadronic vac.-pol. contributions

- Not calculable in perturbative QCD, but from experimental $\sigma(e^+e^- \to \gamma^* \to hadrons)$ data (or from $\tau \to \nu_\tau + hadrons$ spectral functions) via the *dispersion relation*:

$$a_\mu^{\mathrm{had,LO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} \mathrm{d}s\, \sigma_{\mathrm{had}}^0(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s} \cdot (0.63 \ldots 1)$$

- Lowest energies most important, i.e. the hadronic channels $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, K^+K^-, $K_S^0 K_L^0$, 4π, 5π, etc. Have to sum up ~ 24 exclusive hadronic channels to get σ_{had}.

- In each channel: combine data from many experiments (non-trivial w.r.t. error analysis / correlations / different energy ranges, see below).

- Before averaging and summing: check Radiative Corrections of each exp. data set:

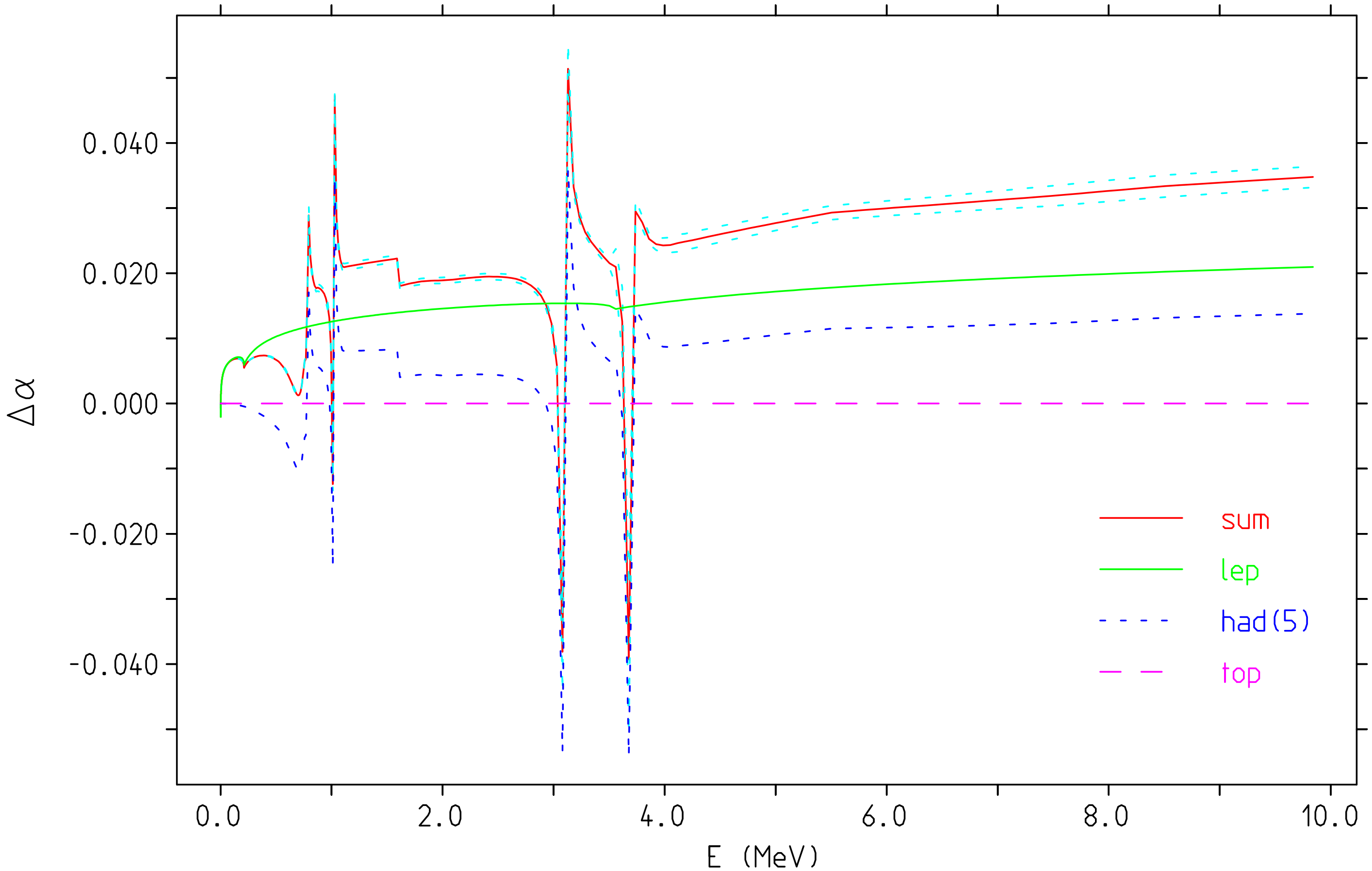
  $\longrightarrow$ additional, *possibly missing* final state photons must be fully *included/estimated*

  $\longrightarrow$ running coupling $\alpha(s) = \alpha/(1 - \Delta\alpha(s))$ (vac.-pol.) effects must be *subtracted* (otherwise double-counting with $a_\mu^{\mathrm{had,NLO}}$), but

  $\longrightarrow$ effects can cancel in $\sigma_{\mathrm{had}}/\sigma_{\mathrm{norm}}$, and corrections often done already partly...

$\longrightarrow$ important detail: use of time-like running of $\alpha(s)$:

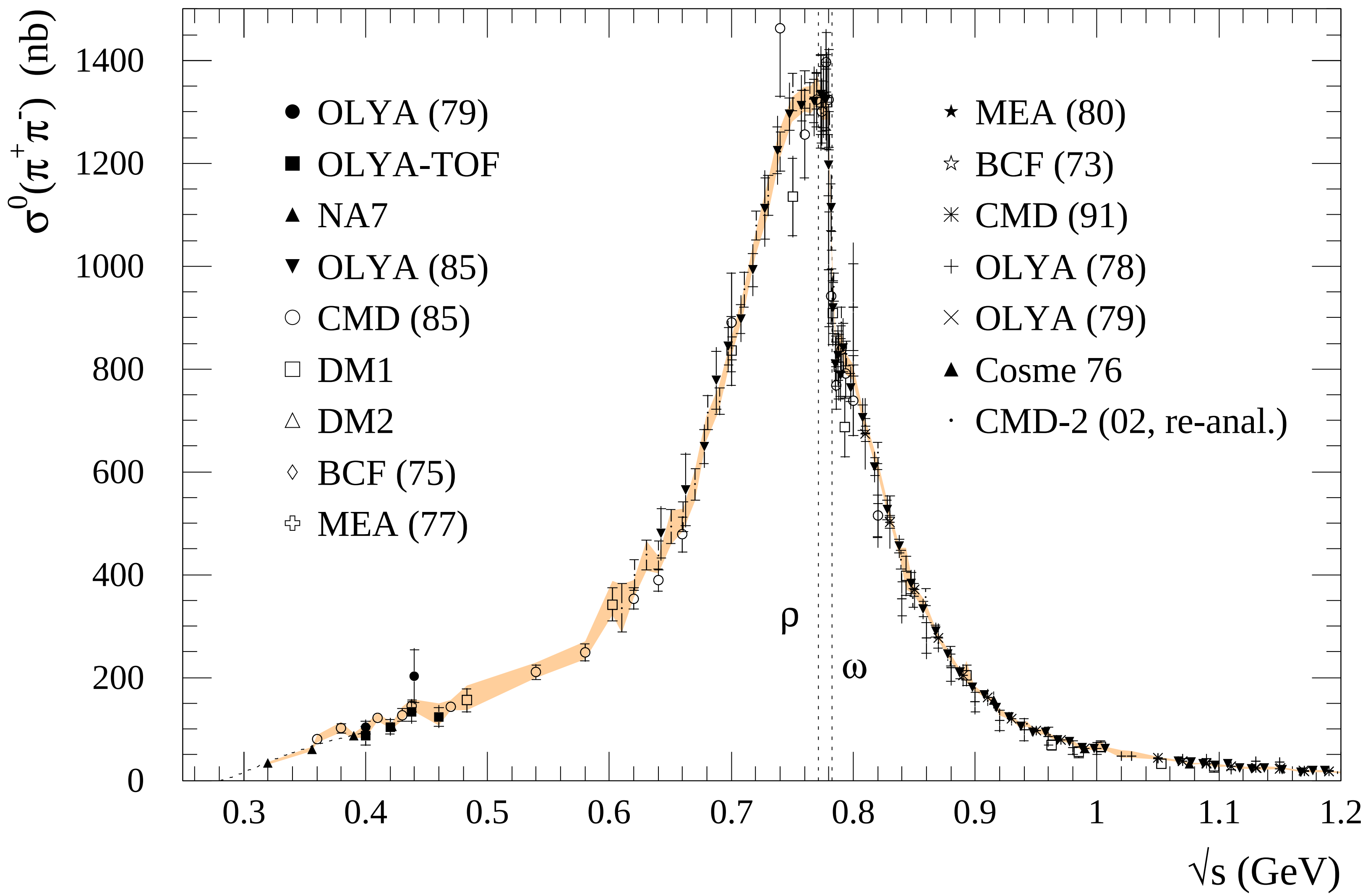
$$\alpha(s) \;=\; \alpha \,/\, \left(1 - \Delta\alpha_{\text{lep}}(s) - \Delta\alpha^{(5)}_{\text{had}}(s) - \Delta\alpha^{\text{top}}(s)\right)$$



Plot from F. Jegerlehner

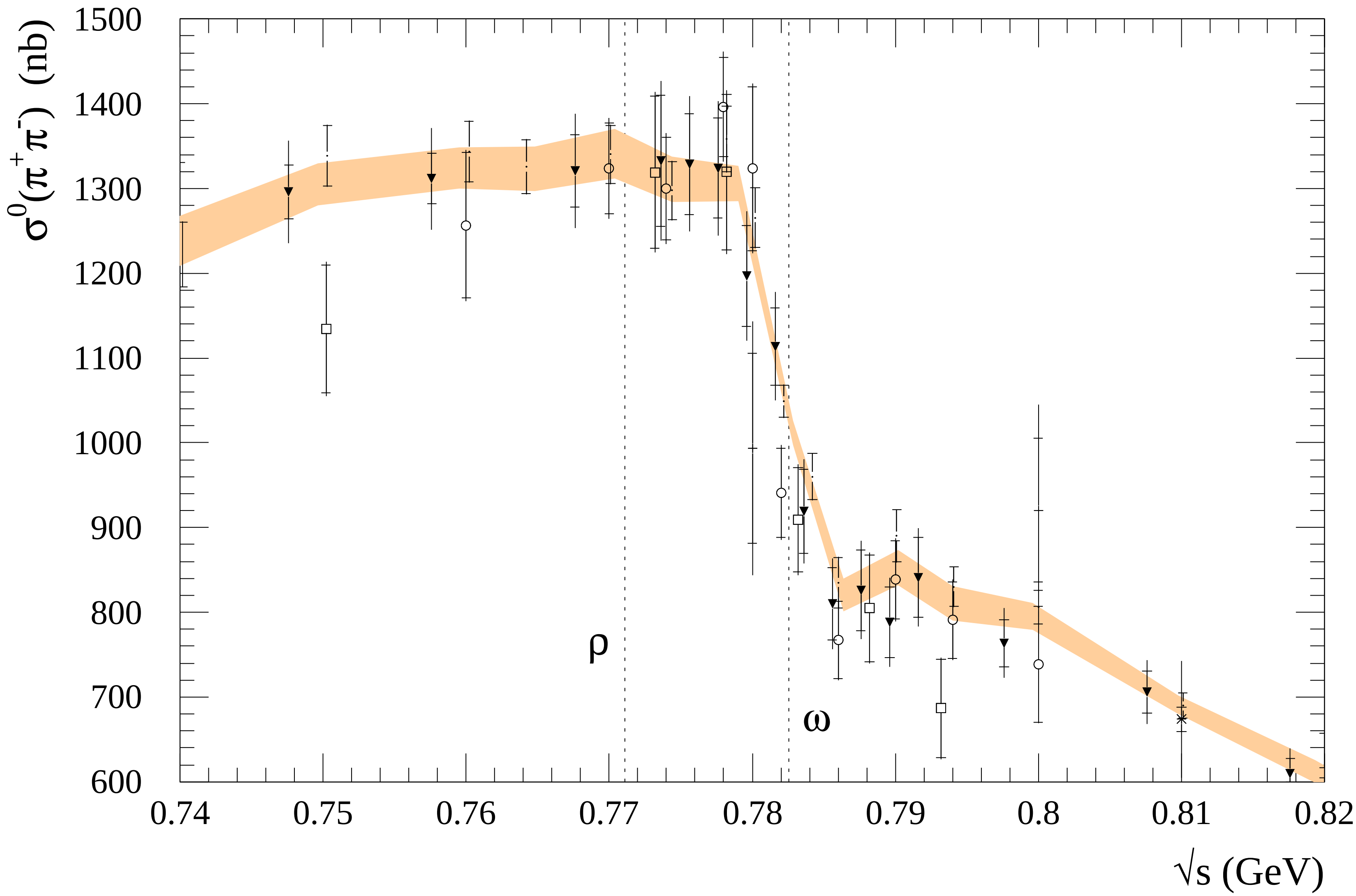
$\longrightarrow$ In total these radiative corrections leads to an additional uncertainty of $\delta a_\mu^{\text{had,VP+FSR}} \simeq 2.4 \times 10^{-10}$.

- The most important channel: $\pi^+\pi^-$ ($\sim 72\%$ of total LO-hadronic!)



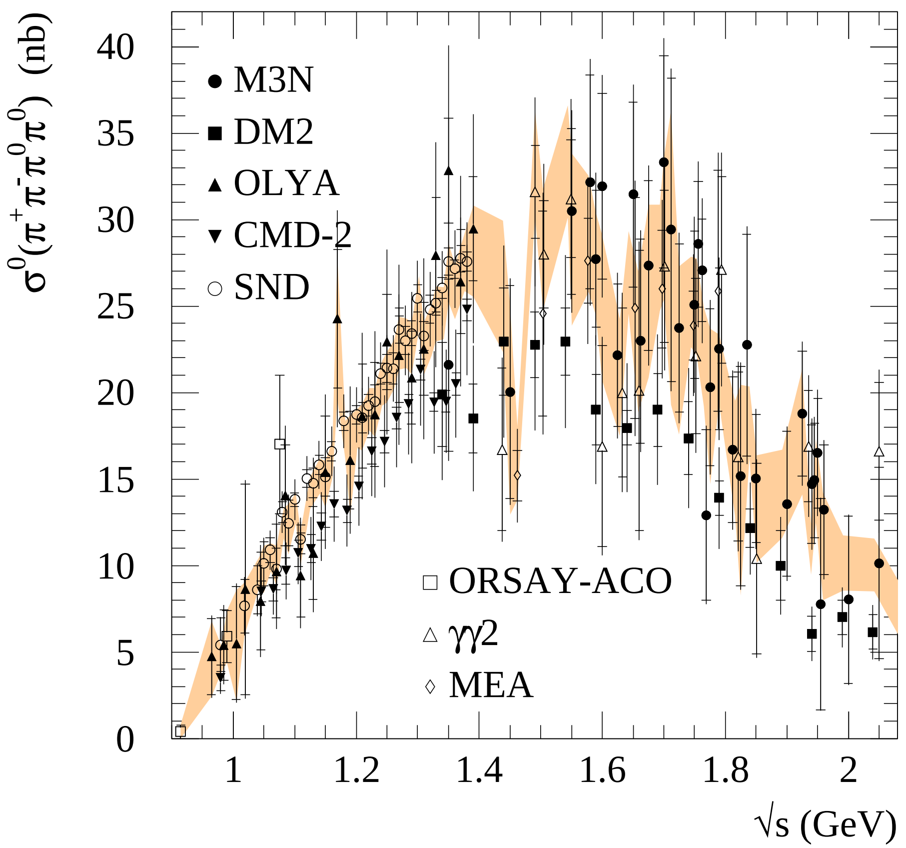
$\longrightarrow$ Prominent role of the CMD-2 data: without it $a_\mu^{\pi^+\pi^-}$ would go *down* by 2.4%, its error *up* by $\sim$ 68% ! (after 2003 re-analysis)

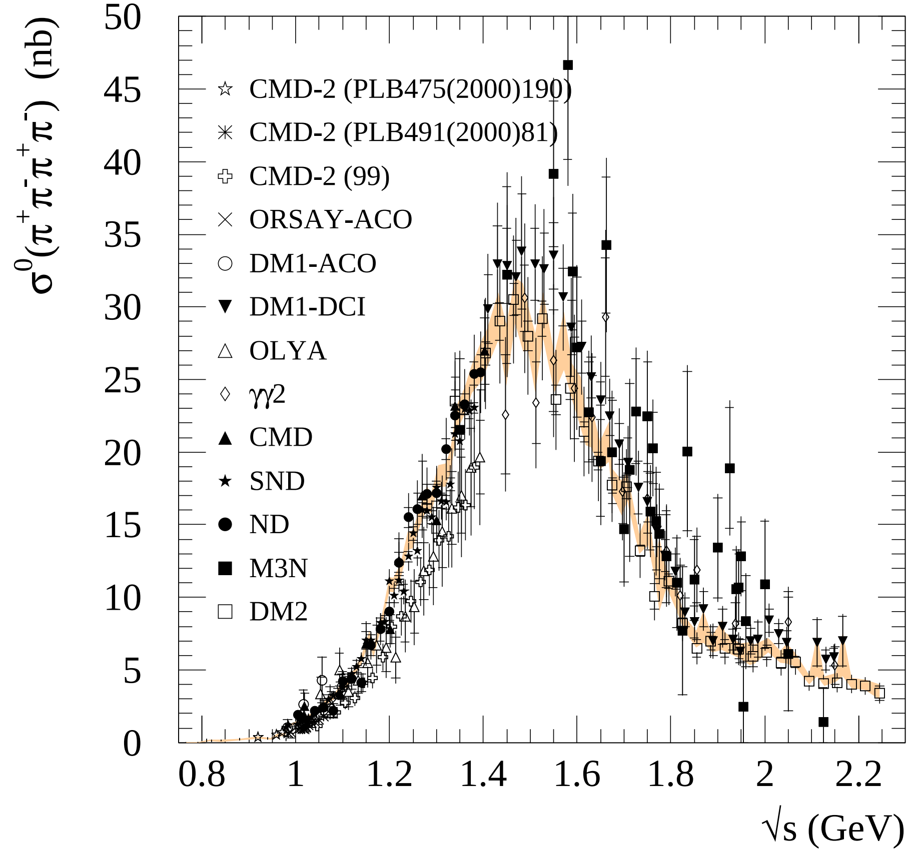
Zoom: $\rho - \omega$ interference:



(Has to be included 'by hand' for the τ spectral function based approach.)

- 'Difficult' channels: $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^+\pi^-$





– Some data-sets poorly consistent with each other (but new analysis will lead to better agreement between accurate SND and CMD-2 data as already reported).

→ And: preliminary Radiative Return data (from BaBar) very promising!

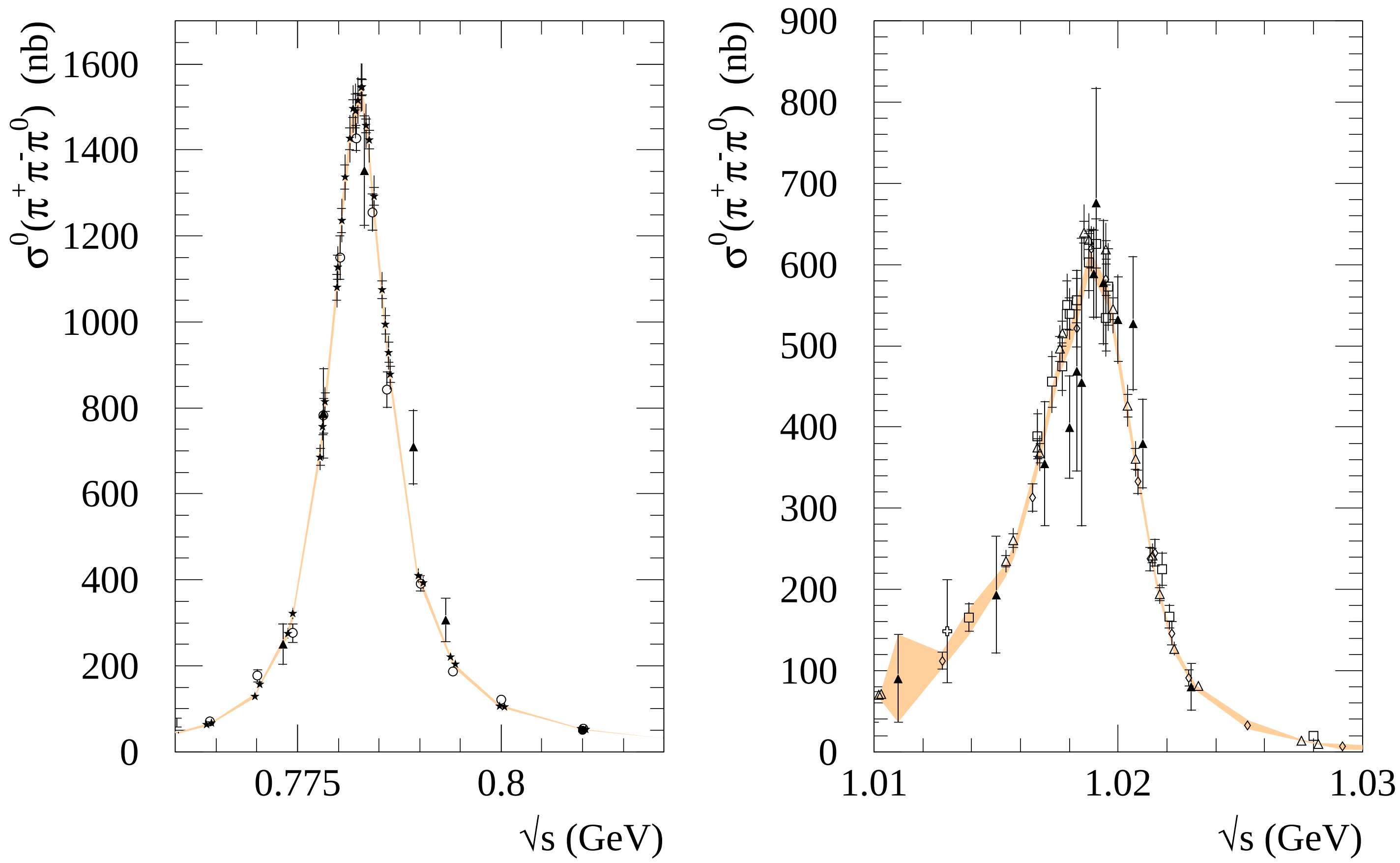
– At present we have to inflate the error in the $\pi^+\pi^-\pi^+\pi^-$ channel by $\sqrt{(\chi^2/d.o.f.) = 2.0}$.

• Narrow resonances: ω, $\phi \to \pi^+\pi^-\pi^0$

$\longrightarrow$ the same data-driven approach is applied: integration of the $e^+e^- \to hadrons$ data,

$\longrightarrow$ NO parametrizations of the resonance-shapes (with problematic error analysis),

$\hookrightarrow$ NO problem with interference effects, 'tails', missing or double-counting of backgrounds.



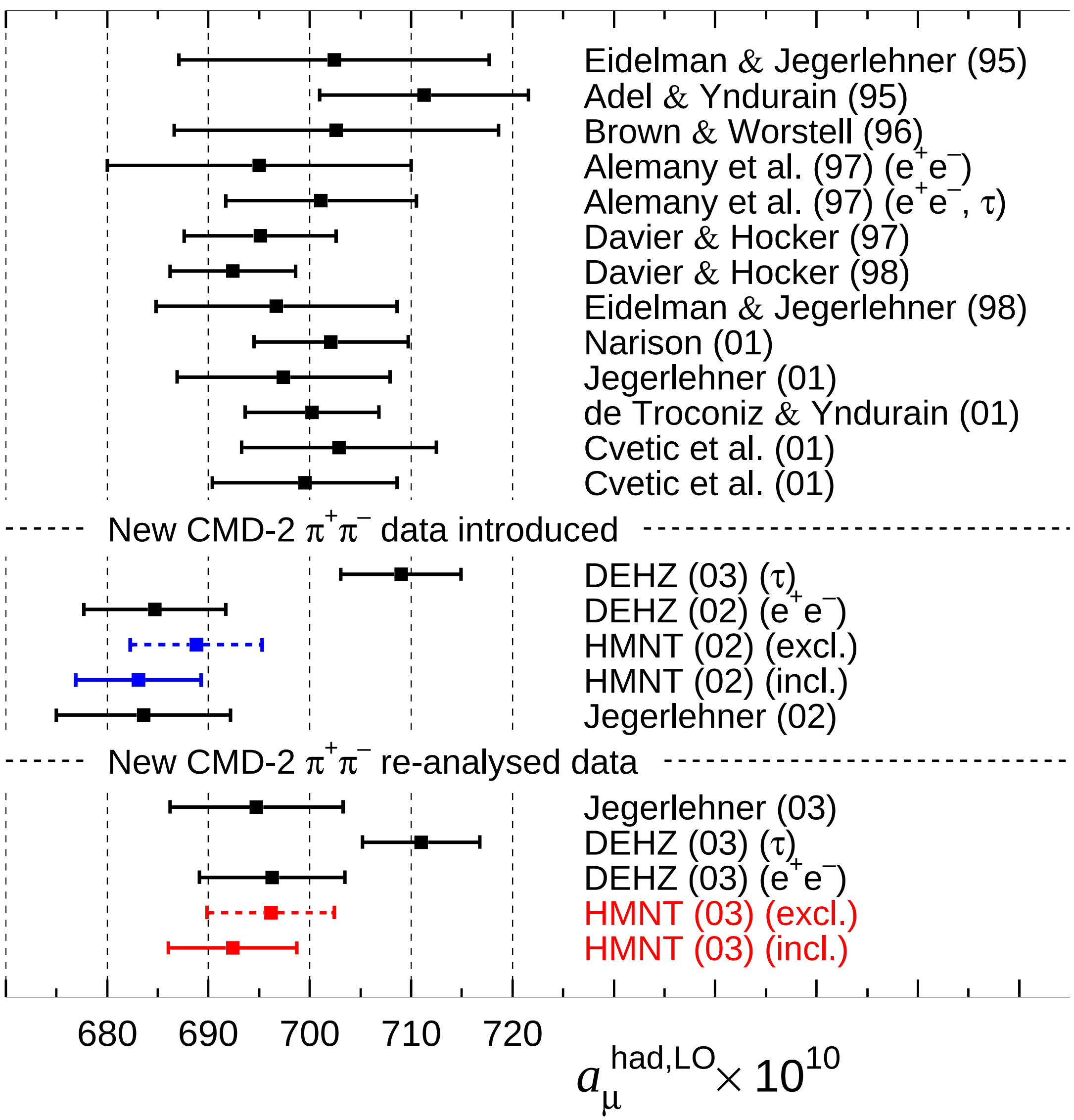
Note the energy scales!

Results. Contributions from the different energy regimes:

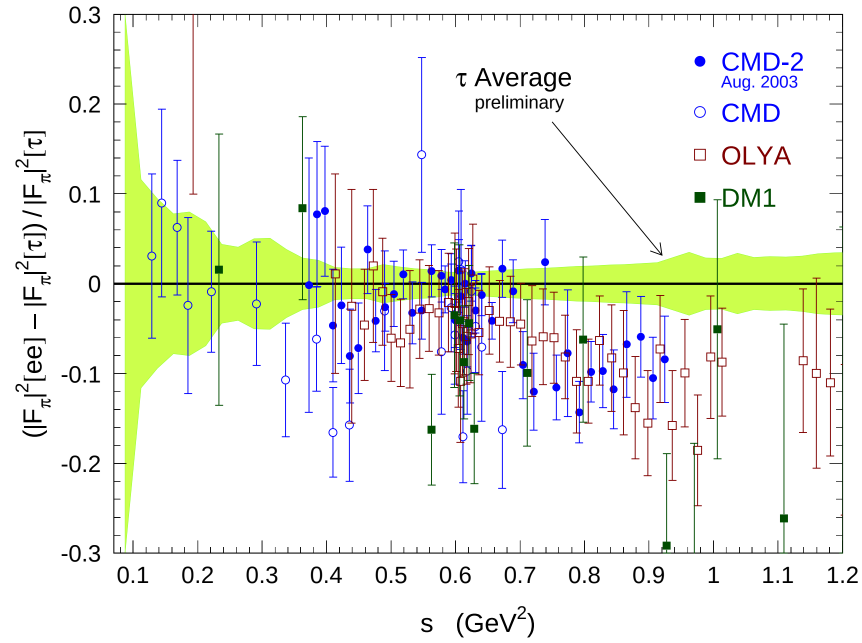
energy range	comments	$a_\mu^{\text{had, LO}} \times 10^{10}$
$2m_\pi \ldots 0.32$	chiral PT	2.36 ± 0.05
$0.32 \ldots 1.43$	excl. only	606.55 ± 5.22
$1.43 \ldots 2.00$	excl. only	35.68 ± 1.71
	incl. only	31.91 ± 2.42
$2.00 \ldots 11.09$	incl. only	42.05 ± 1.14
$J/\psi + \psi(2S)$	NW	7.30 ± 0.43
$\Upsilon(1-6S)$	NW	0.10 ± 0.00
$11.09 \ldots \infty$	pQCD	2.11 ± 0.01
$\sum$ of all	'excl.'	696.15 ± 5.68
	'incl.'	692.38 ± 5.88

♢ Extended use of pQCD from $3.00 \ldots 3.73$ and $5.50 \ldots \infty$ now gives only slightly smaller values and errors.

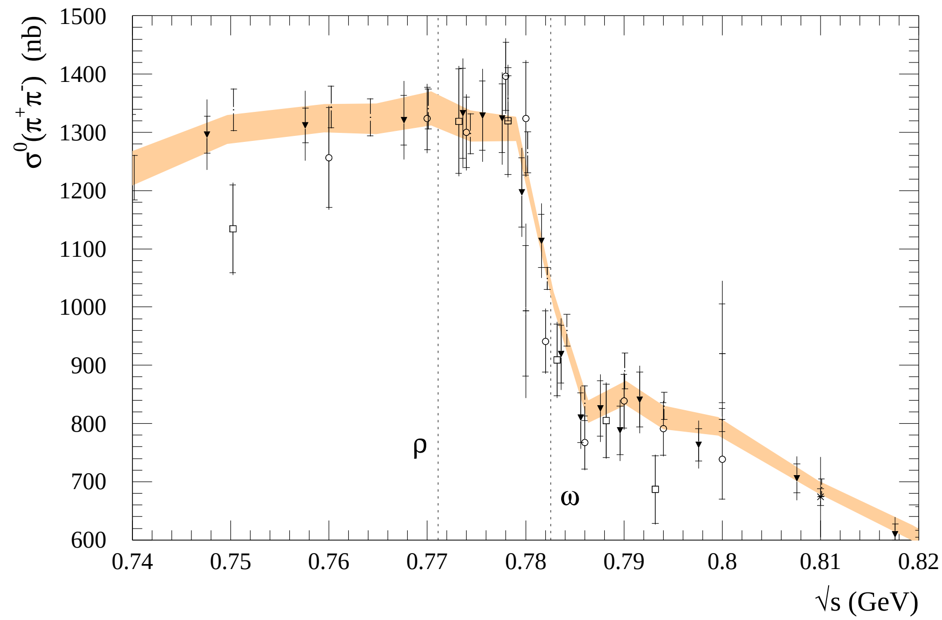
Overview of different evaluations of $a_\mu^{\text{had, LO}}$:



DEHZ (03a): e^+e^- compared to τ data:



'Zoom': $\rho - \omega$ interference (HMNT):



▶ (Un)resolved puzzle: **Why are τ spectral function data not compatible with e^+e^- ?**

!Connection of $\tau^- \to \nu_\tau \pi^- \pi^0, \nu_\tau \pi^- 3\pi^0, \nu_\tau 2\pi^- \pi^+ \pi^0$ and $e^+e^- \to n\,\pi's$ not direct:

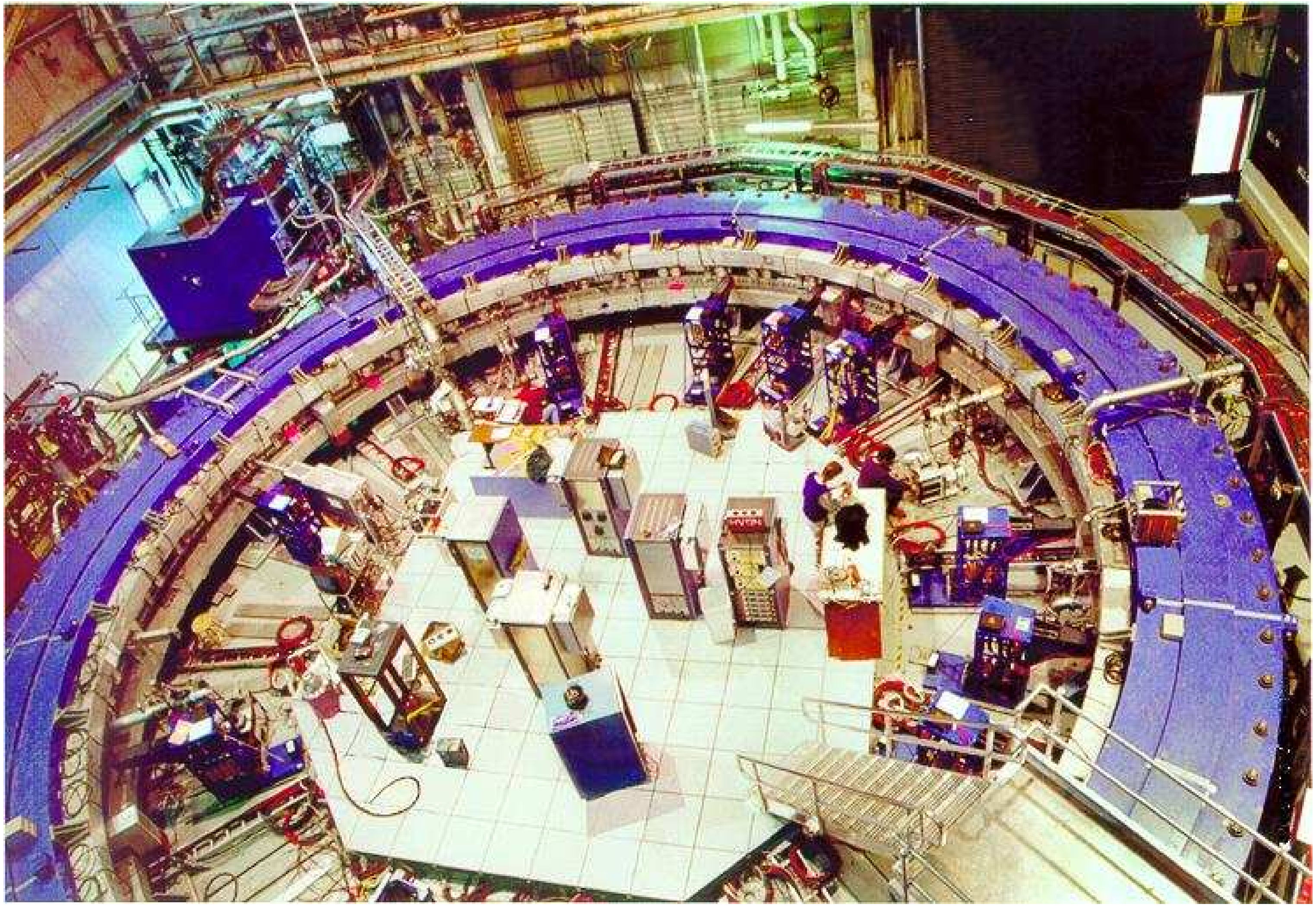
- Violation of CVC hypothesis (Isospin-symmetry) $\leadsto$ complicated corrs. ($\to$ Cirigliano+Ecker+Neufeld) due to short+long distance rad. corrs., mass differences ($m_{\pi^-} \neq m_{\pi^0}$ etc.), $\rho - \omega$ interference, ...
- Is everything under control *at the % level of accuracy*? Is something wrong with data?

$\longrightarrow$ independent check: KLOE Radiative Return analysis agrees with e^+e^-.

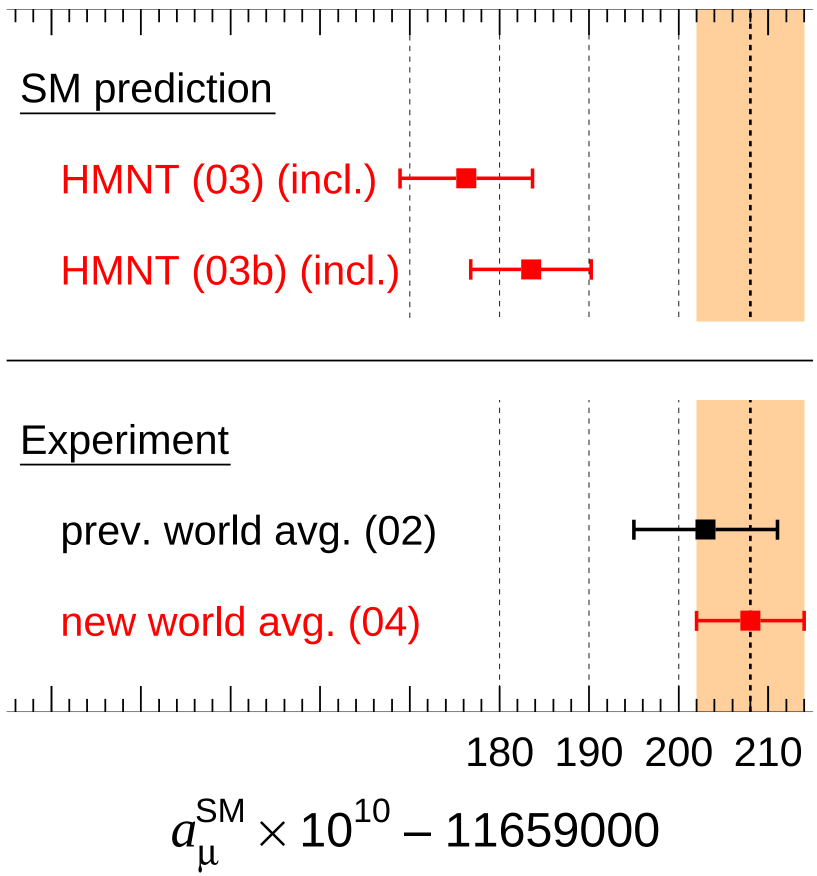
$\longrightarrow$ One possible explanation: $\rho^0 - \rho^\pm$ mass difference.

IV. The new BNL measurement, a sign of Physics beyond SM?

The experiment E821 at Brookhaven (Picture of the magnet from the $g-2$ homepage)



$a_\mu^{\rm SM}$ comp. to new BNL 04 world av.:



Latest changes:

- Improved Light-by-Light estimate:

  $(8 \pm 4) \cdot 10^{-10} \longrightarrow (13.6 \pm 2.5) \cdot 10^{-10}$.

- 'Update' of 4- and 5-loop QED:

  was: $116\,584\,705.7 \pm 1.8) \cdot 10^{-11}$

  $\rightarrow$ is now: $(116\,584\,719.35 \pm 1.43) \cdot 10^{-11}$.

⋆ `hep-ex/0401008`: BNL's 2001 μ^- data:

$a_{\mu^-} = 11\,659\,214(8)(3) \times 10^{-10}\ (0.7ppm)$

$\rightarrow a_\mu = 11\,659\,208(6) \times 10^{-10}\ (0.5ppm)$.

▶ With all these changes we (HMNT) get:

$a_\mu^{\rm EXP} - a_\mu^{\rm TH} = (24.5 \pm 9) \cdot 10^{-10}$, i.e. $\sim \mathbf{2.7\,\sigma}$.

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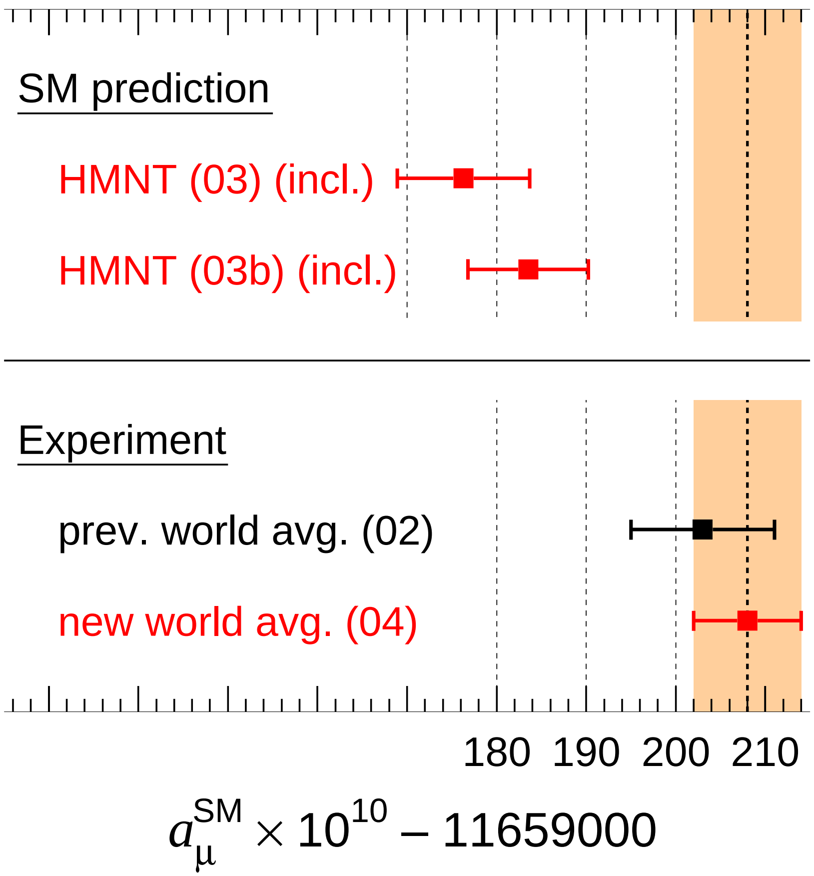

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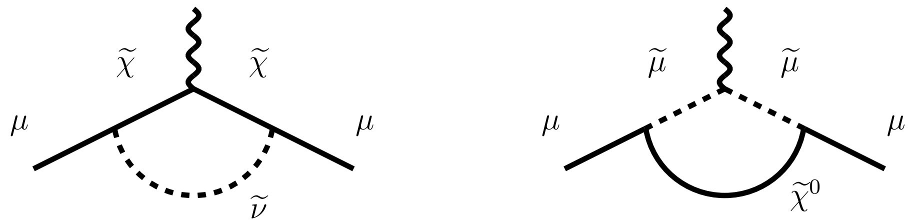
Space left for speculations

SUSY contributions in a_μ ?

$$a_\mu^{\mathrm{SUSY,1-loop}} \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \tan\beta \ \mathrm{sign}(\mu) \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}$$

They mainly come from:



SUSY is a good candidate to explain $\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$ (Little Higgs not):

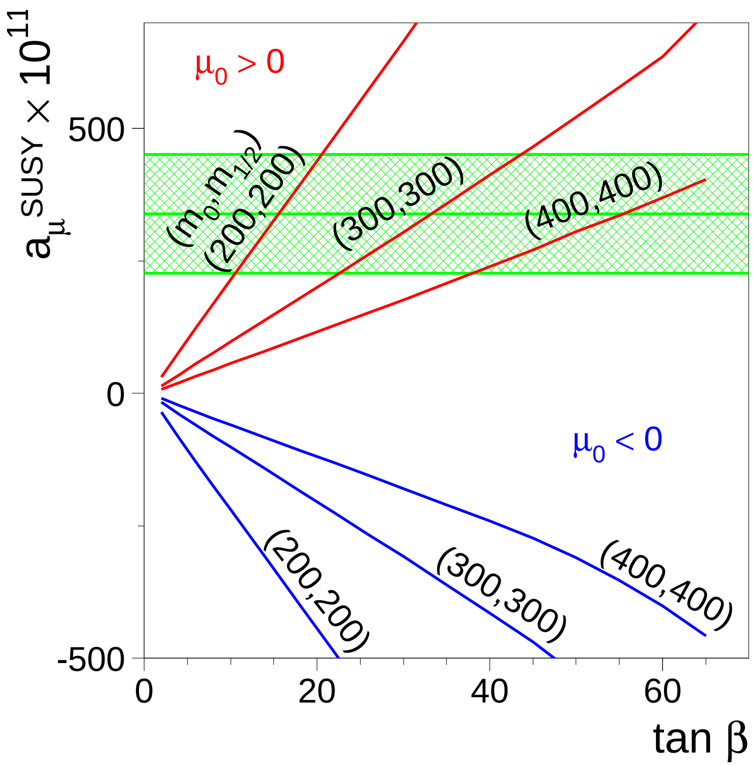


Figure from W. de Boer + C. Sander, hep-ph/0307049

$$\Delta a_\mu = (34 \pm 11) \cdot 10^{-10}$$

In order to be in the $2\,\sigma$ range,

$$14 \le a_\mu^{\mathrm{SUSY}} \cdot 10^{10} \le 58$$

$\rightsquigarrow$ *constrain* $M_{\mathrm{SUSY}} = 150 - 670$ GeV

for $\tan\beta = 10 - 50$

V. $\alpha_{\rm QED}(M_Z)$: A new result and its impact on m_H

- $\alpha_{\rm QED}$ the least well known parameter of (G_μ, M_Z and $\alpha(M_Z^2)$).
  Precision Electro-Weak fits affected!

- PDG2002: $\alpha^{-1} \equiv \alpha(0)^{-1} = 137.03599976(50)$,

$$\alpha(s)^{-1} = \Big(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\Big)\,\alpha^{-1}$$

- $\Delta\alpha_{\rm lep}(M_Z^2) = 0.03149769$ (3 loop, $\to$ M. Steinhauser), $\Delta\alpha^{\rm top}(M_Z^2) = -0.00007$.

- Use of similar dispersion relation:

$$\boxed{\Delta\alpha^{(5)}_{\rm had} = -\frac{\alpha s}{3\pi}\,P\int_{s_{\rm th}}^{\infty}\frac{R(s')\,ds'}{s'(s'-s)}}$$

- HMNT find:

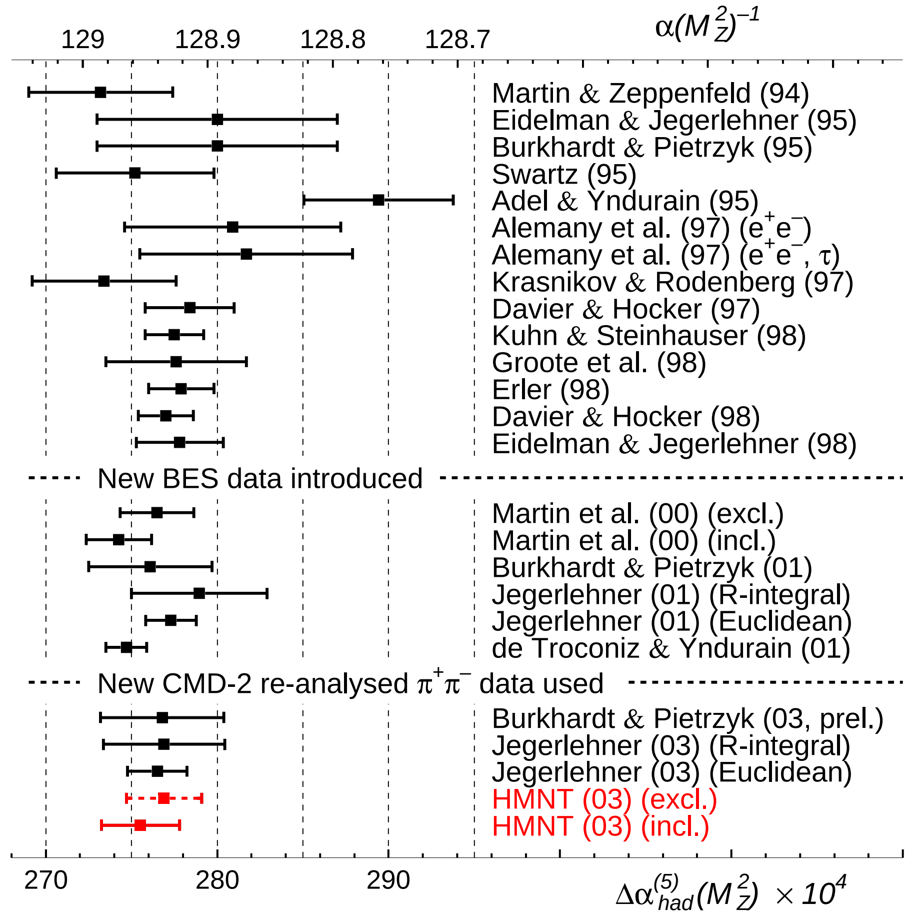
$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02755 \pm 0.00023\,, \quad \alpha(M_Z^2)^{-1} = 128.954 \pm 0.031\,.$$

Contr. from the different energy regimes:

(HMNT)

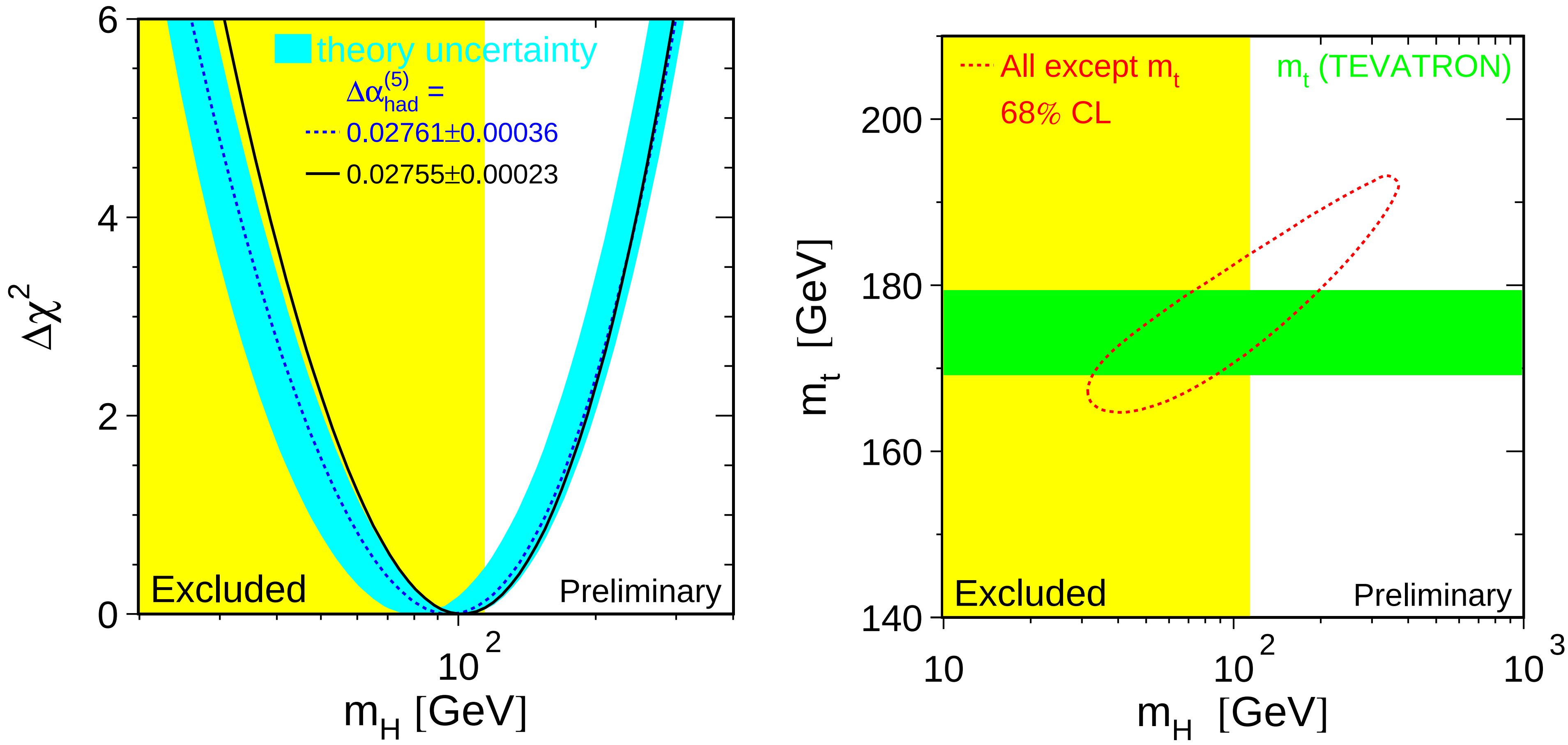
energy (GeV)	comment	$\Delta\alpha^{(5)}_{\rm had} \times 10^4$
$2m_\pi$—0.32	ch. PT	0.04 ± 0.00
0.32—1.43	excl.	47.34 ± 0.35
1.43—2.00	incl.	10.78 ± 0.81
(1.43—2.00	excl.	12.17 ± 0.59)
2.00—11.09	incl.	81.97 ± 1.53
$J/\psi + \psi(2S)$	NW appr.	8.90 ± 0.51
$\Upsilon(1-6S)$	NW appr.	1.16 ± 0.04
11.09—∞	pQCD	125.32 ± 0.15
$\sum$	'incl.'	275.52 ± 1.85

Comparison of different evaluations:



HMNT: Sim. value but smaller err. despite data-driven.

Fitting the Higgs mass: (Plots from the LEP EWWG using our $\Delta\alpha$)



$m_H = 102\,^{+58}_{-38}$ GeV, $m_H < 221$ GeV at 95% confidence.

($m_H = 88\,^{+50}_{-30}$ GeV, $m_H < 190$ GeV with old value from BP.)

Where is improvement needed most urgently?

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors^2:

Critical regions:

$\rightarrow a_\mu$:

low(est) energy regime,

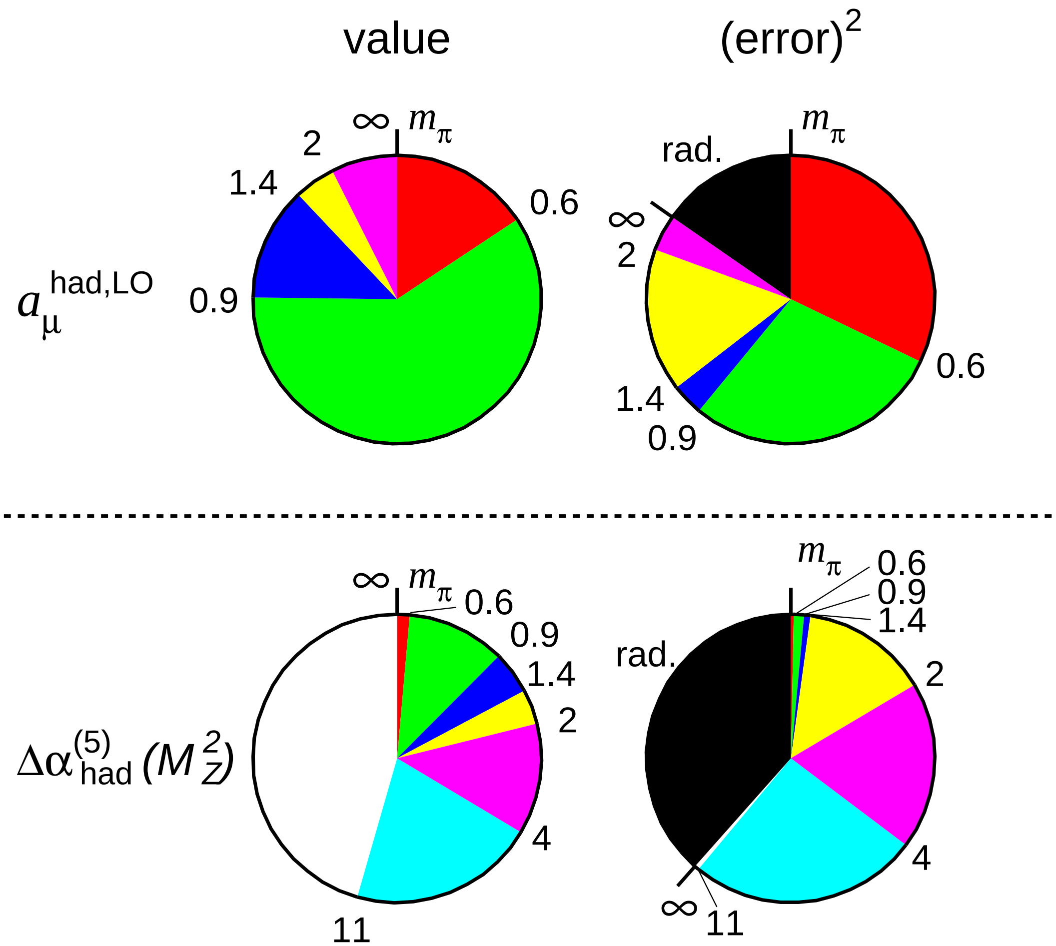
region below 2 GeV

$\rightarrow \alpha(M_Z)$:

again below 2 GeV,

intermediate energies,

with *better* radcors.



VI. Future prospects for Theory + Experiment

⋆ TH + Exp: Further improvement possible for radiative corrections and correlations.

⋆ TH *or* Exp?: Solving the τ vs. e^+e^- puzzle. ($\to$ *Radiative Return* exp. soon!)

⋆ TH + Exp. (+ Lattice ?): better determination of $a_\mu^{\mathrm{had,Light-by-Light}}$?!

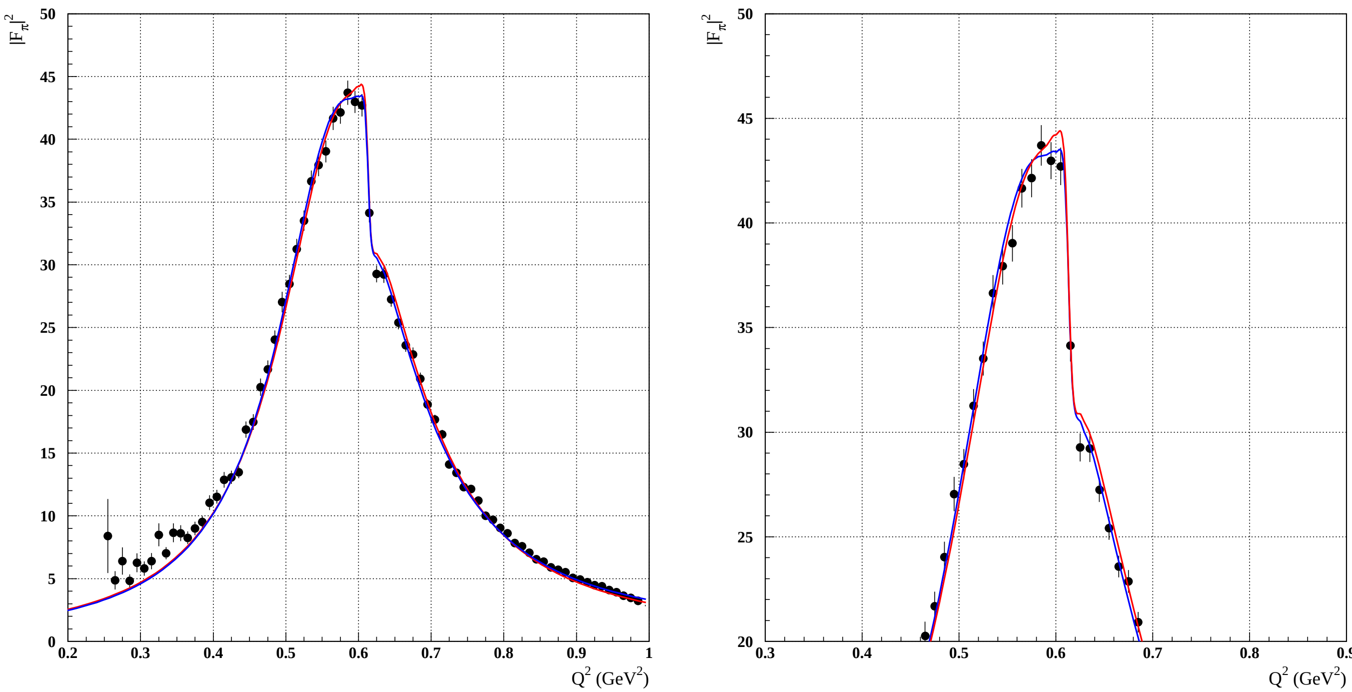
⋆ Lattice: a_μ^{had} from 'first principles'? (First results, but unquenching needed..)

• Expect more precise e^+e^- data from Novosibirsk (CMD-2 and SND), Beijing (BES II+III) and Cornell (CLEO(c)).

Prospects on R measurement at low energy (Table taken from G. Venanzoni, hep-ex/0211005)

Energy Range (GeV)	Current Error	Expected Results	Expected error
$0.3 - 0.6$	1-2%	CMD-2, VEPP-2000, KLOE, B-factories	1%?
$0.6 - 1$	0.6%	KLOE, B-factories, VEPP-2000	$< 0.6\%?$
$1 - 1.4$	5-10%	CMD-2, SND, B-factories, VEPP-2000	$< 5\%?$
$1.4 - 2$	15-20%	B-factories, VEPP-2000	5%?
$2 - 5$	7%	BEPCII, CLEO-C, B-factories	3%

♢ Radiative Return ($e^+e^- \to hadrons + \gamma$) analyses from KLOE, BABAR and BELLE at ϕ and B-factories will provide independent cross-section measurements over a wide energy region. (MonteCarlos from TH most important.)



Preliminary KLOE measurement of the pion form factor, G. Venanzoni, `hep-ex/0210013`.

♡ Future J-PARC (new high intensity proton accelerator facility under construction near KEK) could host a new $g-2$ exp. which may improve a_μ by a factor 5-10!

Is TH up to that?

▶ STAY TUNED!

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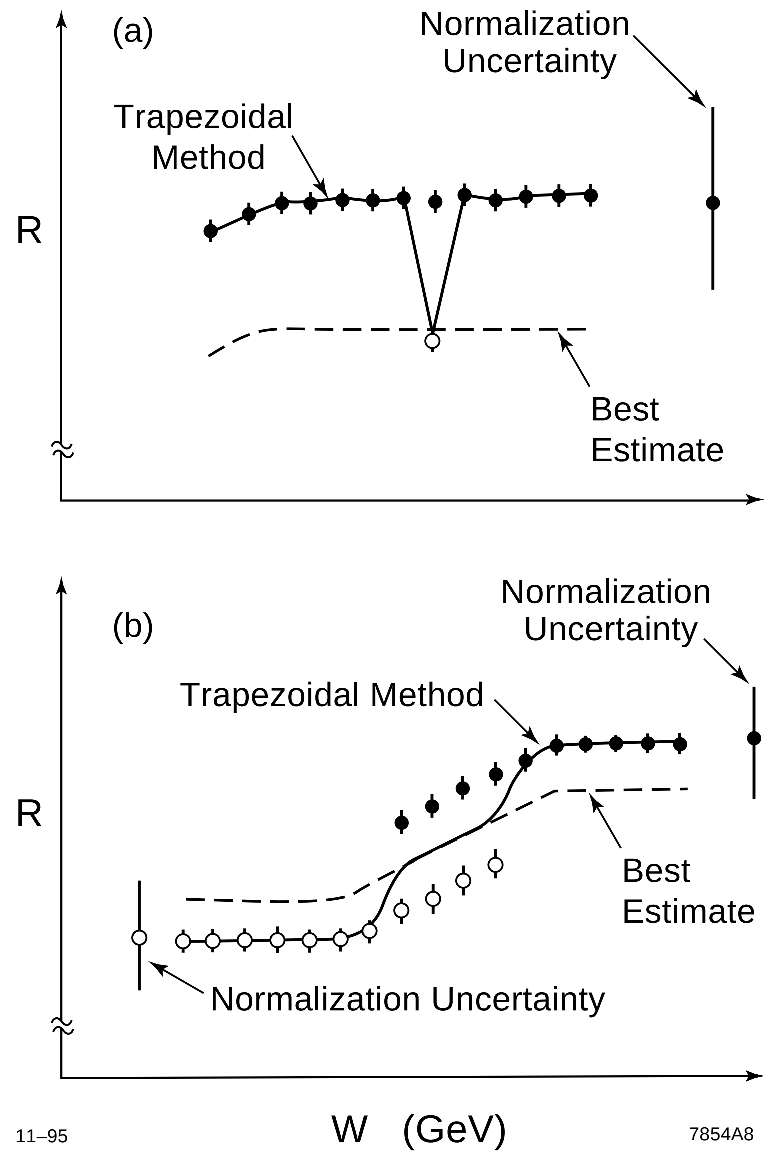
$\sum$	$(11659183.5 \pm 6.7) 10^{-10}$	with HMNT (e^+e^-)
Exp.:	$(11\,659\,208 \pm 6) \cdot 10^{-10}$	New BNL E821 world average 2004
$a_\mu^{EXP} - a_\mu^{TH}$	$(24.5 \pm 9) \cdot 10^{-10}$	$\sim 2.7\,\sigma$ with e^+e^-, $< 1\sigma$ using τ (DEHZ)

Processing the hadronic data: Clustering

- Need: combination of data from different experiments (for the same channel) with very different stat.+sys. errors and different energy ranges.
- Aims:
  - $\longrightarrow$ Make maximal use of (normalization of) precise data.
  - $\longrightarrow$ Don't suppress shape info. of older data.
  - $\longrightarrow$ Have as few theor. constraints on R as possible (like pQCD, BW resonance shapes, fit by polynomials...).
- Solution: 'A fit that's not a fit' $\rightsquigarrow$

Figure from M.L. Swartz:



- Our fit-model: piecewise constant R within a *Cluster* of a given (min.) size.
- Realization: Non-linear (numerical, iterative) χ^2-minimalization of:

$$\chi^2(\bar{R}_m, f_k) = \sum_{k=1}^{\#Exp} \left[(1 - f_k)/\mathrm{d}f_k\right]^2 + \sum_{m=1}^{\#Cl} \sum_{i=1}^{N_{\{k,m\}}} \left[\left(R_i^{\{k,m\}} - f_k \bar{R}_m\right)/\mathrm{d}R_i^{\{k,m\}}\right]^2$$

Given a binning of the R measurements $R_i^{\{k,m\}} \pm \mathrm{d}R_i^{\{k,m\}} \pm \mathrm{d}f_k \cdot R_i^{\{k,m\}}$
(from k experiments) in m clusters

the fit returns the mean values $\bar{R}_m$ (and the renormalizations f_k).

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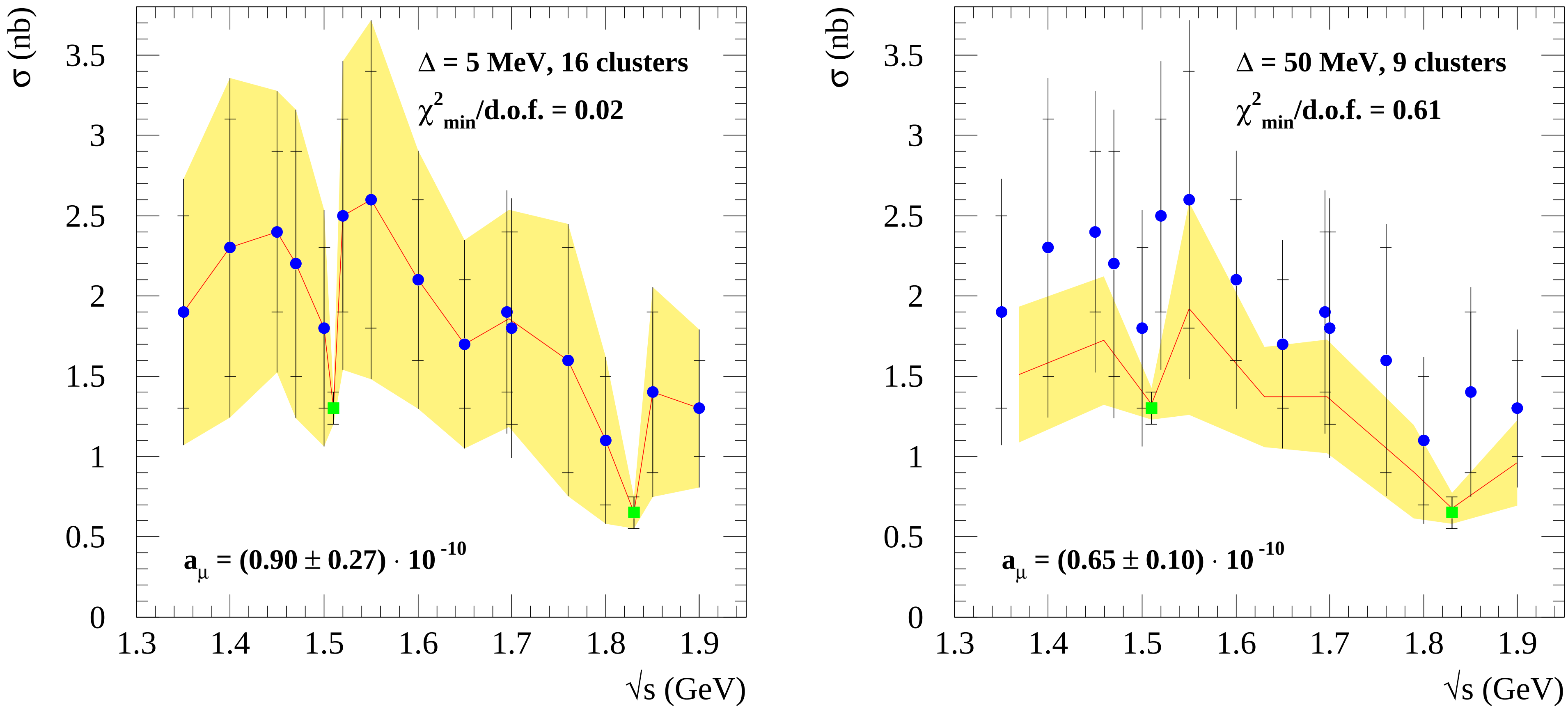
Advantages:

- All data contribute, weighted by their uncertainty^{-1}, hence no loss of information.
- Error estimate using the complete covariance matrix, taking into account statistical and systematic (p.t.p. and overall) errors from all different experiments
  $\longrightarrow$ correlations over different energies taken into account!
- Data-sets with a large (overall-) normalization uncertainty can still contribute shape information without leading to artificial pile-up of (asymmetric) fluctuations.
- Automatic check of data-consistency and fit-quality; minimal $\chi^2_{\mathrm{min}}/(d.o.f.)(\delta)$ determines choice of cluster size δ.
- Amazing stability of $a_\mu^{\mathrm{had,LO}}$ and its error.

How does it work? A few examples:

- Artificial 'demo' data for illustration:



The two green 'high quality' points have 1% sys. error, the other set 30%

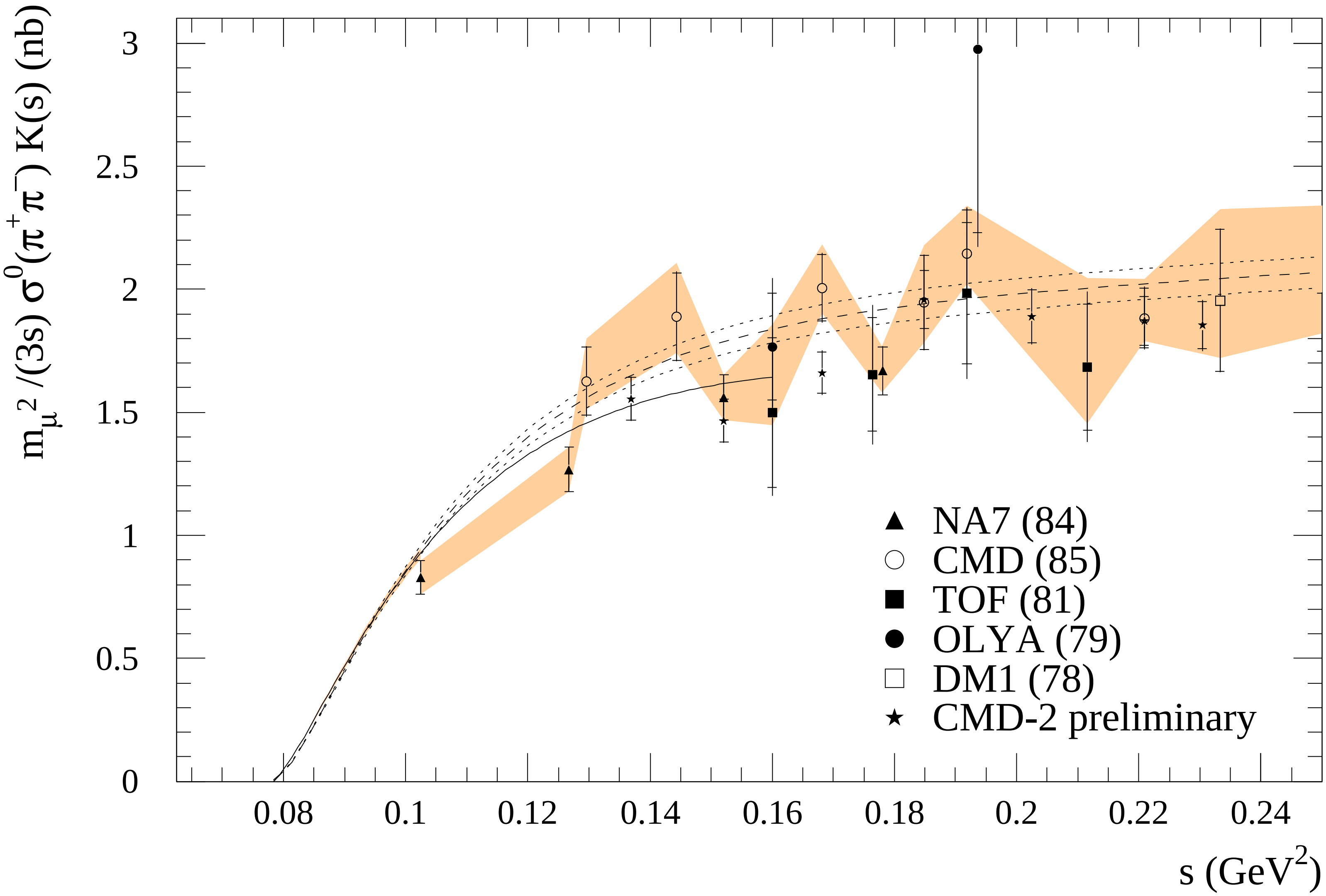
$\hookrightarrow$ 'maximal' adjustment of normalization of the latter by $1/1.35$;

$\hookrightarrow$ smaller a_μ value and error, fit still good, $\chi^2_{\rm min}/d.o.f. < 1$.

→ Data close to threshold poor, therefore use of chiral PT.

→ Important role of few points from NA7 (one of the differences DEHZ – HMNT).

→ Analysis of CMD-2 underway; expected error improvement of $a_\mu^{\rm had,LO}$ of about $1 \cdot 10^{-10}$!

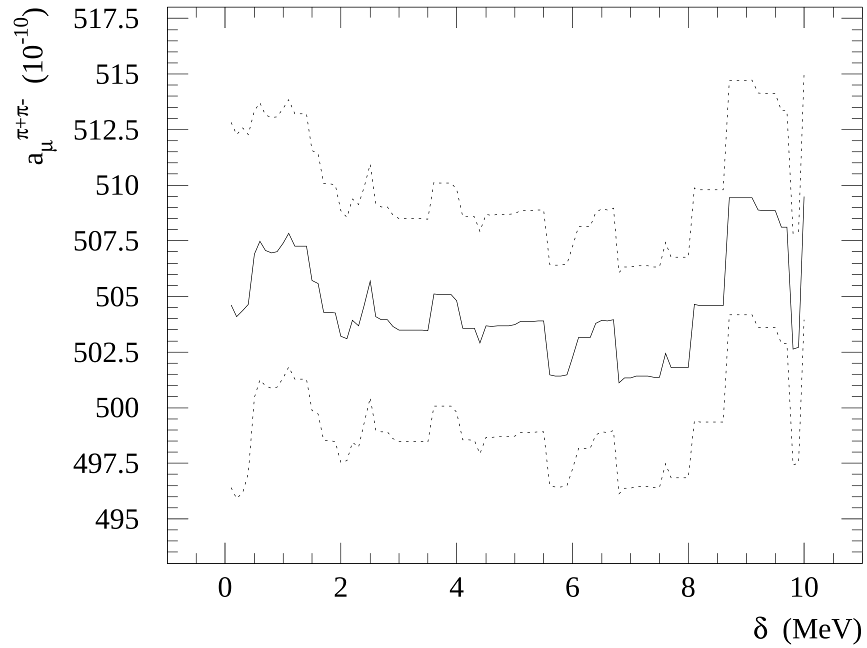


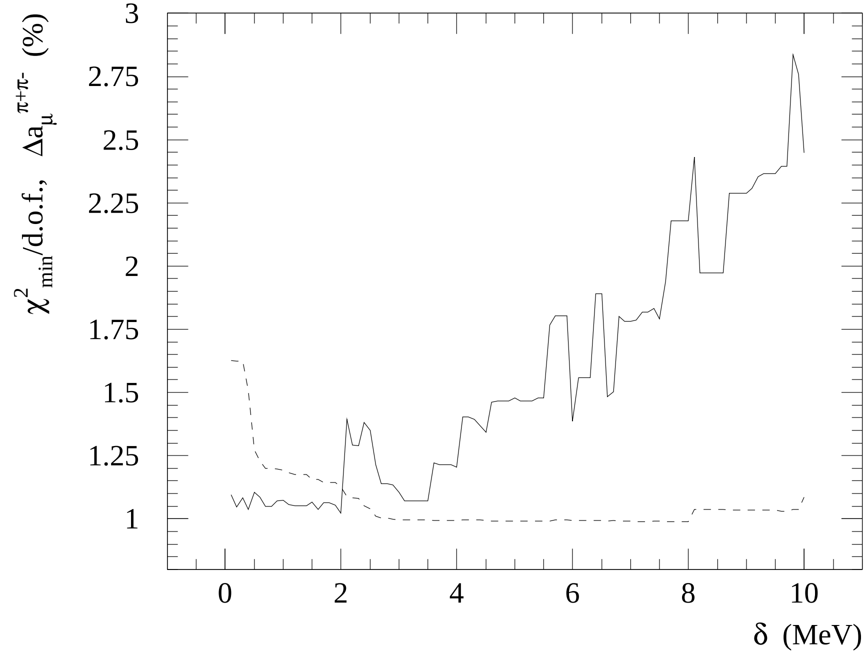
Stability of the fit for $\pi^+\pi^-$

contribution with error band to a_μ $\longrightarrow$

- a_μ and its error stable w.r.t. variation of the cluster size
- significant improvement of error through clustering with $\delta \approx 3.5$ MeV
- finer clustering would worsen error but not much improve fit quality, which is already good: $\chi^2_{\text{min}}/d.o.f. \approx 1$

solid: $\chi^2_{\text{min}}/d.o.f.$, dashed: Δa_μ in % $\rightarrow$





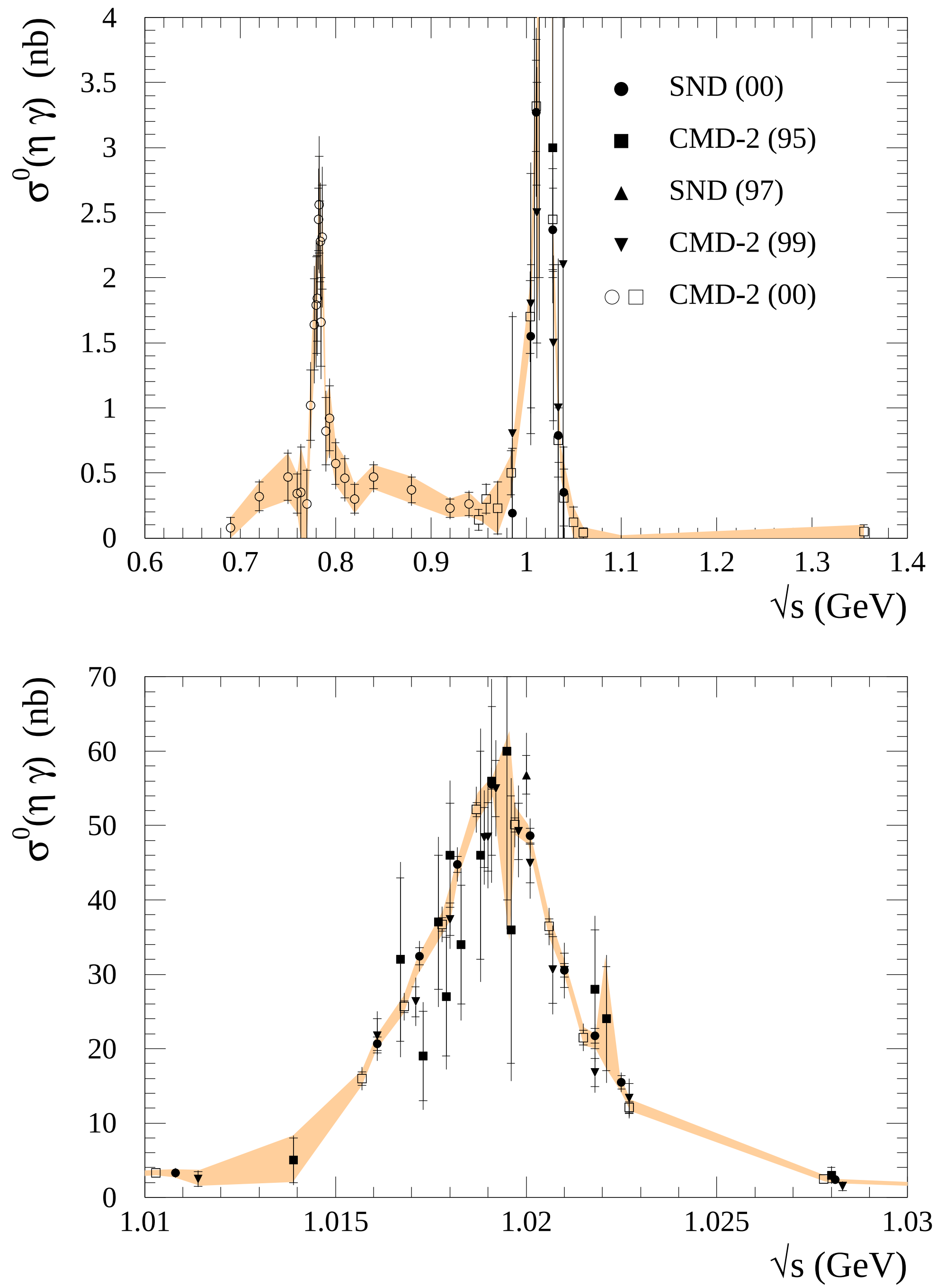
- At the edge (of significance):

  ω and ϕ in the channel $\eta\gamma$

- Cluster sizes:

  10 MeV in the continuum,

  0.2 MeV through the resonances;
- 93 data points $\longrightarrow$ 58 clusters;
- $\chi^2/(d.o.f.) = 1.02$,
- $a_\mu^{\eta\gamma} = (0.73 \pm 0.03) \cdot 10^{-10}$.



- Details of the clustering and fit for the dominant channels

channel	data range	δ (MeV)	$\chi^2/d.o.f.$	range used	a_μ	Δa_μ	w/out fit
$\pi^+\pi^-$	$0.32 - 3$	3.5	1.07	$0.32 - 1.425$	502.76	5.01	500.10
$\pi^+\pi^-\pi^0$	$0.483 - 2.4$	$20, 0.2, 0.2$	1.44	$0.66 - 1.425$	46.42	0.76	47.38
$\pi^+\pi^-\pi^+\pi^-$	$0.765 - 2.245$	11	2.00	$0.765 - 1.432$	6.18	0.23	5.70
$\pi^+\pi^-\pi^0\pi^0$	$0.915 - 2.4$	10	1.28	$0.915 - 1.438$	9.89	0.57	9.44
K^+K^-	$1.009 - 2.1$	$5, 0.6$	1.00	$1.009 - 1.421$	21.58	0.76	21.31
$K_S^0K_L^0$	$1.004 - 2.14$	$10, 0.1$	0.86	$1.004 - 1.442$	13.16	0.16	13.11
inclusive	$1.432 - 3.035$	20	0.28	$1.432 - 2.05$	32.95	2.58	31.99
	$2 - 11.09$	20	0.74	$2 - 11.09$	42.02	1.14	41.51

- Fits use data with energy ranges as indicated in the second column.
- Clustering sizes δ as displayed in the third column.

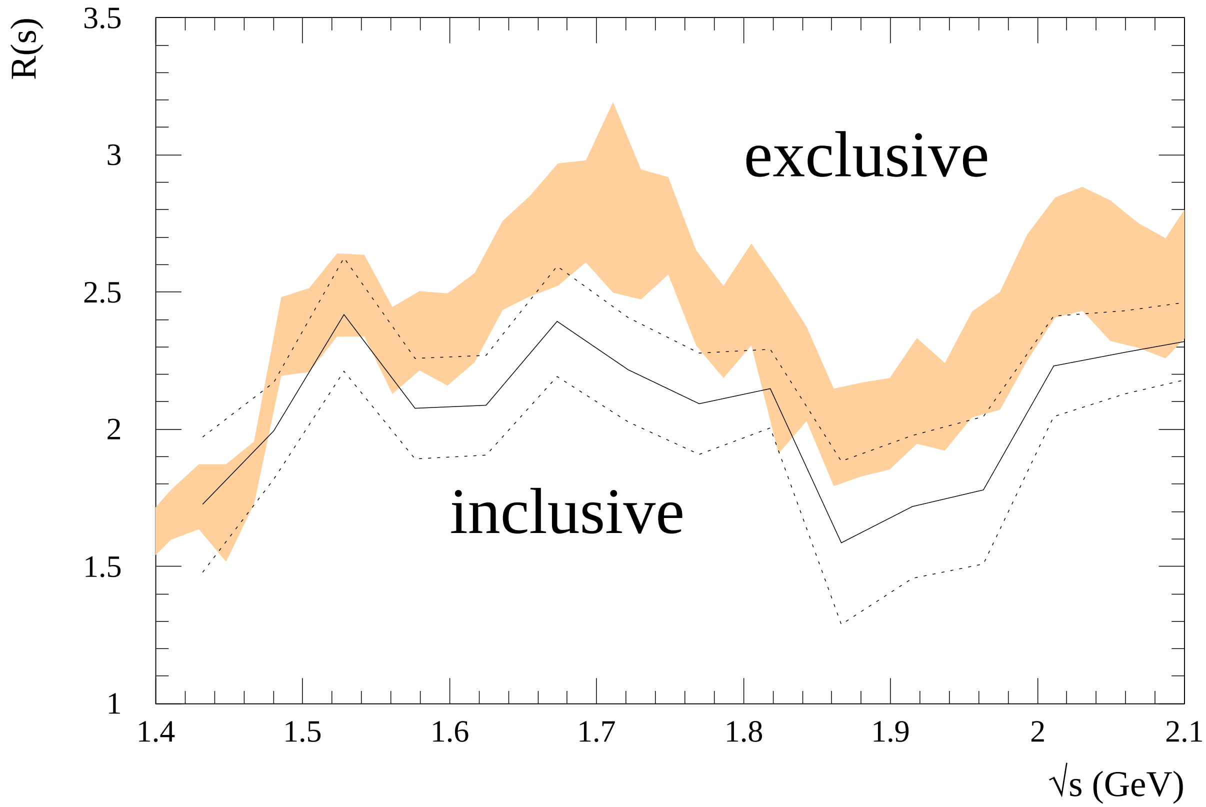
$\longrightarrow$ Very fine binning for $\pi^+\pi^-\pi^0$, K^+K^- and $K_S^0K_L^0$ in the resonance regimes of the ω and ϕ, see additional entries for δ.

$\longrightarrow$ 'Good' $\chi^2_{\min}/d.o.f. < 1.2$, otherwise error inflation.

$\longrightarrow$ Last column demonstrates the importance of *correlations*.

The puzzle exclusive vs. inclusive

Compilation of inclusive and $\sum$ over exclusive data after clustering and fitting:

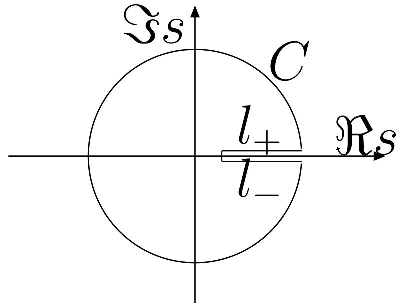


? 'Missing' channels in incl., missing (or overdone) radiative corrections (e.g. from vac.-pol. or extra γ emission) $\longrightarrow$ Our studies indicate: very unlikely at this size.

- Probably pure data-problem that will be clarified with more (and *better*) data.

Resolving the ambiguity: QCD $\sum$-rule analysis

- Evaluate QCD $\sum$-rules of the form:



$$\int_{s_{\rm th}}^{s_0} \mathrm{d}s\, R(s) f(s) = \int_C \mathrm{d}s\, D(s) g(s)\,, \qquad \text{with} \quad D(s) \equiv -12\pi^2 s \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\Pi(s)}{s}\right)$$

- The Adler D function is calculable in pQCD: $D(s) = D_0(s) + D_{\rm m}(s) + D_{\rm np}(s)$
- Take $f(s) = (1 - s/s_0)^m (s/s_0)^n$ to maximize sensitivity to the required region, $g(s)$ follows.
- Choose s_0 below the open charm threshold ($n_f = 3$ for pQCD).
- For $m = 1$, $n = 0$ one gets e.g.

$$\int_{s_{\rm th}}^{s_0} \mathrm{d}s\, R(s)\left(1 - \frac{s}{s_0}\right) = \frac{i}{2\pi}\int_C \mathrm{d}s\left(-\frac{s}{2s_0} + 1 - \frac{s_0}{2s}\right) D(s)\,.$$

Results for two typical $\sum$-rules:

($\sqrt{s_0} = 3.7$ GeV)

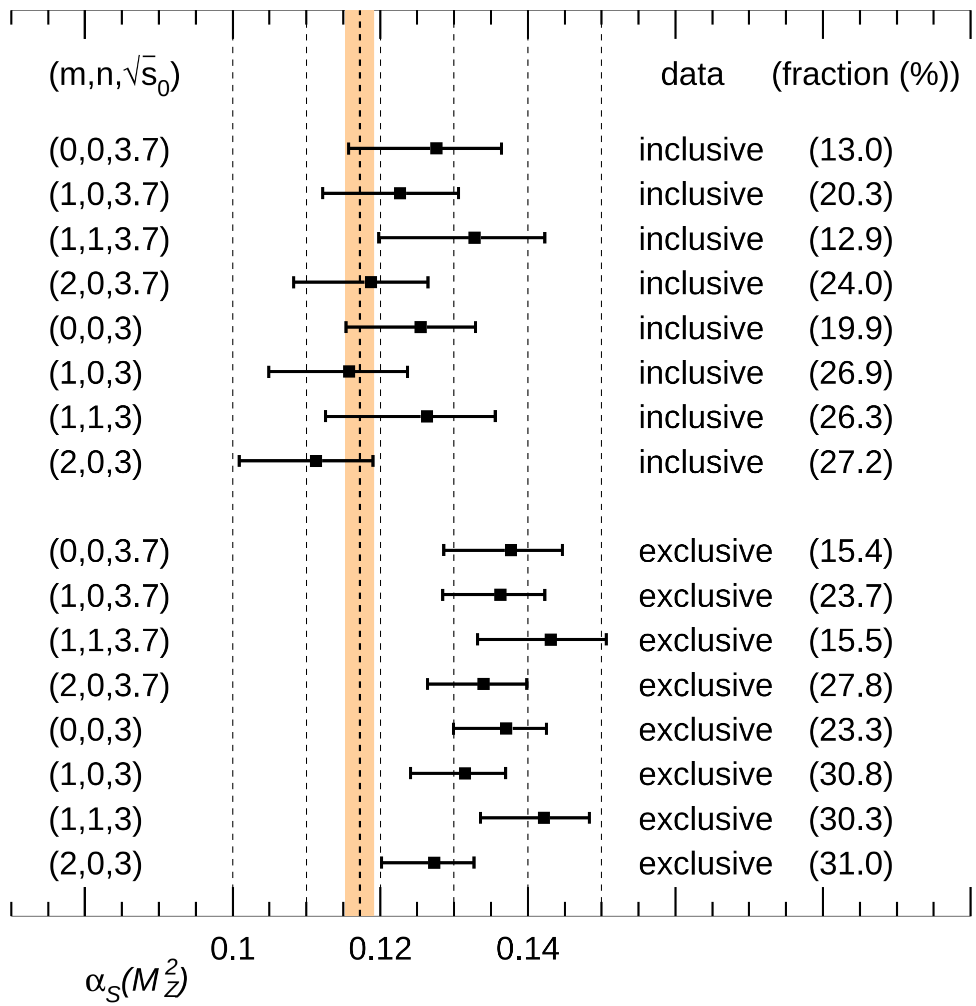
sum rule	l.h.s. (data)	r.h.s. (QCD)
$m = 2, n = 0$	10.34 ± 0.24 (incl) 10.87 ± 0.30 (excl)	10.30 ± 0.06
$m = 0, n = 0$	31.08 ± 0.76 (incl) 31.98 ± 0.82 (excl)	30.40 ± 0.12

- The main QCD error comes from $\alpha_S(M_Z^2) = 0.117 \pm 0.002$.

▶ Inclusive is selected,

× exclusive not consistent.

Sum rules 'determining' α_S:



Similar conclusion independently in earlier analysis of $\Delta\alpha^{\text{had}}$ $\longrightarrow$

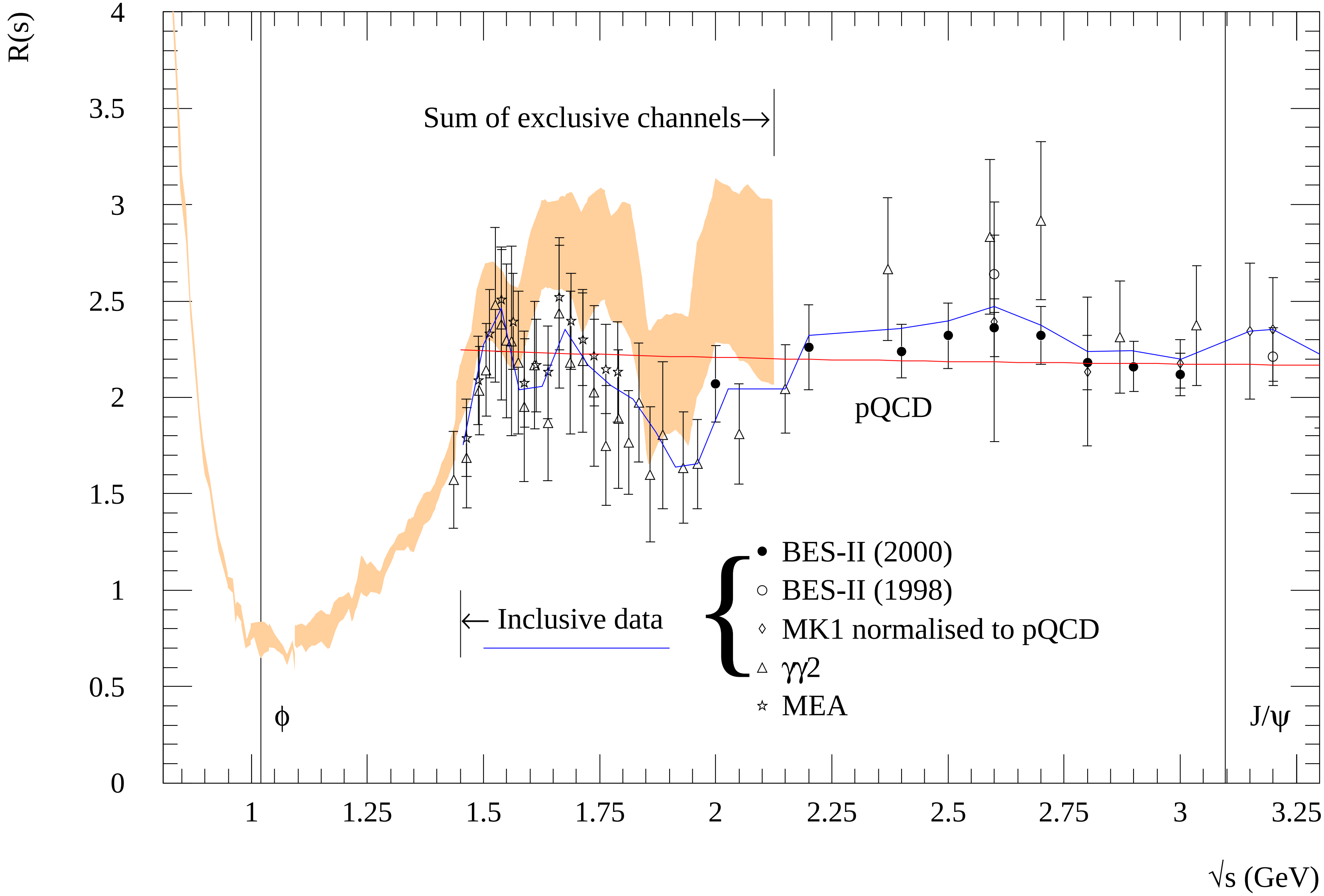


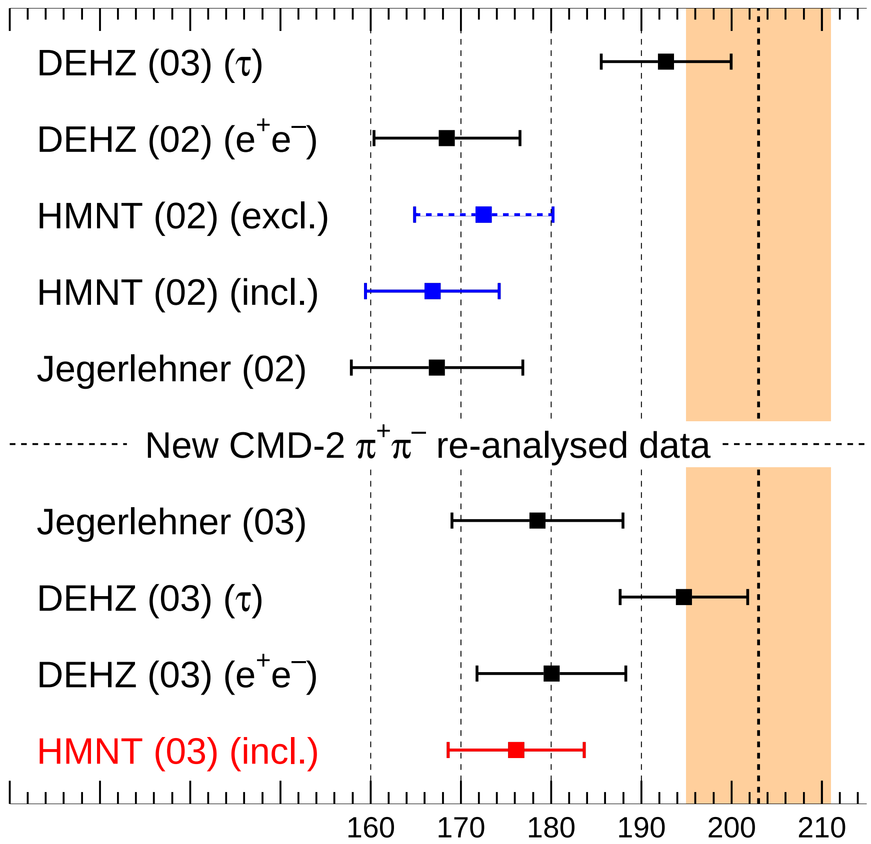
Figure taken from Martin+Outhwaite+Ryskin, EPJC19(2001)681.

Features of the Hagiwara + Martin + Nomura + T. analysis:

- e^+e^- data driven. Very complete data collection ($\sim$ 100 data references).
- Important improvements through recent data from Beijing (BES, inclusive) and Novosibirsk (CMD-2 and SND, exclusive channels).
- Recent re-analyis from CMD-2 moves a_μ slightly upwards (again)!

  Who will check their most important $\pi^+\pi^-$ data at the 0.6% (!) level?
- Comprehensive analysis of radiative corrections.
- Careful treatment of threshold regions and channels with poor or lacking data ($\longrightarrow$ use of isospin relations).
- Data 'clustering' using non-linear χ^2 fit makes best use of stat. and sys. error information.
- Straightforward $\int$ over combined data for all energies (even over ω and ϕ resonances).
- Correlations between different energies.
- not used: τ spectral functions ($\tau \longrightarrow \nu_\tau + hadrons$) ...

$a_\mu^{\rm SM}$ compared to the BNL 02 world av.



Th. and Exp. accuracy comparable!

Is soon Light-by-Light limiting?

Summary a_μ (up to $\sim$ 2003):

- τ 'puzzle'? Stay with e^+e^-, as also..
- first KLOE results from Radiative Return for the ρ region agree with CMD-2 (not τ).
- New results from a CMD-2 re-analysis move a_μ slightly upwards;

$\rightsquigarrow$ a discrepancy of $2.4\,\sigma$ remains!
Not much space for 'physics beyond'!

- Recent e^+e^- based evaluations agree well (though details vary).
- More data (and theoretical work) is needed (and expected) to squeeze the error!