

April 2004

XII International Workshop on  
Deep Inelastic Scattering  
DIS 2004 – Slovakia

# “W Boson Production at Large Transverse Momentum”

**Agustín Sabio Vera**  
(University of Hamburg)

In collaboration with **Nikolaos Kidonakis**  
(University of Cambridge)

Results published in:  
**J. High Energy Phys. JHEP 02 (2004) 027**  
**hep-ph/0311266**

## Main Idea:

- To study the production of  $W$  bosons at large transverse momentum in  $p\bar{p}$  collisions
- The exact NLO differential cross sections predictions can be improved if higher order **threshold soft-gluon corrections** are taken into account.

## Results:

- The transverse momentum distribution of the  $W$  at the Tevatron is slightly enhanced and the factorisation and renormalisation scale dependence is reduced.

## Motivation:

- $W$  boson production in hadron colliders as a test of the Standard Model and to estimate backgrounds to new physics.

(e.g.  $Wb\bar{b}$  production as background

to the associated Higgs boson production

$p\bar{p} \rightarrow H(\rightarrow b\bar{b})W$  at the Tevatron

Ellis–Veseli(1999))

## Background:

- The NLO cross section for  $W$  hadroproduction at large transverse momentum done by Arnold–Reno(1989) Gonsalves–Pawlowski–Wai(1989).

## Threshold Resummation:

Near partonic threshold limited phase space for real gluon emission. Large logarithms arising from incomplete cancellations between real emission and virtual graphs.

Threshold corrections, in the eikonal approximation, exponentiate as a result of the factorization properties of the cross section .

Kidonakis–Sterman(1996) Kidonakis–Oderda–Sterman (1998)

Laenen–Oderda–Sterman(1998)

The cross section is factorized into functions that describe gluons collinear to the incoming partons, hard quanta, and noncollinear soft-gluons.

The renormalization group properties of these functions result in resummation. Threshold corrections have been resummed for many processes.

## **In this talk:**

Phenomenological analysis of the results in Kidonakis–Del Duca (2000) where threshold logarithms were resummed for electroweak boson hadroproduction at NNLL accuracy.

NNLO soft–gluon corrections for  $W$ -boson production at large transverse momentum are studied.

It is possible to go beyond NNLL as shown in N. Kidonakis (2003).

## Kinematics and notation:

Hadronic production of  $W$  boson with  $m_W$ :

$$h_A(P_A) + h_B(P_B) \longrightarrow W(Q) + X$$

Factorisation:

$$E_Q \frac{d\sigma_{h_A h_B \rightarrow W(Q) + X}}{d^3Q} = \sum_f \int dx_1 dx_2 \phi_{f_a/h_A}(x_1, \mu_F^2) \phi_{f_b/h_B}(x_2, \mu_F^2) \\ \times E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow W(Q) + X}}{d^3Q}(s, t, u, Q, \mu_F, \alpha_s(\mu_R^2))$$

$$E_Q = Q^0$$

$\phi_{f/h}$ : parton distribution

$\hat{\sigma}$ : perturbative parton-level cross section

Initial-state collinear singularities

factorized into  $\phi_{f/h}$  at factorization scale  $\mu_F$

$\mu_R$ : renormalization scale.

Parton level, lowest-order subprocesses:

$$\begin{aligned} q(p_a) + g(p_b) &\longrightarrow W(Q) + q(p_c), \\ q(p_a) + \bar{q}(p_b) &\longrightarrow W(Q) + g(p_c) \end{aligned}$$

Hadronic & partonic kinematical invariants:

$$\begin{aligned} S &= (P_A + P_B)^2 \\ T &= (P_A - Q)^2 \\ U &= (P_B - Q)^2 \\ S_2 &= S + T + U - Q^2 \end{aligned}$$

$$\begin{aligned} s &= (p_a + p_b)^2 \\ t &= (p_a - Q)^2 \\ u &= (p_b - Q)^2 \\ s_2 &= s + t + u - Q^2 \end{aligned}$$

$S_2, s_2$ : invariant mass recoiling against the  $W$ .

$s_2 = (p_a + p_b - Q)^2$ : inelasticity parton scatterg.  
 $s_2 = 0$  for one-parton production.

$p_a = x_1 P_A$  and  $p_b = x_2 P_B$ , therefore relations:

$$\begin{aligned} s &= x_1 x_2 S \\ t - Q^2 &= x_1 (T - Q^2) \\ u - Q^2 &= x_2 (U - Q^2) \end{aligned} \tag{1}$$

Partonic  $\hat{\sigma}$  includes  $+$  distributions at  $\alpha_s^n$  remnants of cancellations between real and virtual contributions:

$$\left[ \frac{\ln^m(s_2/Q_T^2)}{s_2} \right]_+, \quad m \leq 2n - 1$$

defined by:

$$\int_0^{s_{2max}} ds_2 f(s_2) \left[ \frac{\ln^m(s_2/Q_T^2)}{s_2} \right]_+ \equiv \int_0^{s_{2max}} ds_2 \frac{\ln^m(s_2/Q_T^2)}{s_2} (f(s_2) - f(0)) + \frac{1}{m+1} \ln^{m+1} \left( \frac{s_{2max}}{Q_T^2} \right) f(0)$$

In this work use of  $Q_T$  instead of  $Q$ .

Terminology at order  $\alpha_s^n$ :

$$\text{LL} \rightarrow m = 2n - 1$$

$$\text{NLL} \rightarrow m = 2n - 2$$

$$\text{NNLL} \rightarrow m = 2n - 3$$

$$\text{NNNLL} \rightarrow m = 2n - 4$$

- $qg \longrightarrow Wq$  **subprocess:**  $\overline{\text{MS}}$  scheme.

Born differential cross section:

$$E_Q \frac{d\sigma_{qg \rightarrow Wq}^B}{d^3Q} = F_{qg \rightarrow Wq}^B \delta(s_2)$$

where

$$F_{qg \rightarrow Wq}^B = \frac{\alpha \alpha_s(\mu_R^2) C_F}{s(N_c^2 - 1)} A^{qg} \sum_f |L_{ff_a}|^2$$

$$A^{qg} = - \left( \frac{s}{t} + \frac{t}{s} + \frac{2uQ^2}{st} \right)$$

$L$ : left-handed couplings of  $W$  to quarks  
 $\sum_f$ : flavour sum allowed by CKM mixing & energy threshold.

NLO soft + virtual corrections:

$$E_Q \frac{d\hat{\sigma}_{qg \rightarrow Wq}^{(1)}}{d^3Q} = F_{qg \rightarrow Wq}^B \frac{\alpha_s(\mu_R^2)}{\pi} \left\{ c_3^{qg} \left[ \frac{\ln(s_2/Q_T^2)}{s_2} \right]_+ + c_2^{qg} \left[ \frac{1}{s_2} \right]_+ + c_1^{qg} \delta(s_2) \right\}$$

$$c_3^{qg} = C_F + 2C_A$$

$$c_2^{qg} = -(C_F + C_A) \ln \left( \frac{\mu_F^2}{Q_T^2} \right) - \frac{3}{4}C_F - C_A \ln \left( \frac{tu}{sQ_T^2} \right)$$

$$c_1^{qg} = \frac{1}{2A^{qg}} [B_1^{qg} + B_2^{qg} n_f + C_1^{qg} + C_2^{qg} n_f] + \frac{c_3^{qg}}{2} \ln^2 \left( \frac{Q_T^2}{Q^2} \right) + c_2^{qg} \ln \left( \frac{Q_T^2}{Q^2} \right)$$

NNLO soft + virtual:  $E_Q \frac{d\hat{\sigma}_{gg \rightarrow Wq}^{(2)}}{d^3Q} = F_{gg \rightarrow Wq}^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \hat{\sigma}'_{gg \rightarrow Wq}^{(2)}$

$$\begin{aligned}
\hat{\sigma}'_{gg \rightarrow Wq}^{(2)} &= \frac{1}{2}(c_3^{gg})^2 \left[ \frac{\ln^3(s_2/Q_T^2)}{s_2} \right]_+ + \left[ \frac{3}{2}c_3^{gg} c_2^{gg} - \frac{\beta_0}{4}c_3^{gg} + C_F \frac{\beta_0}{8} \right] \left[ \frac{\ln^2(s_2/Q_T^2)}{s_2} \right]_+ \\
&+ \left\{ c_3^{gg} c_1^{gg} + (c_2^{gg})^2 - \zeta_2 (c_3^{gg})^2 - \frac{\beta_0}{2} T_2^{gg} + \frac{\beta_0}{4} c_3^{gg} \ln \left( \frac{\mu_R^2}{s} \right) + (C_F + C_A) K \right. \\
&\quad \left. + C_F \left[ -\frac{K}{2} + \frac{\beta_0}{4} \ln \left( \frac{Q_T^2}{s} \right) \right] - \frac{3}{16} \beta_0 C_F \right\} \left[ \frac{\ln(s_2/Q_T^2)}{s_2} \right]_+ \\
&+ \left\{ c_2^{gg} c_1^{gg} - \zeta_2 c_2^{gg} c_3^{gg} + \zeta_3 (c_3^{gg})^2 - \frac{\beta_0}{2} T_1^{gg} + \frac{\beta_0}{4} c_2^{gg} \ln \left( \frac{\mu_R^2}{s} \right) + \mathcal{G}_{gg}^{(2)} \right. \\
&\quad \left. + (C_F + C_A) \left[ \frac{\beta_0}{8} \ln^2 \left( \frac{\mu_F^2}{s} \right) - \frac{K}{2} \ln \left( \frac{\mu_F^2}{s} \right) \right] - C_F K \ln \left( \frac{-u}{Q_T^2} \right) - C_A K \ln \left( \frac{-t}{Q_T^2} \right) \right. \\
&\quad \left. + C_F \left[ \frac{\beta_0}{8} \ln^2 \left( \frac{Q_T^2}{s} \right) - \frac{K}{2} \ln \left( \frac{Q_T^2}{s} \right) \right] - \frac{3}{16} \beta_0 C_F \ln \left( \frac{Q_T^2}{s} \right) \right\} \left[ \frac{1}{s_2} \right]_+ \\
&+ \left\{ \frac{1}{2}(c_{1\mu}^{'gg})^2 + c_{1\mu}^{'gg} T_1^{'gg} + \frac{\beta_0}{4} c_{1\mu}^{'gg} \ln \left( \frac{Q_T^2}{s} \right) + \frac{\beta_0}{4} c_1^{gg} \ln \left( \frac{\mu_R^2}{Q_T^2} \right) - (C_F + C_A)^2 \frac{\zeta_2}{2} \ln^2 \left( \frac{\mu_F^2}{Q_T^2} \right) \right. \\
&\quad \left. + (C_F + C_A) \ln \left( \frac{\mu_F^2}{Q_T^2} \right) \left( \zeta_2 T_2^{'gg} - \zeta_2 (C_F + C_A) \ln \left( \frac{Q_T^2}{s} \right) - \zeta_3 c_3^{gg} \right) \right. \\
&\quad \left. - \frac{\beta_0^2}{32} \ln^2 \left( \frac{\mu_R^2}{Q_T^2} \right) - \frac{\beta_0^2}{16} \ln \left( \frac{\mu_R^2}{Q_T^2} \right) \ln \left( \frac{Q_T^2}{s} \right) + \frac{\beta_1}{16} \ln \left( \frac{\mu_R^2}{Q_T^2} \right) \right. \\
&\quad \left. + \frac{\beta_0}{8} \left[ \frac{3}{4} C_F + \frac{\beta_0}{4} - C_F \ln \left( \frac{-u}{Q_T^2} \right) - C_A \ln \left( \frac{-t}{Q_T^2} \right) \right] \left[ \ln^2 \left( \frac{\mu_F^2}{Q_T^2} \right) + 2 \ln \left( \frac{\mu_F^2}{Q_T^2} \right) \ln \left( \frac{Q_T^2}{s} \right) \right] \right. \\
&\quad \left. + C_F \frac{K}{2} \ln \left( \frac{-u}{Q_T^2} \right) \ln \left( \frac{\mu_F^2}{Q_T^2} \right) + C_A \frac{K}{2} \ln \left( \frac{-t}{Q_T^2} \right) \ln \left( \frac{\mu_F^2}{Q_T^2} \right) - (\gamma'_{q/q}{}^{(2)} + \gamma'_{g/g}{}^{(2)}) \ln \left( \frac{\mu_F^2}{Q_T^2} \right) \right. \\
&\quad \left. + R_{gg}^{(2)} \right\} \delta(s_2)
\end{aligned}$$



$$K = C_A(67/18 - \zeta_2) - 5n_f/9 \quad \text{Kodaira-Trentadue (1982)}$$

$$\beta_1 = 34C_A^2/3 - 2n_f(C_F + 5C_A/3)$$

$$\gamma'_{q/q}^{(2)} = C_F^2 \left( \frac{3}{32} - \frac{3}{4}\zeta_2 + \frac{3}{2}\zeta_3 \right) + C_F C_A \left( -\frac{3}{4}\zeta_3 + \frac{11}{12}\zeta_2 + \frac{17}{96} \right) + n_f C_F \left( -\frac{\zeta_2}{6} - \frac{1}{48} \right)$$

$$\gamma'_{g/g}^{(2)} = C_A^2 \left( \frac{2}{3} + \frac{3}{4}\zeta_3 \right) - n_f \left( \frac{C_F}{8} + \frac{C_A}{6} \right)$$

## Two-loop parton anomalous dimensions

Gonzalez-Arroyo-Lopez-Yndurain (1979)

Curci-Furmanski-Petronzio(1980)

$\mathcal{G}_{qg}^{(2)}$  in the NNNLL term:

A set of two-loop contributions

$$\begin{aligned} \mathcal{G}_{qg}^{(2)} &= C_F^2 \left( -\frac{3}{32} + \frac{3}{4}\zeta_2 - \frac{3}{2}\zeta_3 \right) + C_F C_A \left( \frac{3}{4}\zeta_3 - \frac{11}{12}\zeta_2 - \frac{189}{32} \right) \\ &+ C_A^2 \left( \frac{7}{4}\zeta_3 + \frac{11}{3}\zeta_2 - \frac{41}{216} \right) + n_f C_F \left( \frac{1}{6}\zeta_2 + \frac{17}{16} \right) + n_f C_A \left( -\frac{2}{3}\zeta_2 - \frac{5}{108} \right) \end{aligned}$$

No two-loop process-dependent contributions  
In top hadroproduction & direct photon production small.

$$-\zeta_2 c_2^{qg} c_3^{qg} + \zeta_3 (c_3^{qg})^2 \text{ dominates.}$$

In  $\delta(s_2)$  not full virtual corrections

No  $R_{qg}^{(2)}$ : scale-independent virtual corrections

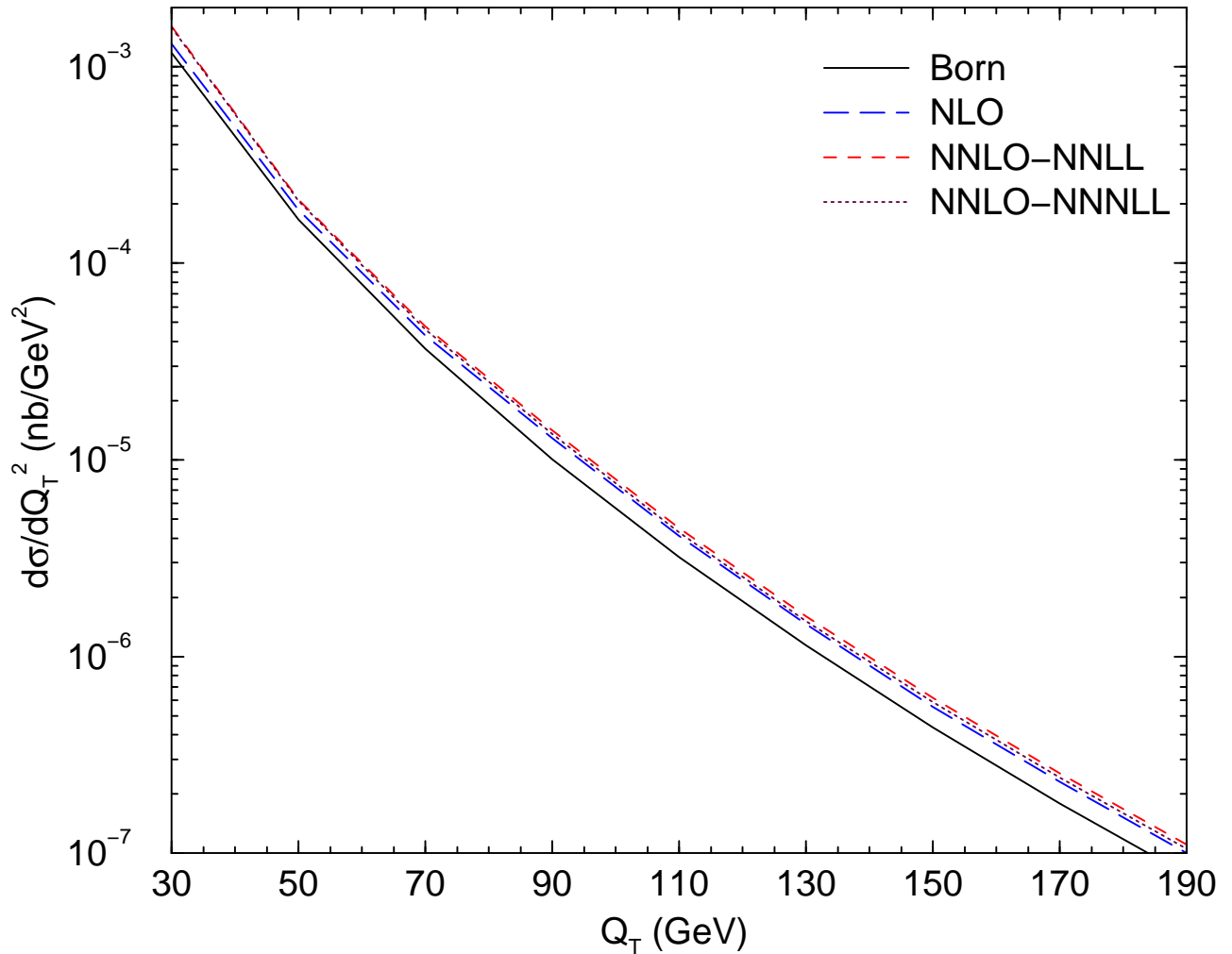
All scale-dependent terms are included.

- $q\bar{q} \rightarrow Wg$  **subprocess:**  
Similar expressions.

- **Numerical results:**

# $p\bar{p} \rightarrow W$

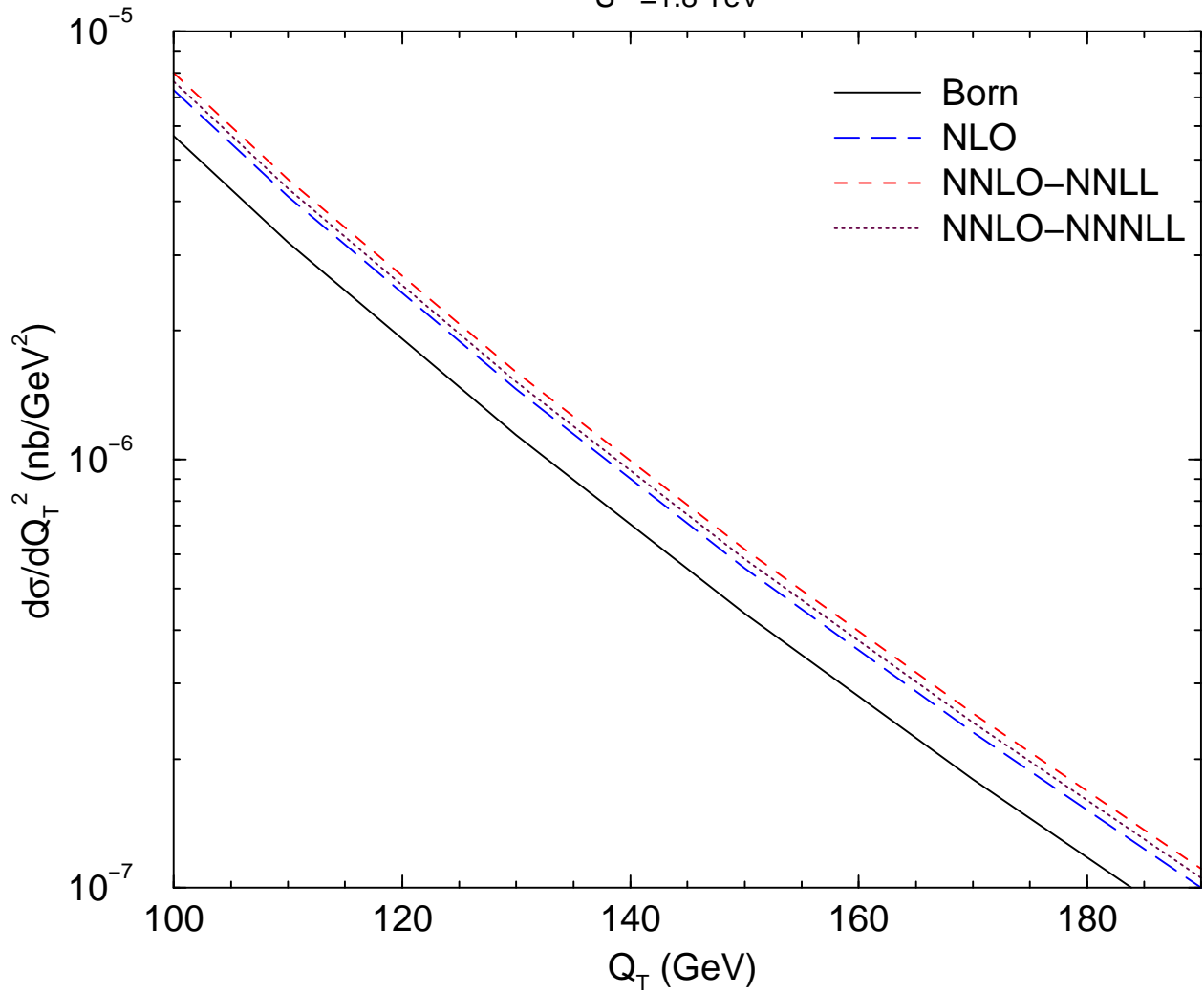
$\sqrt{S} = 1.8 \text{ TeV}$



The differential cross section,  $d\sigma/dQ_T^2$ , for  $W$  hadroproduction in  $p\bar{p}$  collisions at the Tevatron with  $\sqrt{S} = 1.8 \text{ TeV}$  and  $\mu_F = \mu_R = Q_T$ . Shown are the Born (solid line), NLO (long-dashed line), NNLO-NNLL (short-dashed line), and NNLO-NNLL (dotted line) results.

# $p\bar{p} \rightarrow W$

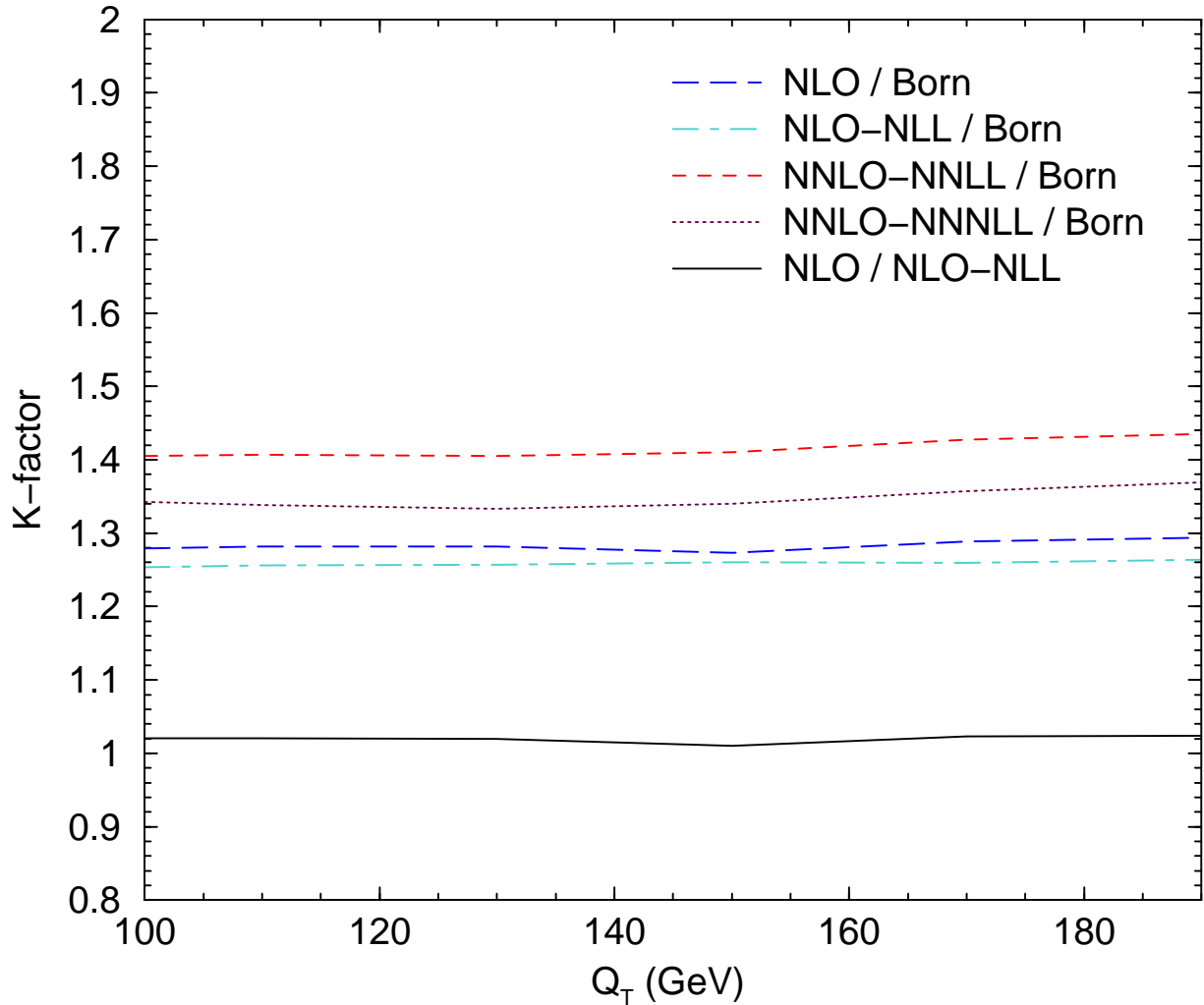
$S^{1/2} = 1.8 \text{ TeV}$



The differential cross section,  $d\sigma/dQ_T^2$  at high  $Q_T$

# $p\bar{p} \rightarrow W$ K-factors

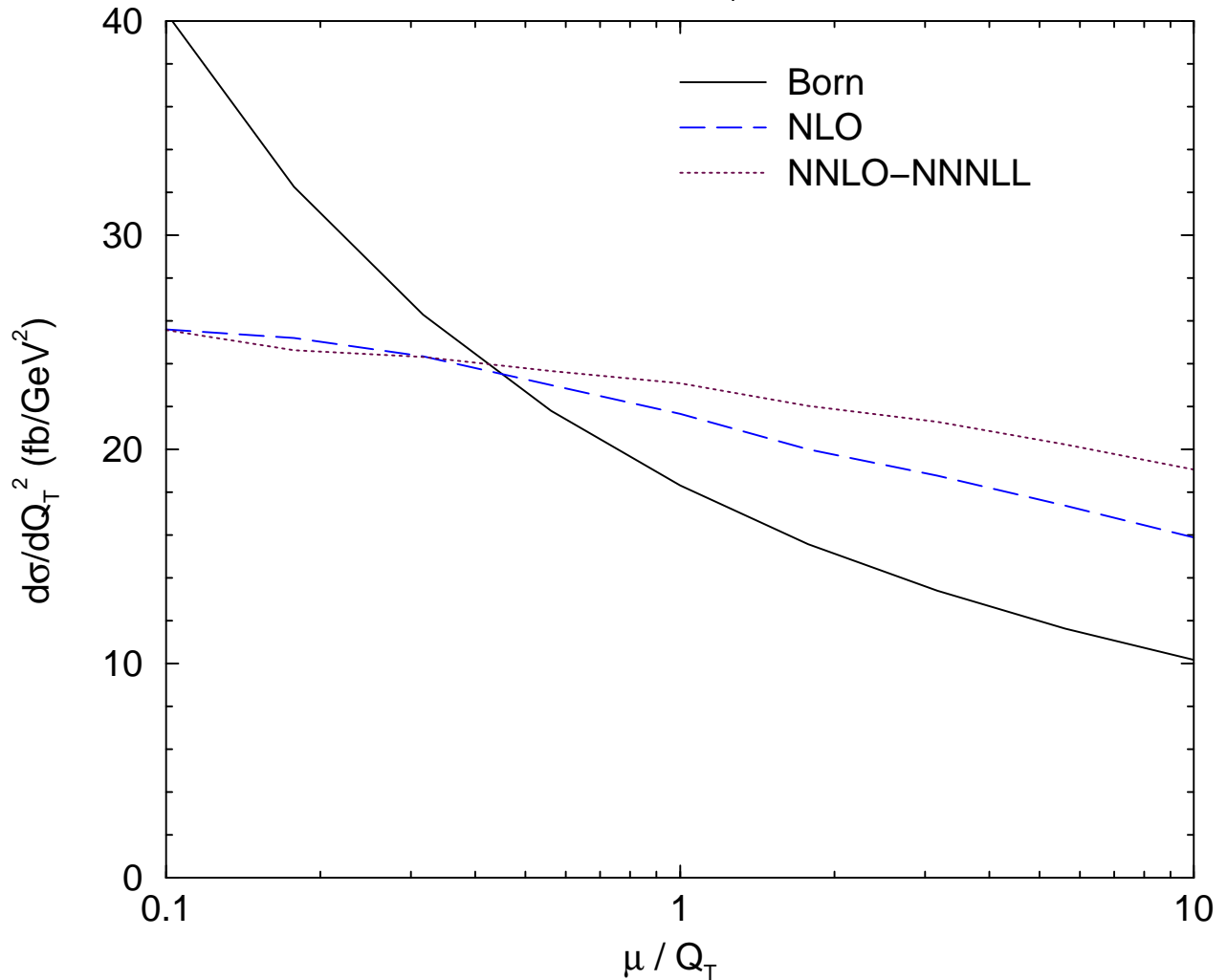
$S^{1/2} = 1.8 \text{ TeV}$



$K$ -factors for  $d\sigma/dQ_T^2$ , for  $W$  hadroproduction with  $\sqrt{S} = 1.8 \text{ TeV}$  and  $\mu_F = \mu_R = Q_T$ . Shown are the  $K$ - factors for exact NLO/Born (long-dashed line), NLO-NLL/Born (dash-dotted line), NNLO-NNLL/Born (short-dashed line), and approximate NNLO-NNLL/Born (dotted line) results. Also shown is the ratio of the exact NLO to the NLO-NLL cross section (solid line).

# $p\bar{p} \rightarrow W$

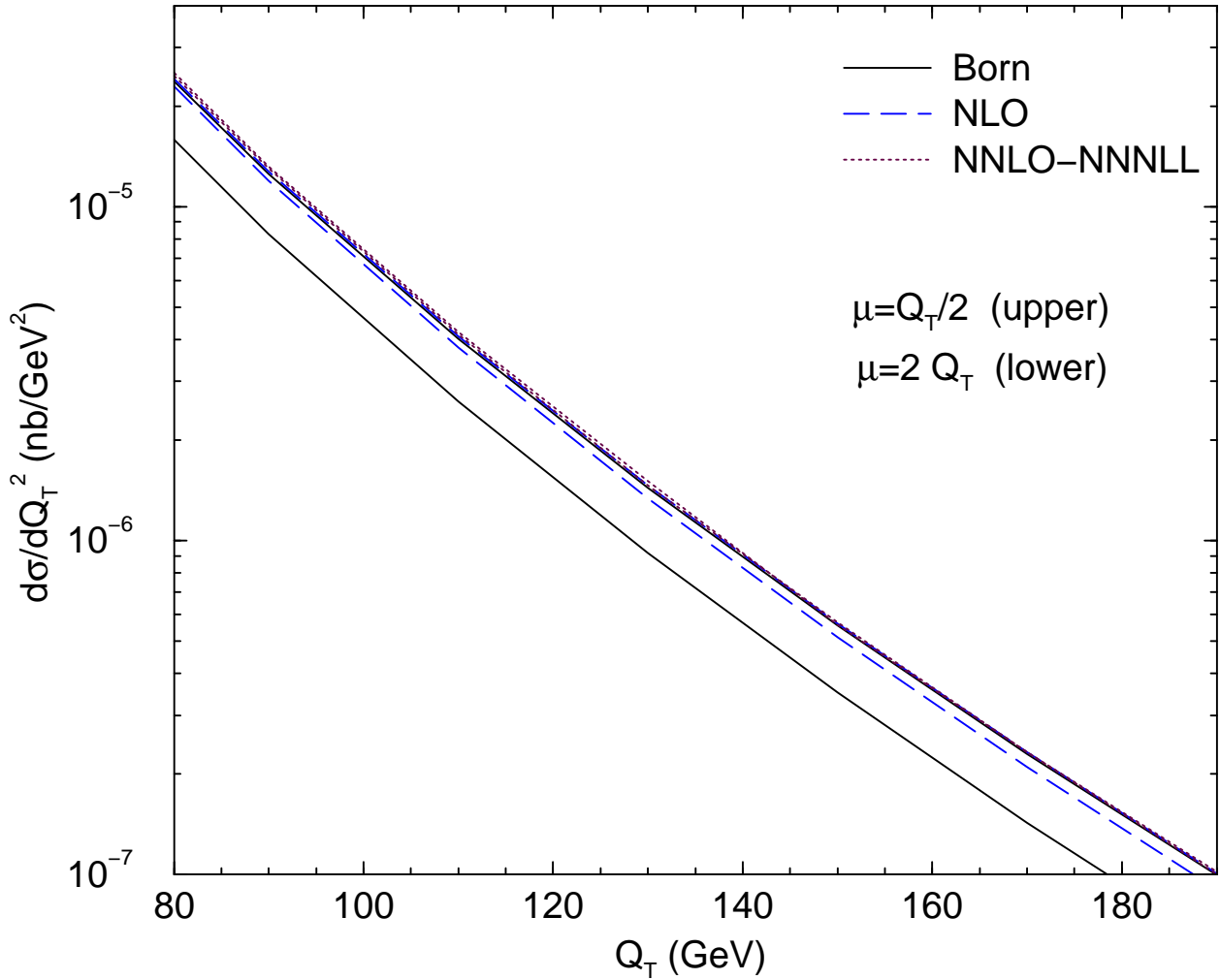
$S^{1/2} = 1.8 \text{ TeV}$     $Q_T = 80 \text{ GeV}$



The differential cross section,  $d\sigma/dQ_T^2$ , for  $W$  hadroproduction in  $p\bar{p}$  collisions at the Tevatron with  $\sqrt{S} = 1.8 \text{ TeV}$ ,  $Q_T = 80 \text{ GeV}$ , and  $\mu \equiv \mu_F = \mu_R$ . Shown are the Born (solid line), exact NLO (long-dashed line), and NNLO-NNLL (dotted line) results.

# $p\bar{p} \rightarrow W$

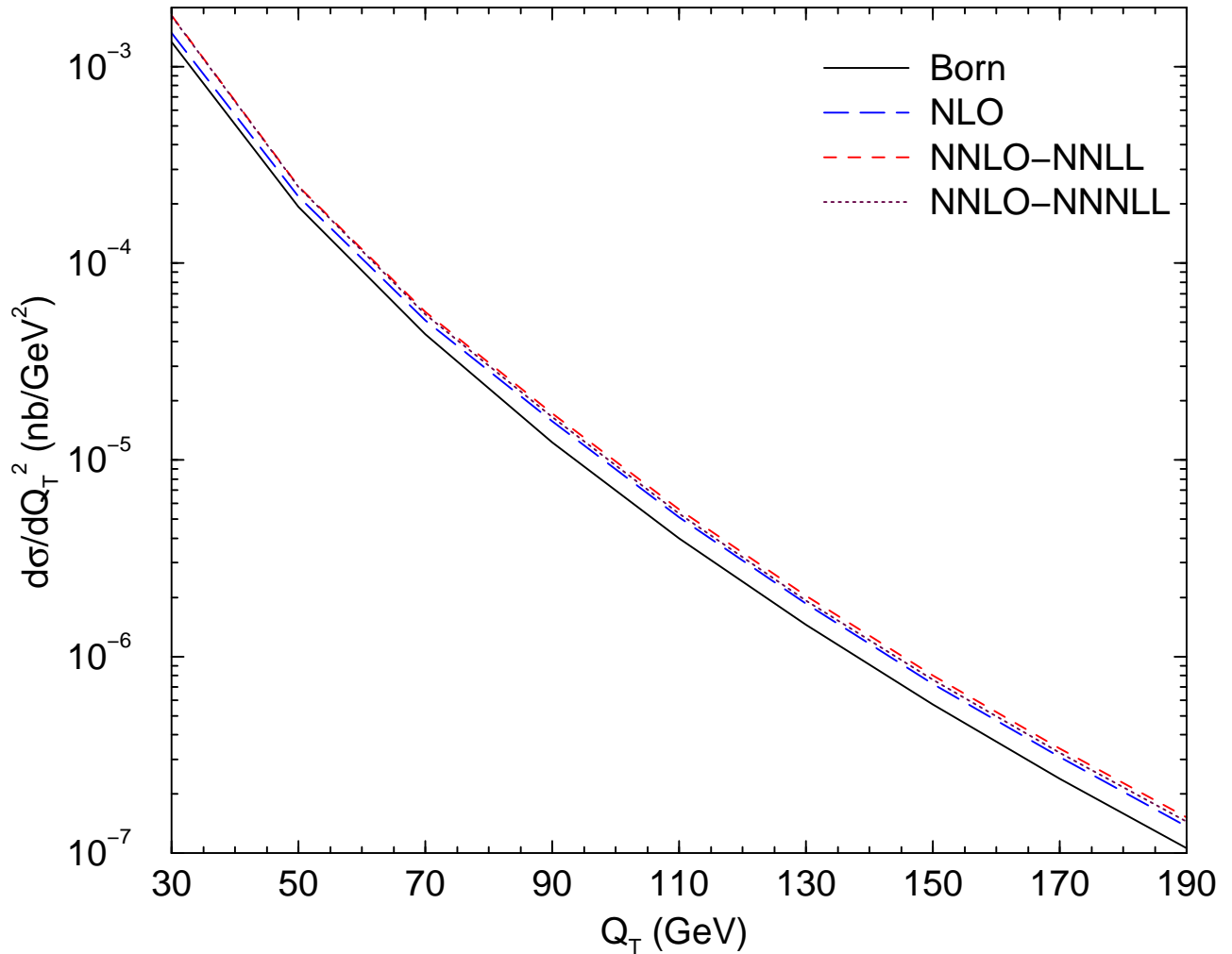
$S^{1/2} = 1.8 \text{ TeV}$



The differential cross section,  $d\sigma/dQ_T^2$ , for  $W$  hadroproduction in  $p\bar{p}$  collisions at the Tevatron with  $\sqrt{S} = 1.8 \text{ TeV}$  and  $\mu \equiv \mu_F = \mu_R = Q_T/2$  or  $2Q_T$ . Shown are the Born (solid lines), NLO (long-dashed lines), and NNLO-NNLL (dotted lines) results. The upper lines are with  $\mu = Q_T/2$ , the lower lines with  $\mu = 2Q_T$ .

# $p\bar{p} \rightarrow W$

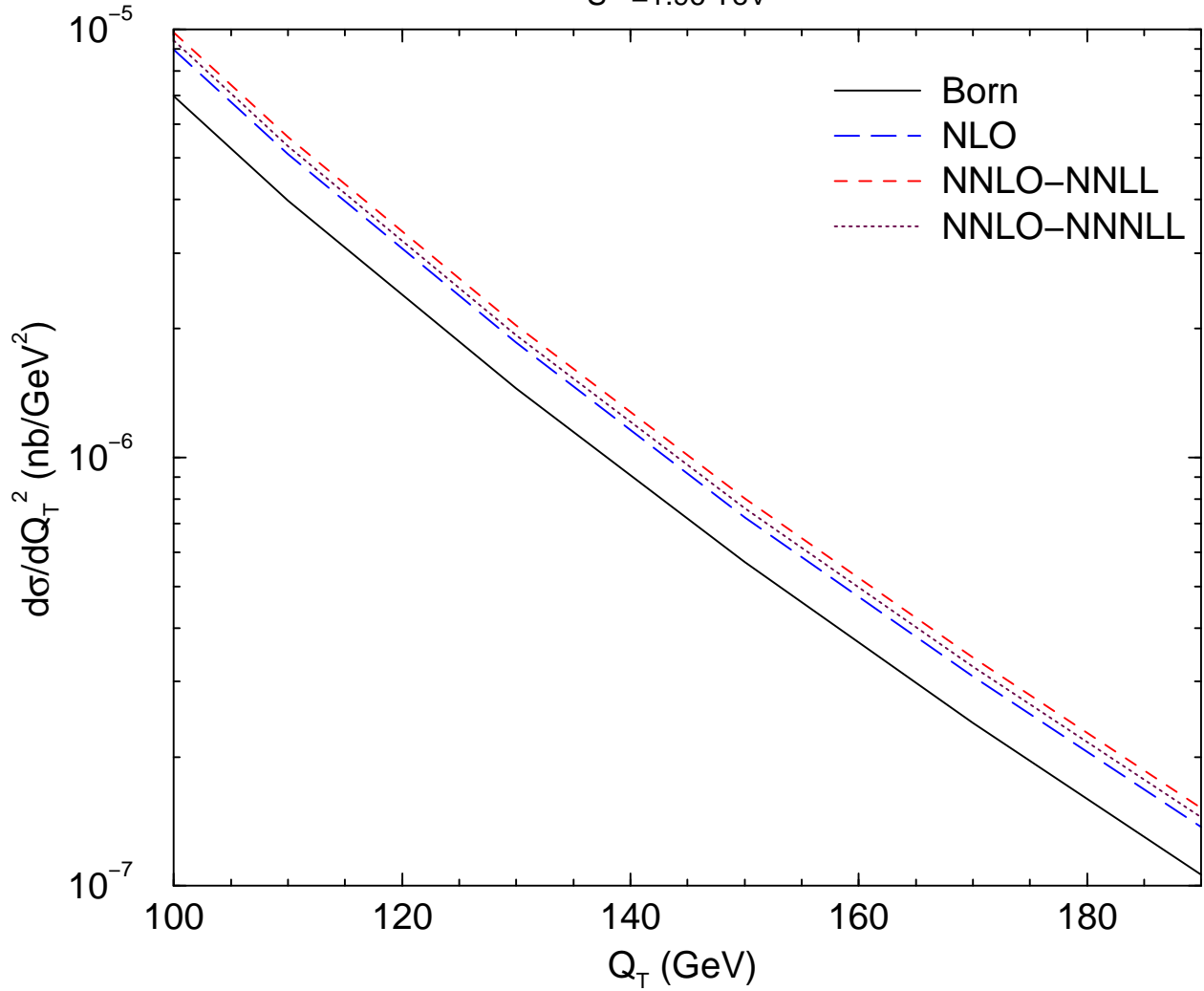
$S^{1/2} = 1.96 \text{ TeV}$



The differential cross section,  $d\sigma/dQ_T^2$ , for  $W$  hadroproduction in  $p\bar{p}$  collisions at the Tevatron Run II with  $\sqrt{S} = 1.96 \text{ TeV}$  and  $\mu_F = \mu_R = Q_T$ . Shown are the Born (solid line), exact NLO (long-dashed line), NNLO-NNLL (short-dashed line), and NNLO-NNNLL (dotted line) results.

# $p\bar{p} \rightarrow W$

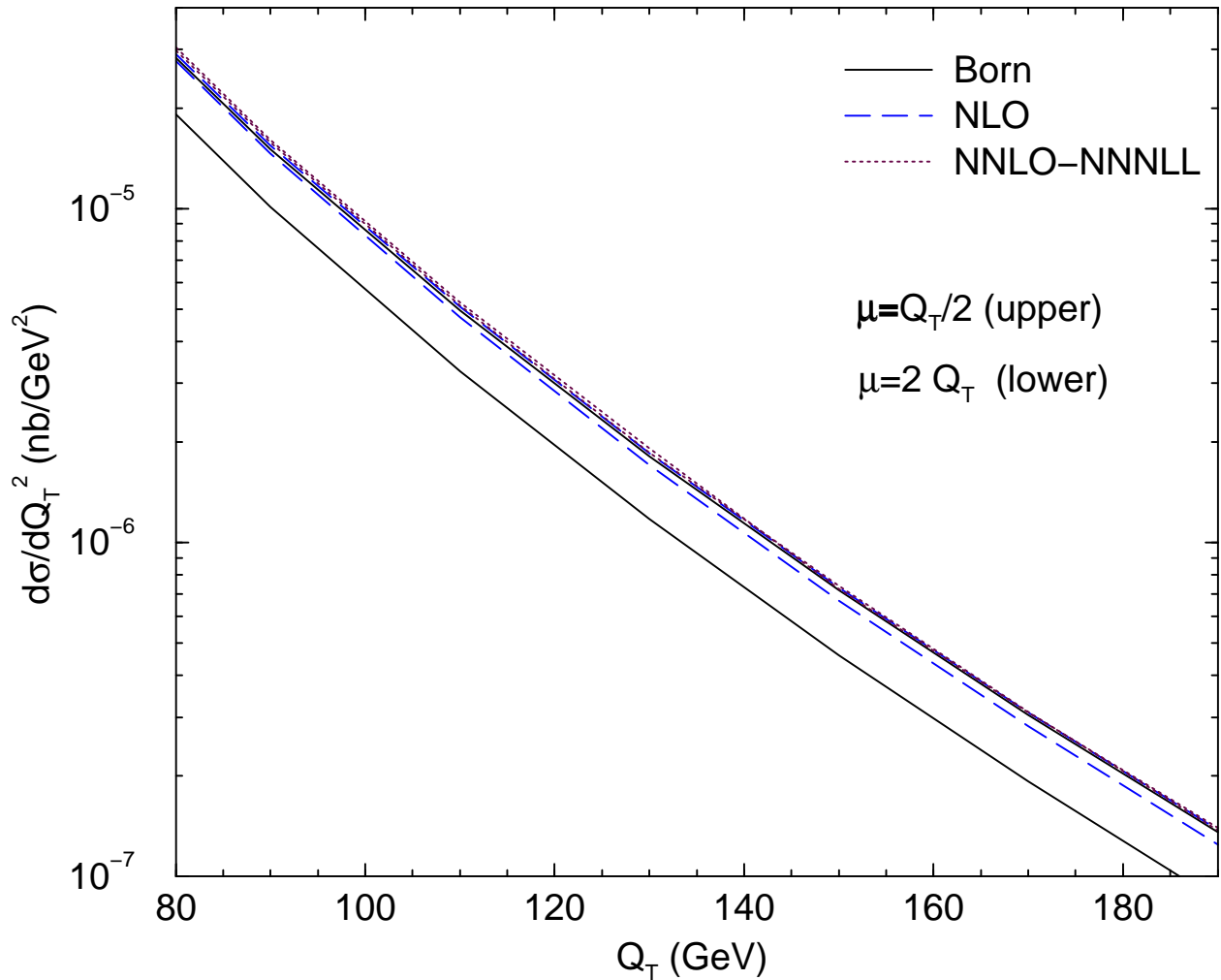
$S^{1/2} = 1.96 \text{ TeV}$



The differential cross section,  $d\sigma/dQ_T^2$  at high  $Q_T$ .

# $p\bar{p} \rightarrow W$

$S^{1/2} = 1.96 \text{ TeV}$



The differential cross section,  $d\sigma/dQ_T^2$ , for  $W$  hadroproduction in  $p\bar{p}$  collisions at the Tevatron with  $\sqrt{S} = 1.96 \text{ TeV}$  and  $\mu \equiv \mu_F = \mu_R = Q_T/2$  or  $2Q_T$ . Shown are the Born (solid lines), NLO (long-dashed lines), and NNLO-NNLL (dotted lines) results. The upper lines are with  $\mu = Q_T/2$ , the lower lines with  $\mu = 2Q_T$ .