Small x effects in heavy quark production

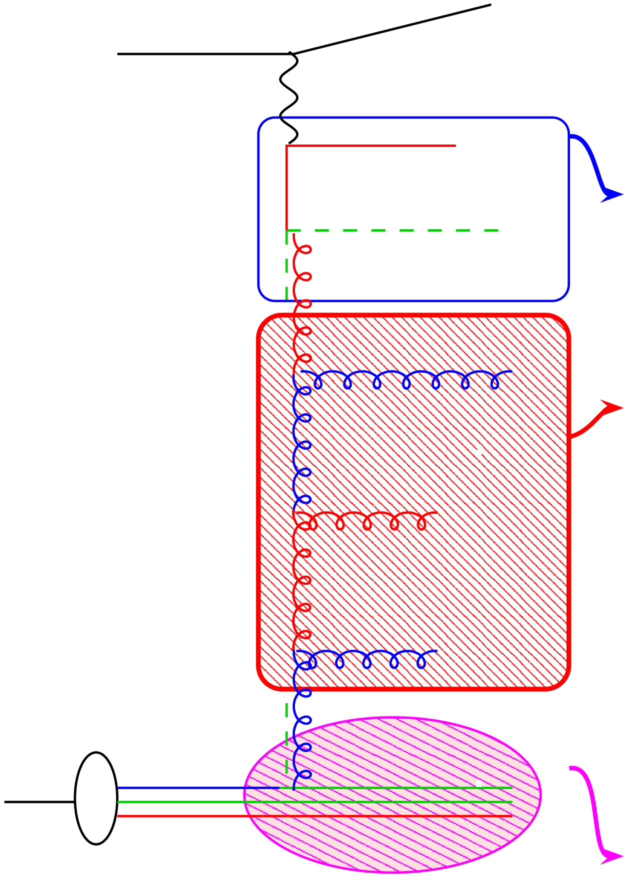
H. Jung, DESY

DIS04, Strbské Pleso, 2004

- factorization
  - collinear vrs k_t factorisation
  - small x effects
- CCFM equation, solution, new hadron level MC CASCADE
- heavy quark production
  - at HERA, Tevatron and at LEP ($\gamma\gamma \rightarrow b\bar{b}$) ???
- conclusion

Basic idea - Collinear factorization

DGLAP

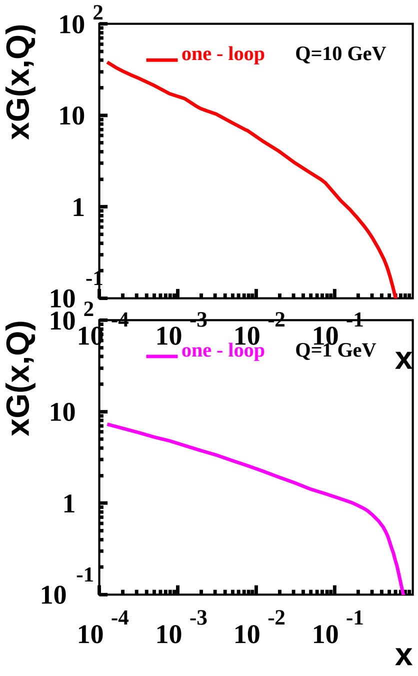


BGF matrix element
on mass shell

evolution of parton cascade
q - ordered
with DGLAP splitting fct.

$$\tilde{P}(z,q,k_t) = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} \right)$$

initial distribution: steep

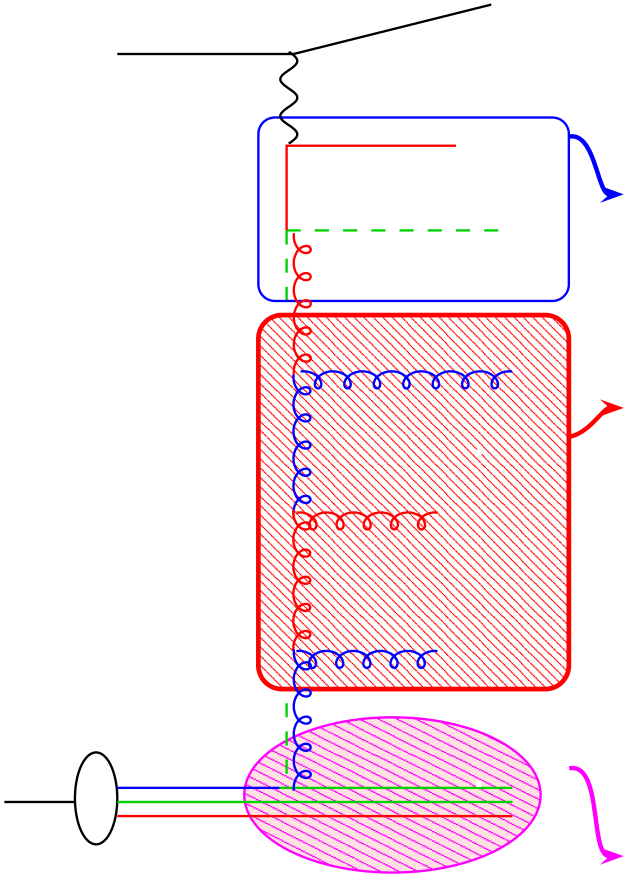


$$\sigma(ep \rightarrow e' q\bar{q}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \hat{\sigma}(\hat{s},0,Q) \int d^2k_t x_g \mathcal{A}(x_g,k_t,\bar{q})$$

with $\int d^2k_t x_g \mathcal{A}(x_g,k_t,\bar{q}) = x_g G(x_g,Q^2)$

Basic idea - Collinear factorization

J.C. Collins, X. Zu JHEP 06 (2002) 018
on shell matrix element
is only assumption without proof
and is unphysical !!!
and not necessary for proof of factorization !!!



DGLAP

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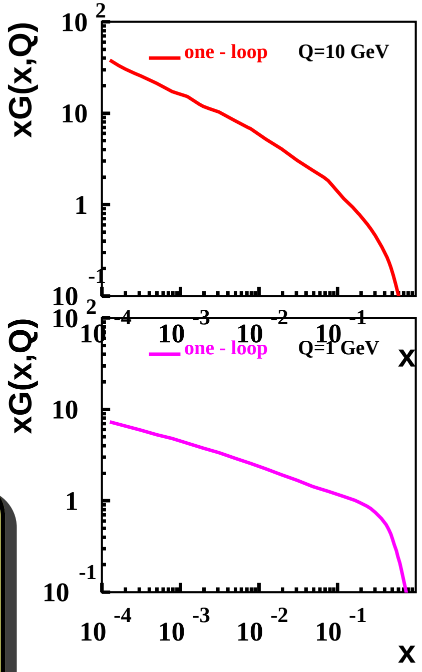


$$\sigma(ep \to e'q\bar q) = \int\int d^2k_t \frac{dy}{y} d^2Q \frac{dx_g}{x_g}\hat\sigma(\hat s, k_t, Q)\, x_g \mathcal{A}(x_g, k_t, \bar q)$$

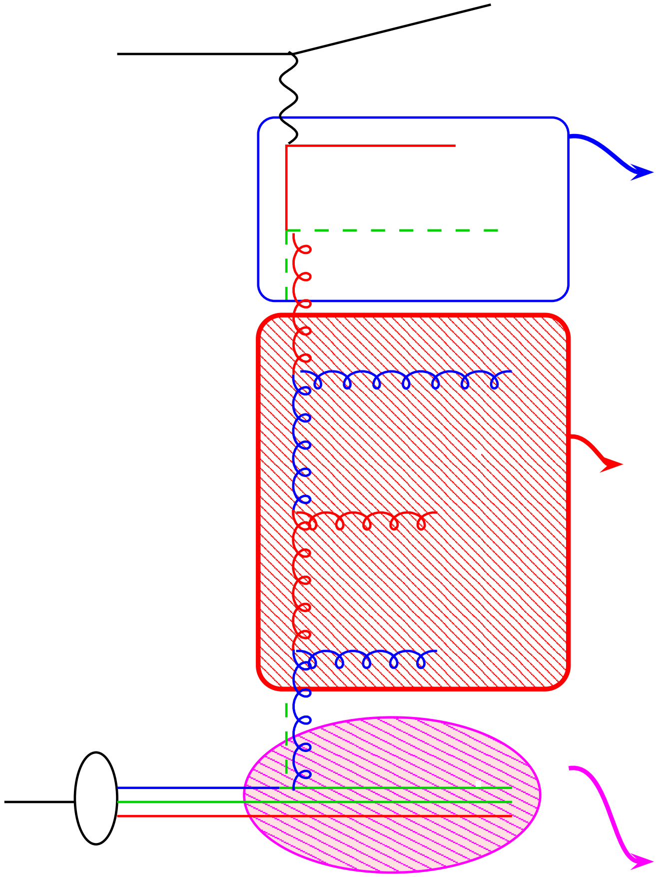
with $\int d^2k_t\, x_g \mathcal{A}(x_g, k_t, \bar q) \overset{?}{=} x_g G(x_g, Q^2)$

Basic idea - k_t factorisation

CCFM

CCFM (one loop)

- **angular ordering**



BGF matrix element
off mass shell

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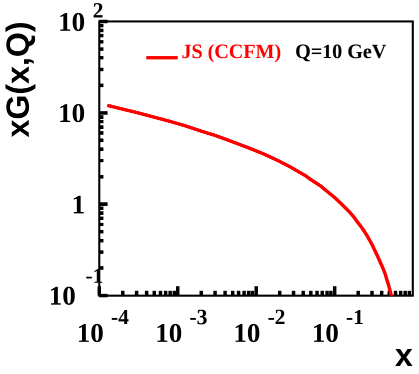
initial distribution:



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Basic idea - k_t factorisation

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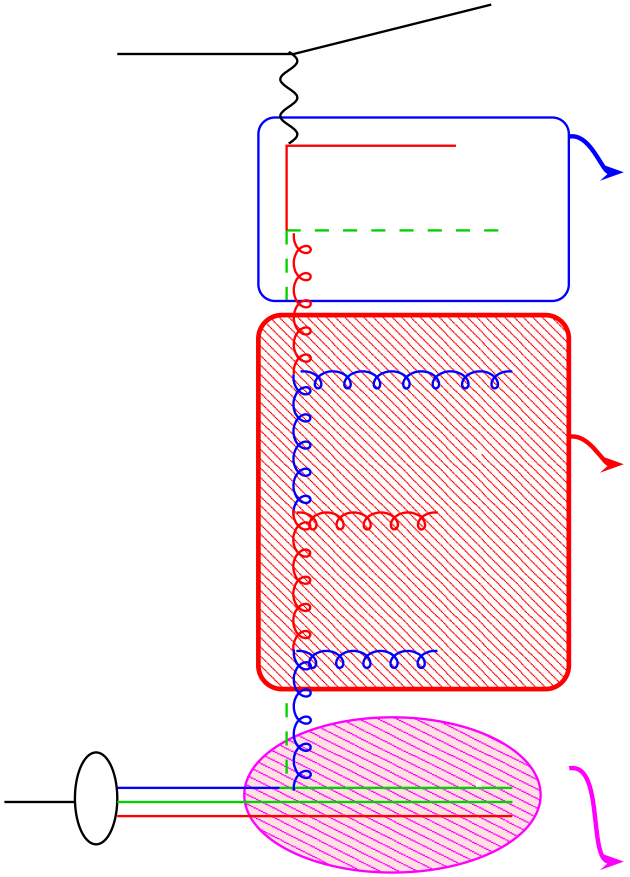


BGF matrix element
off mass shell

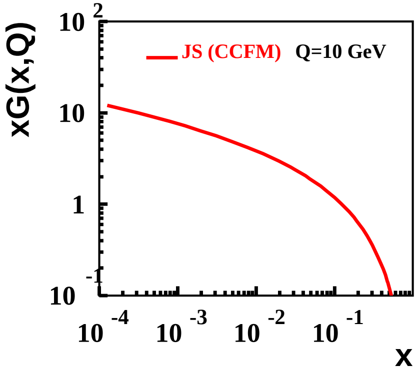
evolution of parton cascade
with CCFM splitting fct.

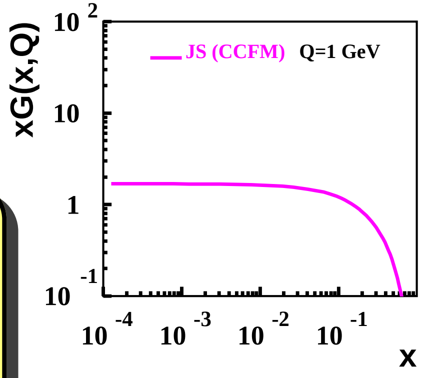
$$\tilde{P} = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} \Delta_n{}^s \right)$$

initial distribution: flat

CCFM (all loops)

- **angular ordering (instead of q_t ordering)**
- **Δ_{ns} (non - Sudakov)**





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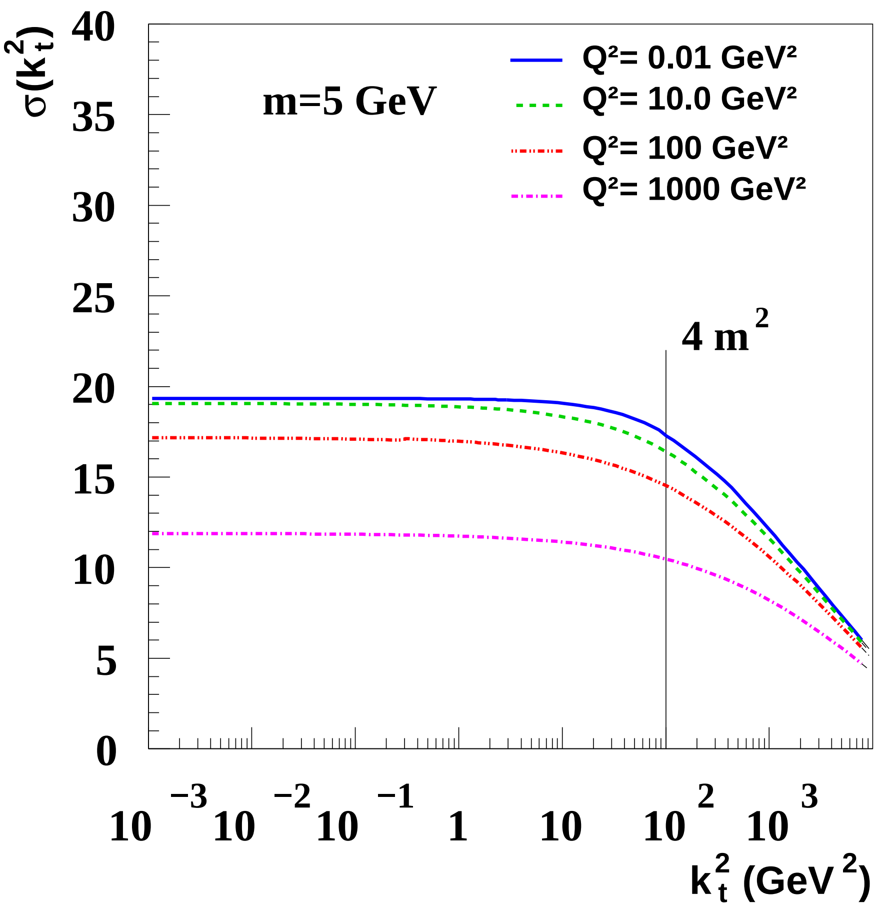
Small x effects for $k_t \sim E_{parton}$

ME: partons off-shell

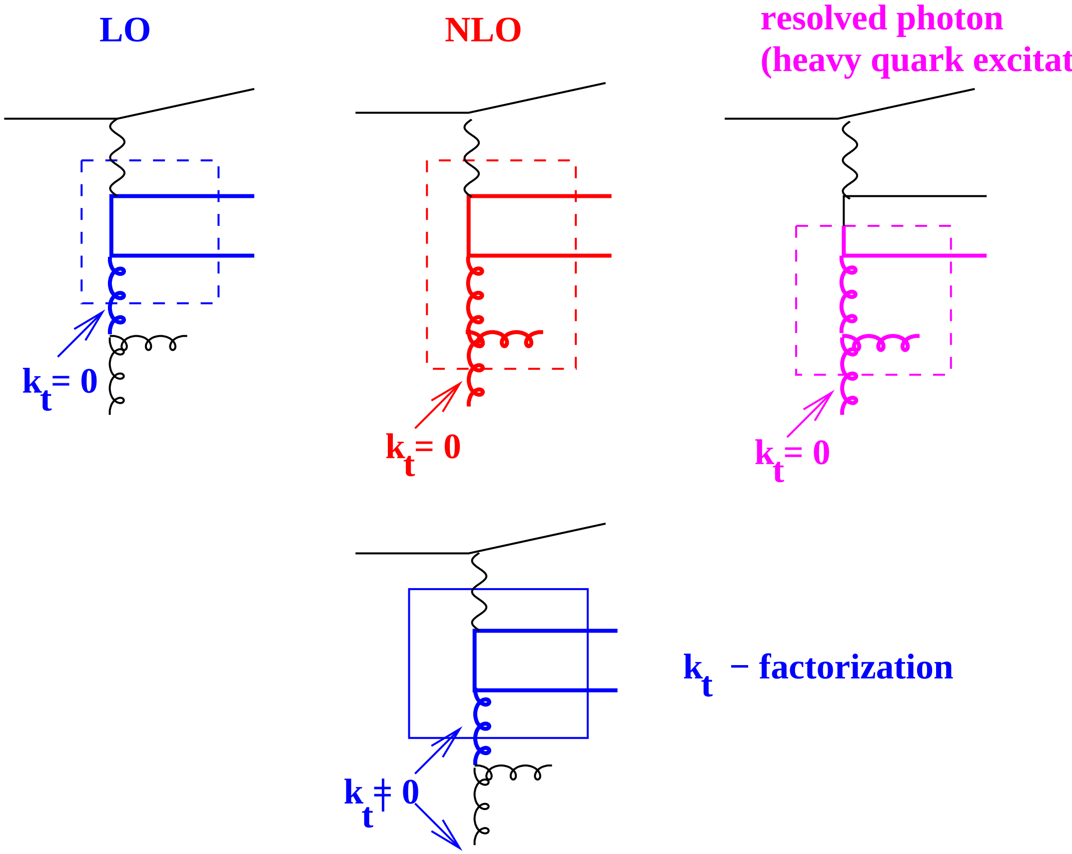


Small x effects for $k_t \sim E_{parton}$

ME: partons off-shell

includes part of NLO

and HQ excitation



Small x effects for $k_t \sim E_{parton}$

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includes part of NLO

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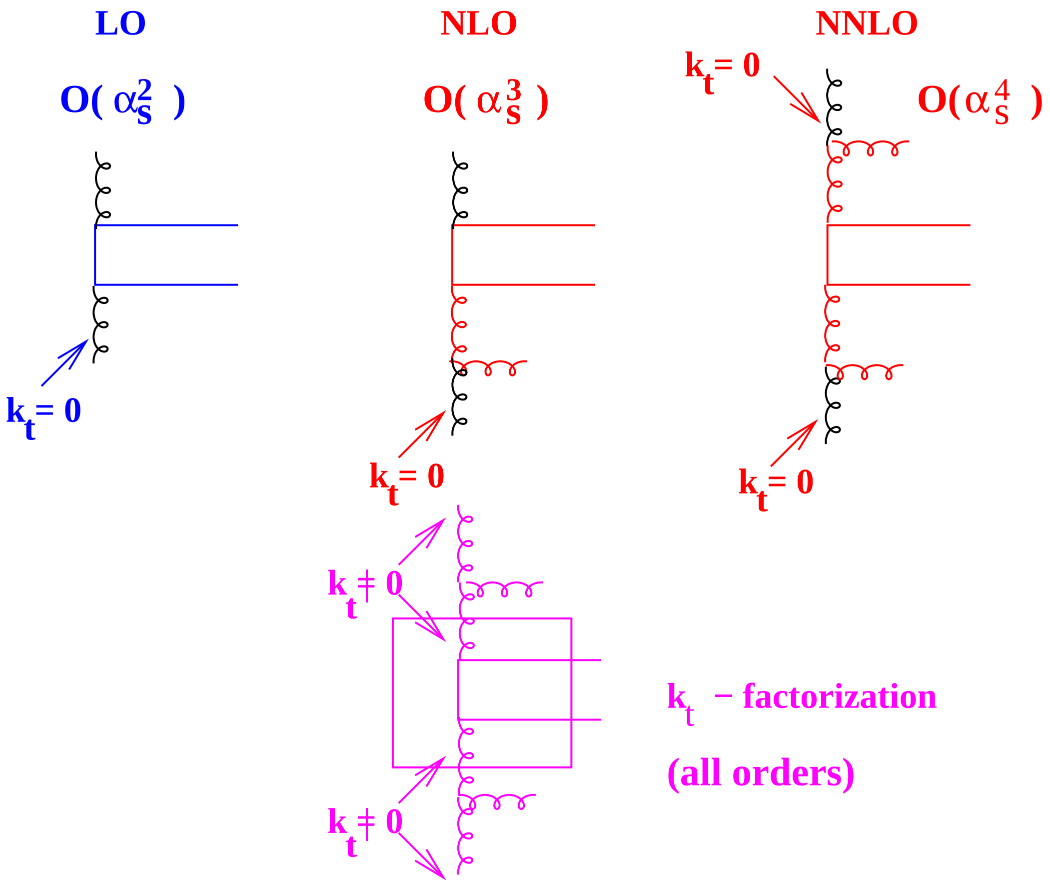
also for $p\bar{p}$



Small x effects for $k_t \sim E_{parton}$

ME: partons off-shell

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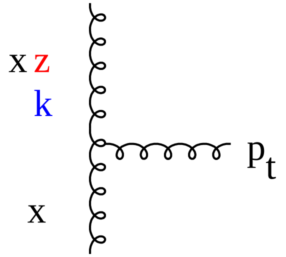
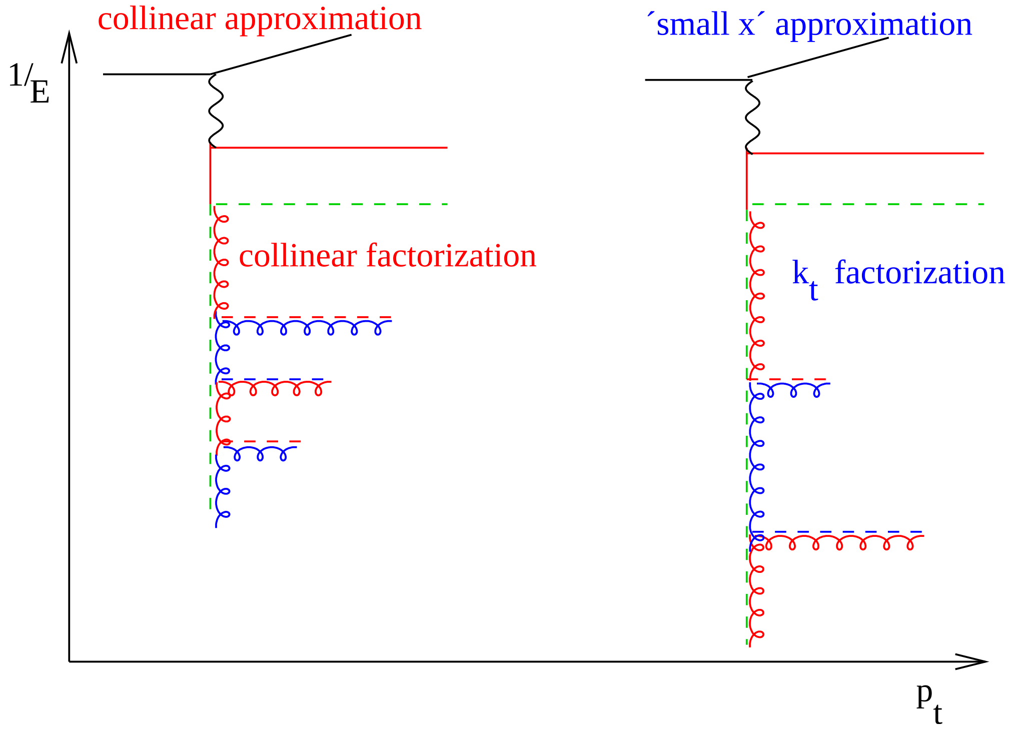
Evolution:

break k_t ordering

Gluon Bremsstrahlung:

$$\sim \frac{1}{k^2}\left(\frac{1}{z}+\cdots\right)$$





Small x effects for $k_t \sim E_{parton}$

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Evolution:

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BFKL/CCFM splitt. fct

uPDFs

ang. ord. ☞ CCFM

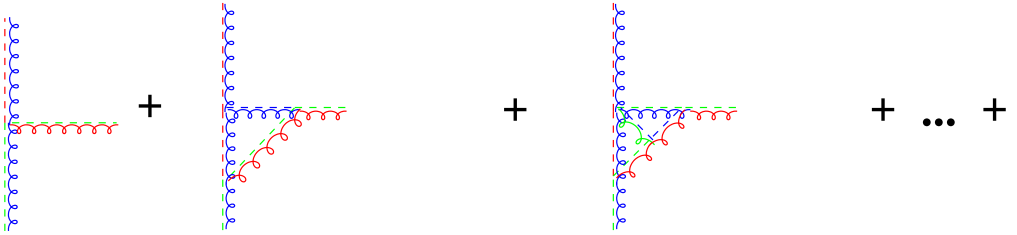
... Only singular parts....

$$\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\text{ns}}(z, q, k_t)$$

Non - Sudakov form factor:

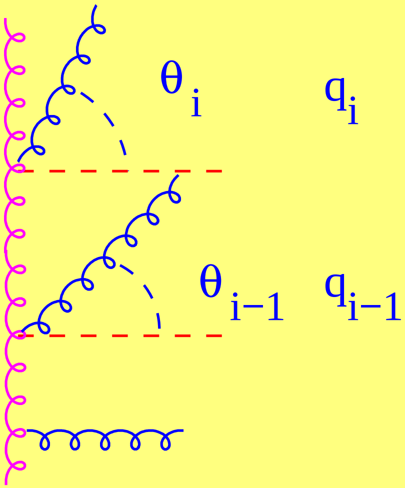
$$\Delta_{\text{ns}} =$$
$$\exp\left(-\bar{\alpha}_s \int \frac{dz}{z} \int \frac{dq}{q} \Theta(k_t - q)\Theta(q - zq_t)\right)$$

$$\Delta_{\text{ns}} =$$
$$1 + \left(-\bar{\alpha}_s \int \frac{dz}{z} \int \frac{dq}{q}\right)^1 + \frac{1}{2!}\left(-\bar{\alpha}_s \int \frac{dz}{z} \int \frac{dq}{q}\right)^2 ..$$



Catani **C**iafaloni **F**iorani **M**archesini (1988 - 1990)

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:



- ordering in q (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL

☛ **WOW**

for small z no restriction in q: ☛ random walk in q

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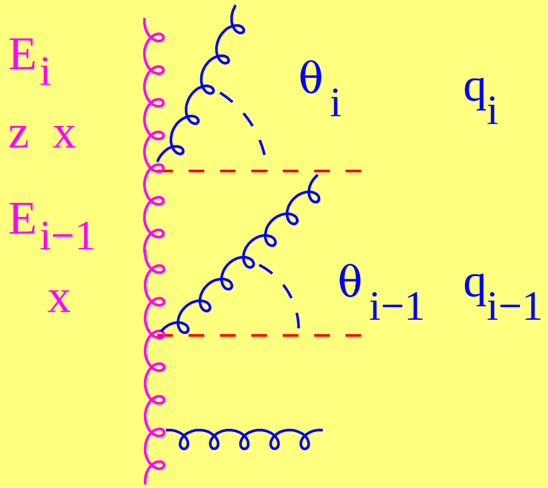

$$p_{ti} = |q_i^0| \sin\Theta_i, \; z = \frac{E_i}{E_{i-1}}$$

$$E_{i-1} = E_i + q_i^0 = zE_{i-1} + q_i^0, ☛ q_i^0 = (1-z)E_{i-1}$$

$$p_{ti} = q_i^0 \sin\Theta_i \simeq (1-z)E_{i-1}\Theta_i$$

$$\frac{p_{ti}}{1-z} \simeq E_{i-1}\Theta_i$$

with: $q_i = \frac{p_{ti}}{1-z_i}$ ☛ $\Theta_i = \frac{q_i}{E_{i-1}}$ and $\Theta_{i+1} = \frac{q_{i+1}}{E_i}$

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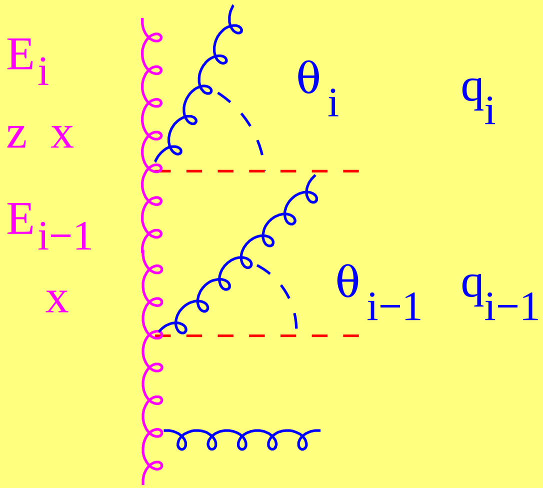


☛

in lab. frame

$$\Theta_{i+1} > \Theta_i$$

$$\boxed{q_{i+1} > z_i q_i}$$

with $q = \frac{p_t}{1-z}$

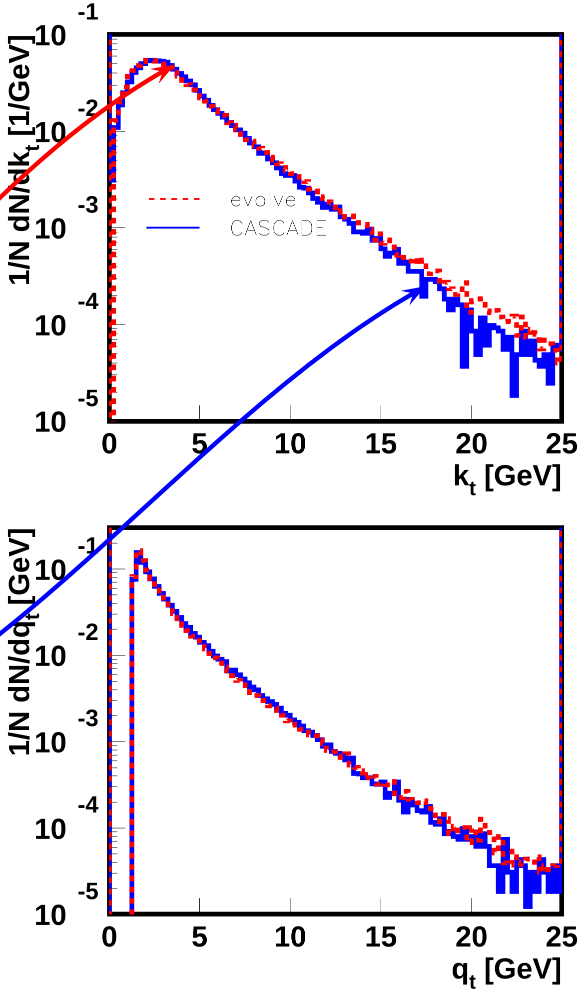
- ordering in q (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL

☛ **WOW**

for small z no restriction in q: ☛ random walk in q

Advantage of CCFM: parton emissions

- DGLAP or BFKL
  - only inclusive predictions
  - no info on emitted partons !!!
- CCFM treats explicitely:
  partons emitted during cascade
  color coherence
  energy momentum conservation
- best to implement in MC generator
  - compare evolution and MC
- CASCADE MC generator



evolution - MC parton shower comparison
never shown for DGLAP type MC's!!!

The Monte Carlo Generator CASCADE

- **CCFM backward evolution implemented in MC generator**
  CASCADE (http://www.quark.lu.se/ hannes/cascade)
- **initial state CCFM cascade with strict angular ordering**
- **off-shell hard scattering processes:**
  - ☞ $\gamma g^* \to q\bar{q}$, $\gamma^* g^* \to Q\bar{Q}$, $\gamma g^* \to J/\psi g$, $\gamma\gamma \to Q\bar{Q}$
  - ☞ $g^* g^* \to q\bar{q}$, $g^* g^* \to Q\bar{Q}$, $g^* g^* \to h$
- P**-remnant treatment like in PYTHIA (**q**-di-**q**, primordial** k_t**)**
- **final state parton showers added to quarks**
  hadronization via JETSET/PYTHIA

CASCADE is MC implementation of CCFM

for $ep, ee, \gamma\gamma$ **and also for** $p\bar{p}$

Status of comparison with theory

status of comparsion in 2001

	collinear-fact.	k_t-fact.
HERA observables		
high Q^2 D* production	OK	OK
direct photoproduction of D*	1/2	OK
resolved photoproduction of D*	NO	1/2
high Q^2 B production	NO,	
direct photoproduction of B	OK?, NO	OK
resolved photoproduction of B	OK?	OK
TEVATRON observables		
high-$p_\perp$ D* production	?	
high-$p_\perp$ B production	OK?,	OK
low-$p_\perp$ B production	OK?,	OK
J/Ψ production	NO	?
LEP observables		
B production (σ_{tot})	NO	?

Status of comparison with theory

status of comparsion in 2001 2004

	collinear-fact.	k_t-fact.
HERA observables		
high Q^2 D* production	OK	OK
direct photoproduction of D*	1/2	OK
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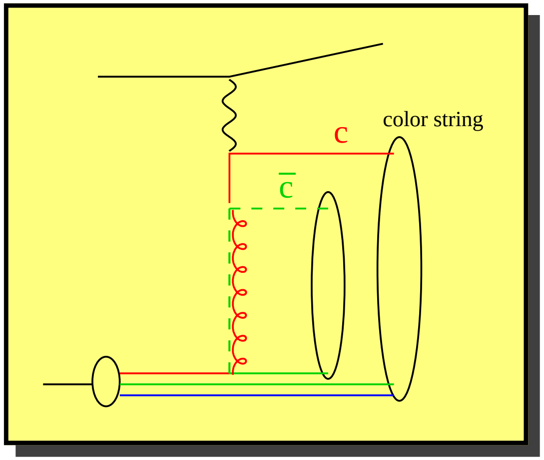
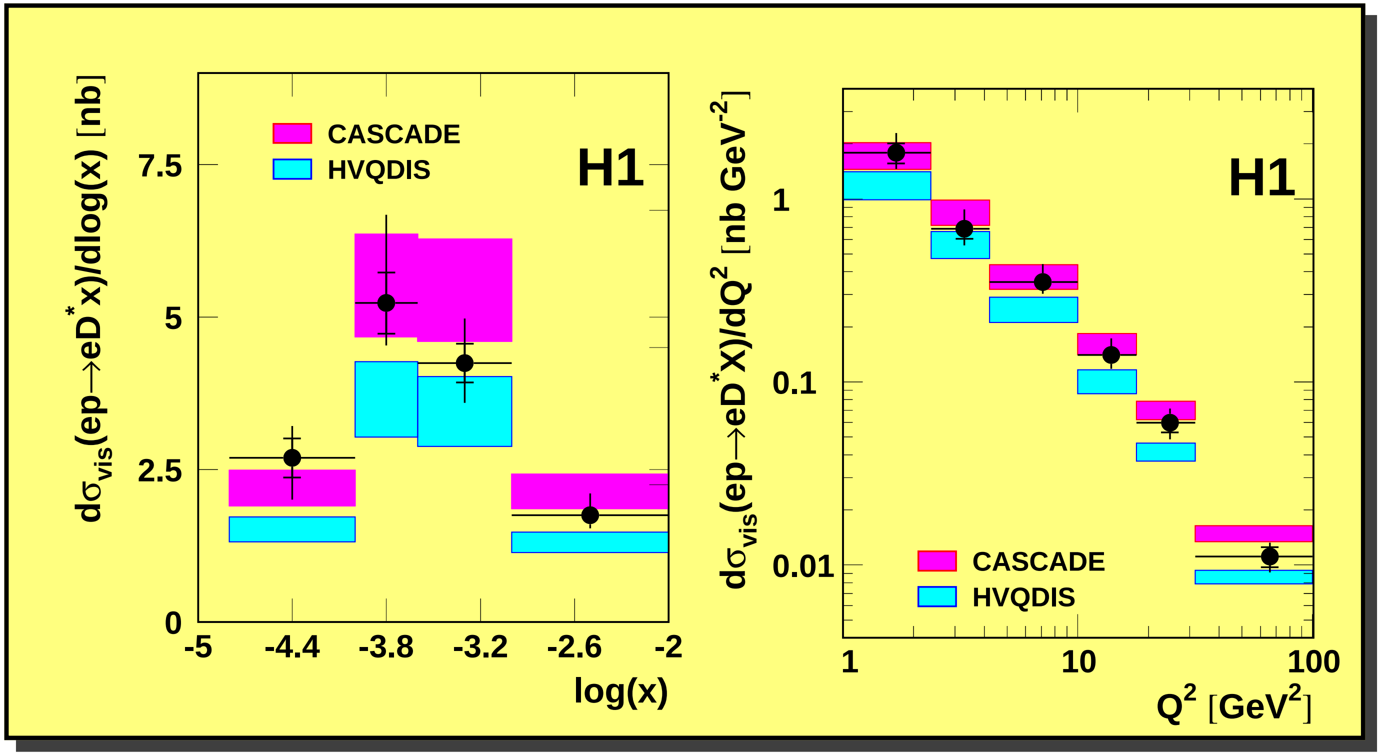
high Q^2 B production	NO, OK	OK
direct photoproduction of B	OK?, NO OK	OK
resolved photoproduction of B	OK? OK	OK
TEVATRON observables		
high-$p_\perp$ D* production	? NO?	OK
high-$p_\perp$ B production	OK?, OK	OK
low-$p_\perp$ B production	OK?, OK	OK
J/Ψ production	NO	? OK ?
LEP observables		
B production (σ_{tot})	NO	? NO ?

Charm production in DIS

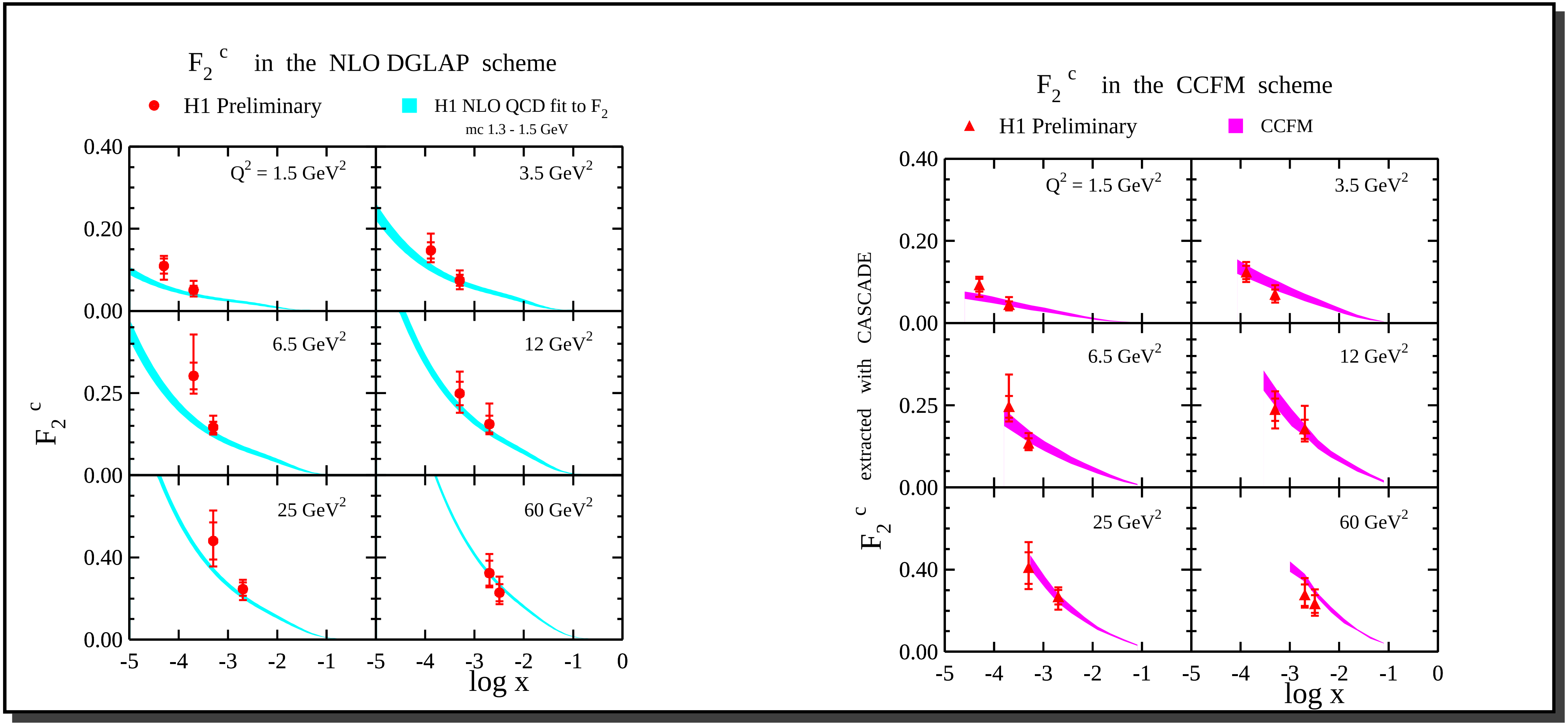




D^{*} production in DIS

- ☞ **standard DGLAP with NLO calculation too small**
- ☞ CASCADE $\sim$ **perfect even at low x and Q^2**
- ☞ **free parameter: m_c**

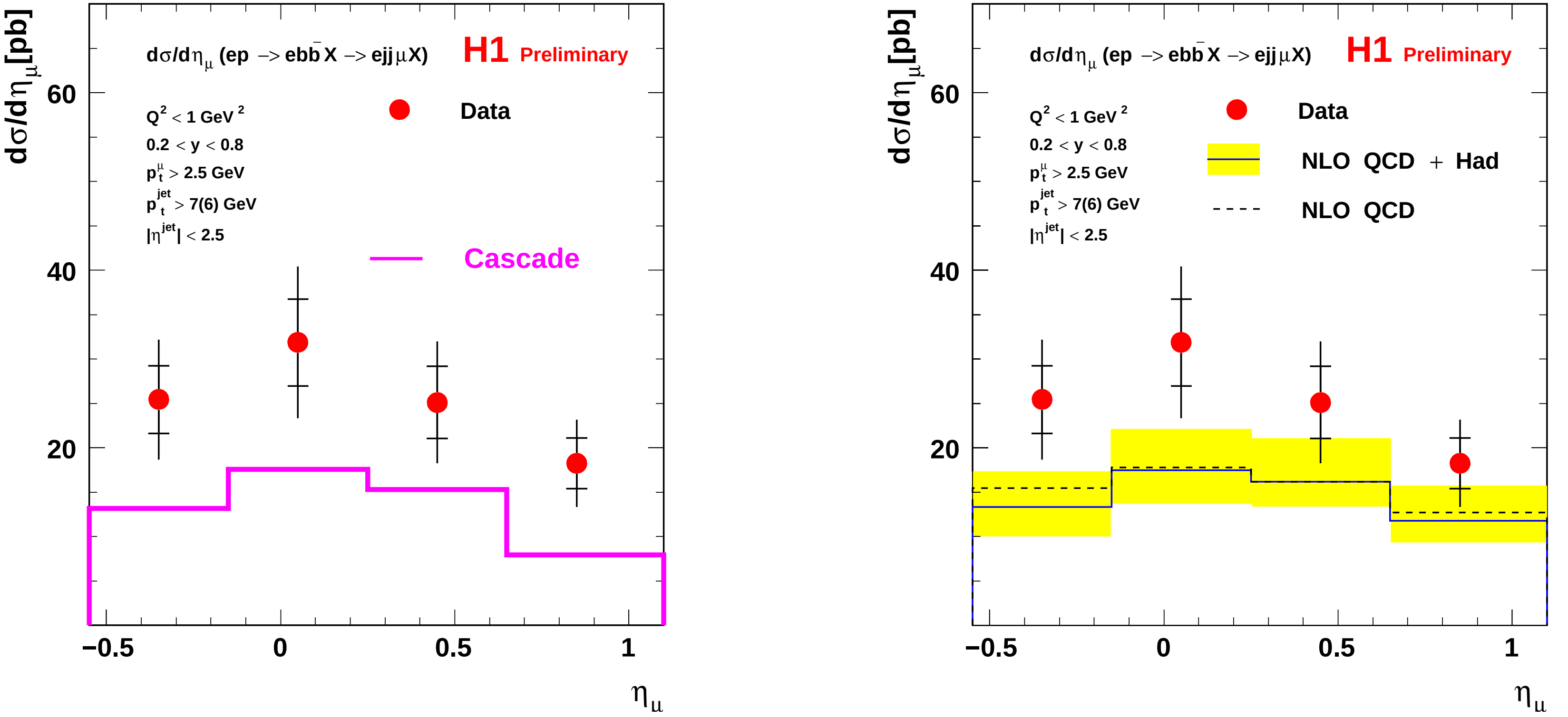
Problem of extrapolation or what is F_2^c ? ! ?



- F_2^c **depends on model for extrapolation... does that make sense ???**
- **whom to believe ????**
- **collinear NLO calculation or k_t factorization with CCFM ?**
- **proper model / MC desperately needed**

$b\bar{b}$ production at HERA

- large extrapolation from visible to total x-section
- look at visible x-section



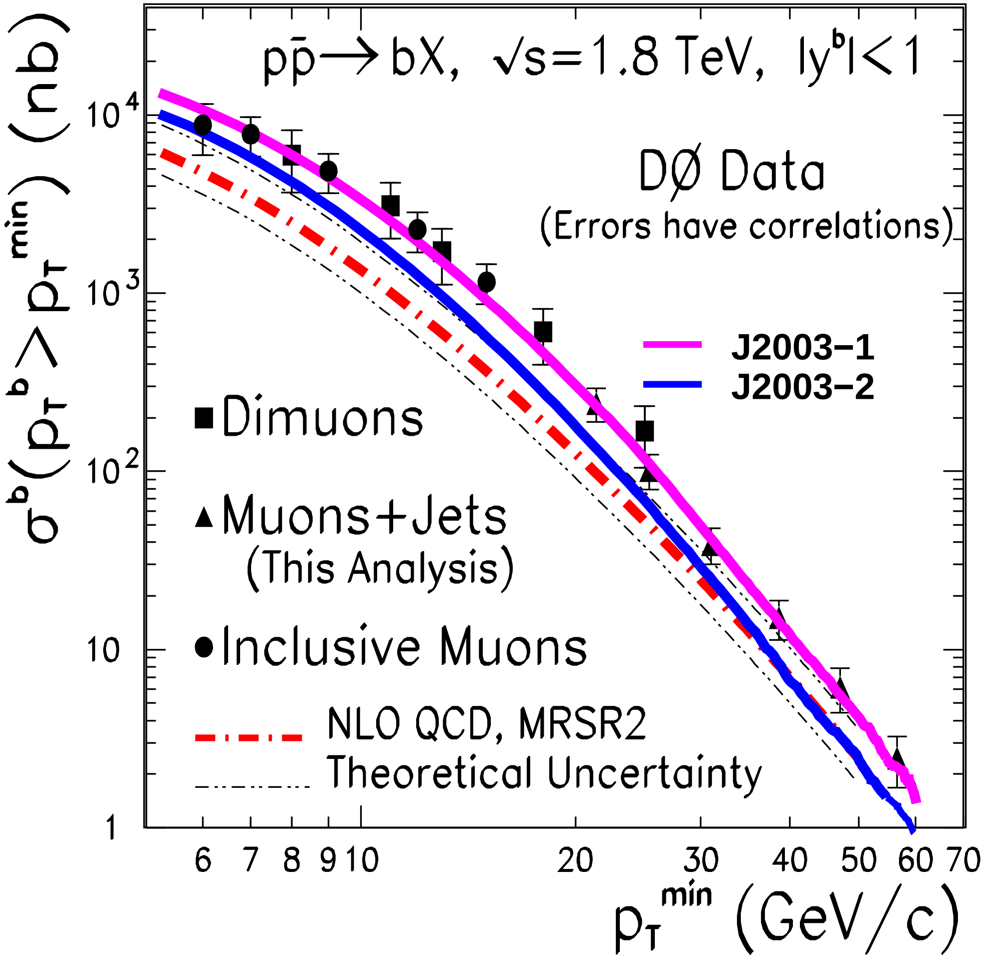
CASCADE $\sim$ ok for visible μ's (similar to NLO)

$b\bar{b}$ production at the Tevatron

Test universality of unintegrated gluon density from HERA

- use unintegrated gluon from F_2 fit at HERA
- use off-shell matrix element for $g^* g^* \to b\bar{b}$ with $m_b = 4.75$ GeV.
- set with singular terms ok
- even with full splitting fct ok

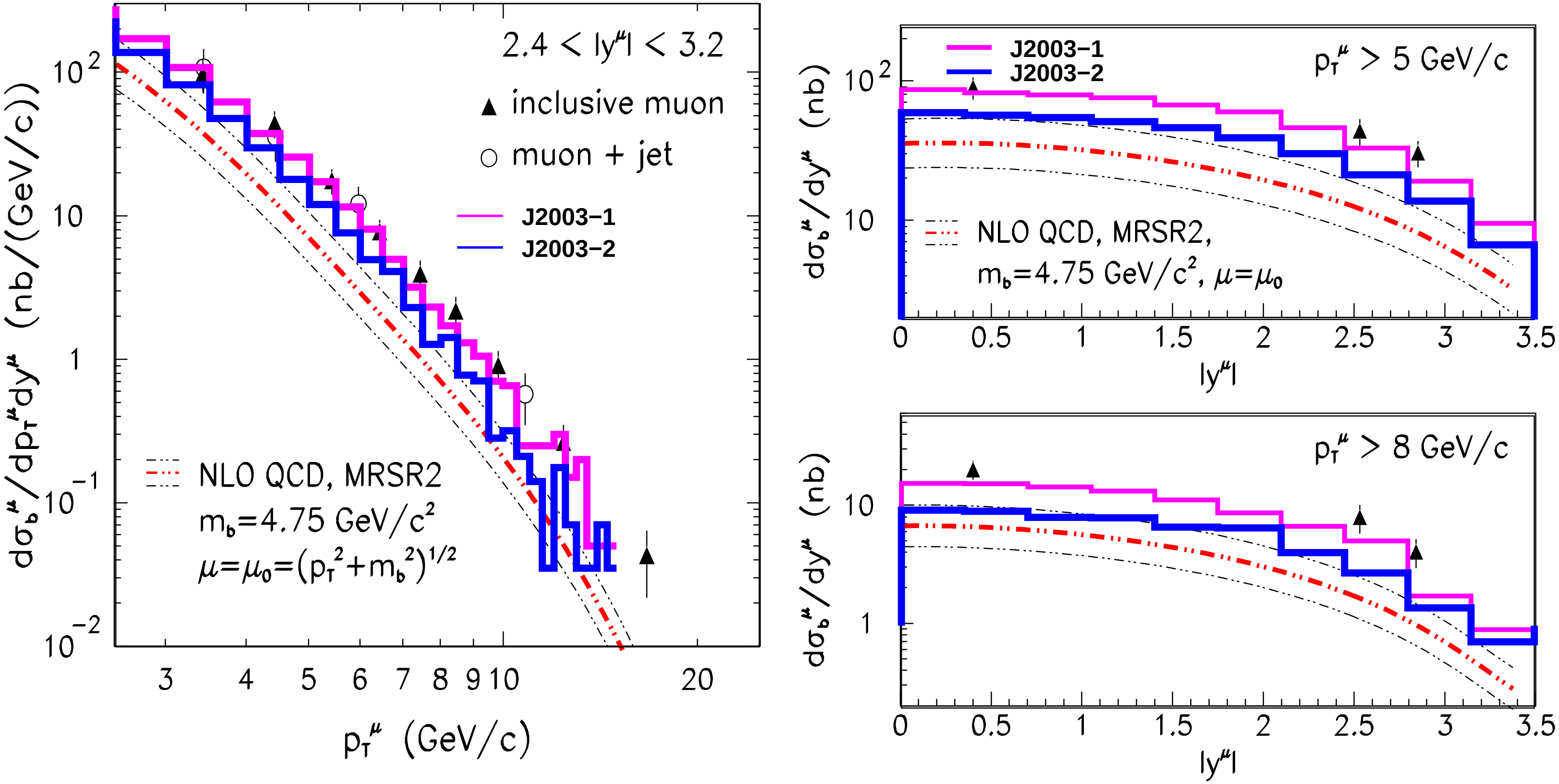
NOTE NLO off by factor 2



CASCADE w/o additional free parameters

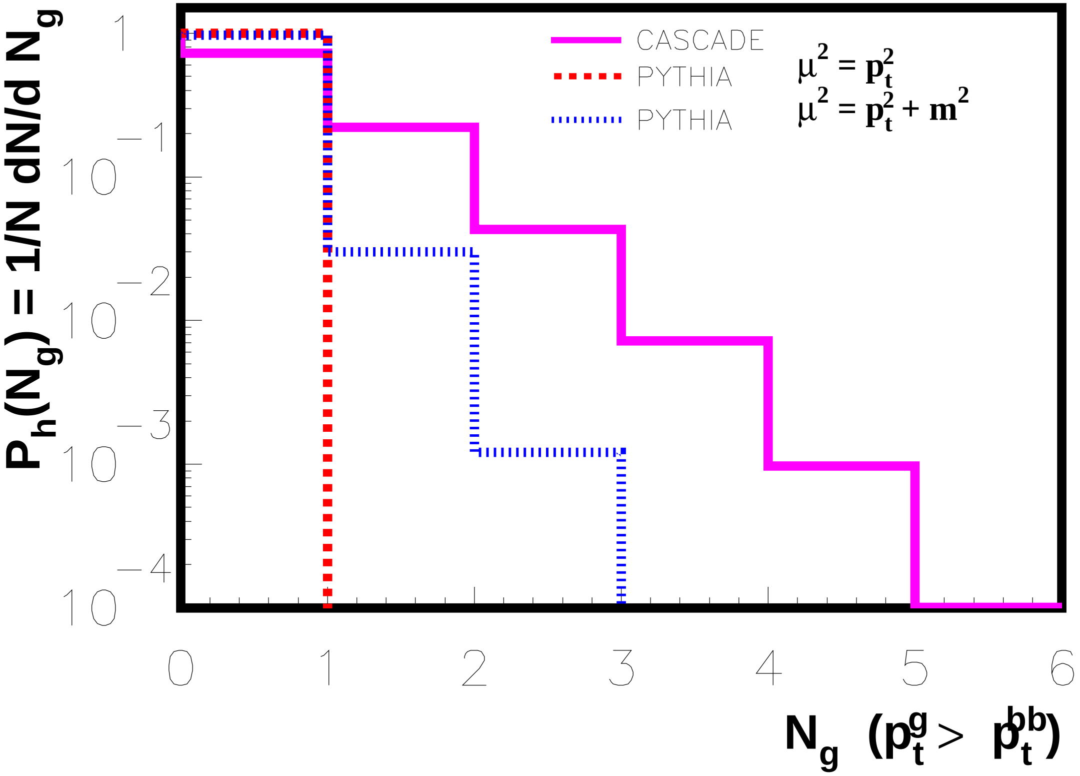
$b\bar{b}$ production at the Tevatron: visible cross section

data from: D0 Collaboration B. Abbott et al., *Phy.Rev.Lett* 84 (2000) 5478



- **CASCADE describes μ spectrum over huge range well**
- **NLO fails by factor $\sim$ 2 (central) and $\sim$ 4 (forward)**

Why does k_t-factorization help for $b\bar{b}$ production at Tevatron



estimate higher order corrections

Nr of gluons with $p_t > p_t^{b\bar{b}}$

LO: $\mathcal{O}(\alpha_s^2) \rightarrow N_g = 0$

NLO: $\mathcal{O}(\alpha_s^3) \rightarrow N_g = 1$

NNLO: $\mathcal{O}(\alpha_s^4) \rightarrow N_g = 2$

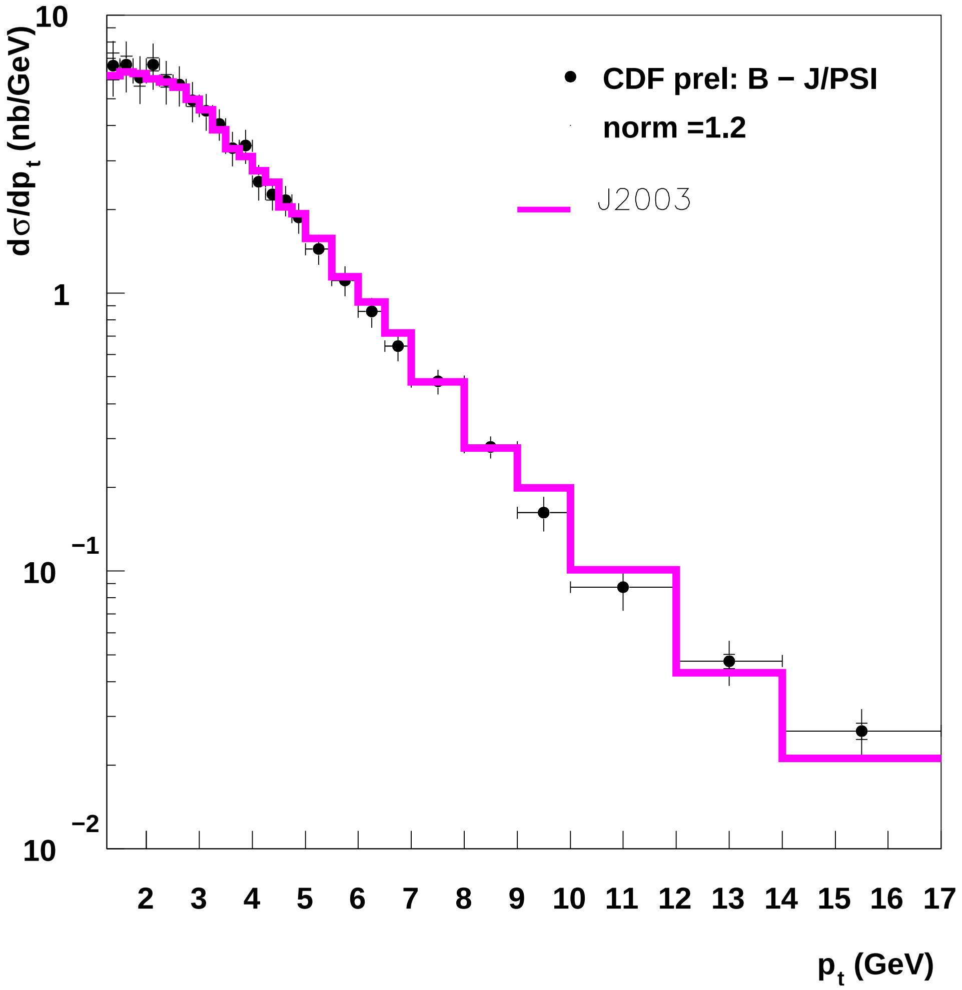
.....

CASCADE $\rightarrow \mathcal{O}(\alpha_s^6)$

CASCADE with k_t factorization for estimation of higher order corrections

$b\bar{b}$ production at CDF:
$b \to J/\psi$

	$\sigma(b, \lvert y \rvert < 1)$
CASCADE	$24.95\ \mu b$
Cacciari et al (FONLL)	$23.6\ \mu b$
CDF	$24.9 \pm 0.6 \pm 6.2\ \mu b$
	$\sigma(b, \lvert y \rvert < 0.6)$ $BR(B \to J/\psi \to \mu)$
CASCADE	$17.2\ nb$
... with PYTHIA BR	$15.2 nb$
MC@NLO	$17.2\ nb$
Cacciari et al (FONLL)	$19.0^{+8.4}_{-6.0}\ nb$
CDF	$19.9^{+3.8}_{-3.2}\ nb$

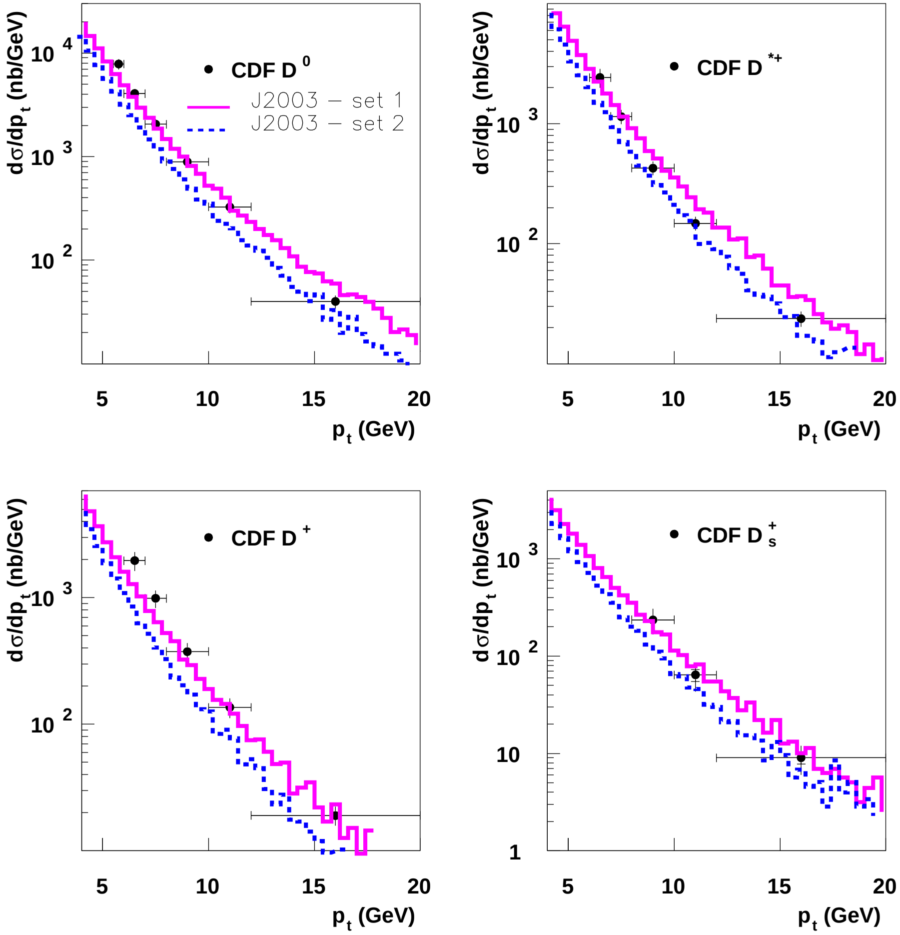


- remarkable agreement CASCADE and MC@NLO !!!
- Extrapolation ok ???

Charm production at the Tevatron

data from: CDF Collaboration D. Acosta et al., hep-ex/0307080

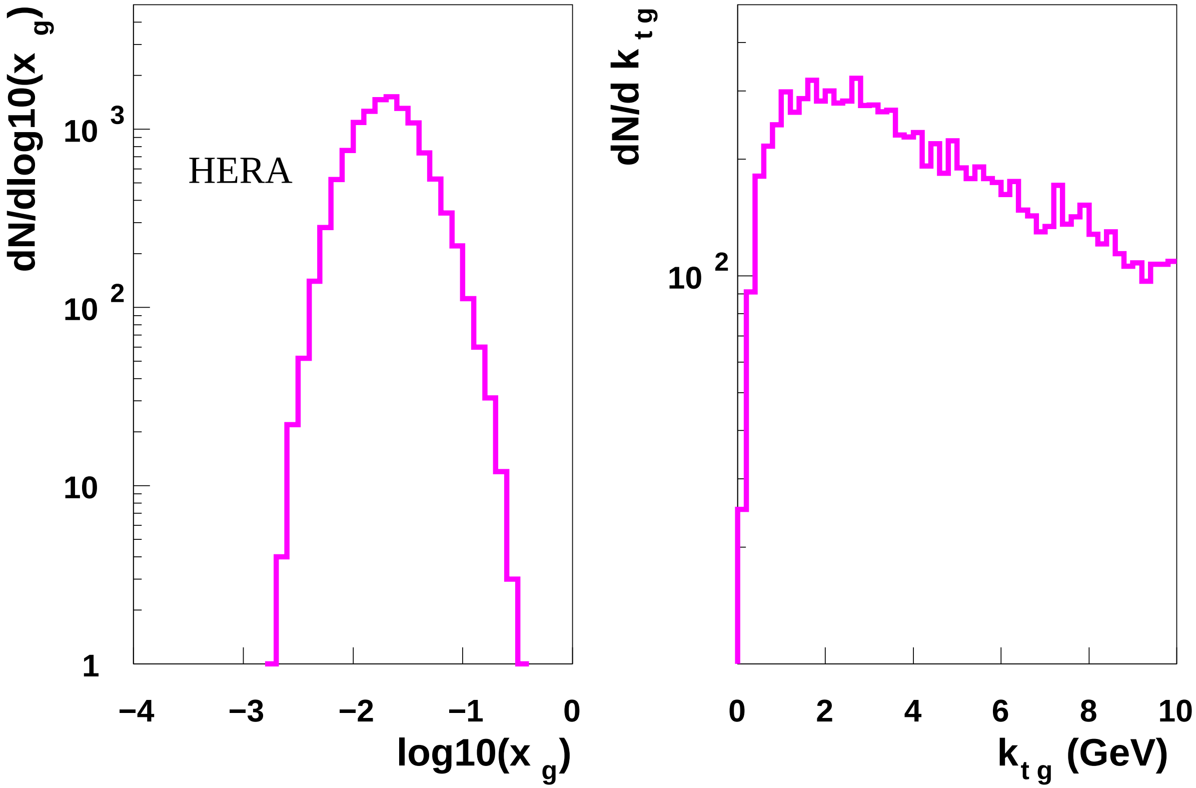
- measure charmed mesons in $|y| < 1$
- NLO by 100 - 50 % below data
- same model as for $b\bar{b}$
- similar problems as in $b\bar{b}$



- set with singular terms ok
- set with full splitting fct smaller but still larger than NLO

Success for CASCADE with k_t factorization !!!

$b\bar{b}$ production at HERA and Tevatron - a typical k_t-factorization process ?????

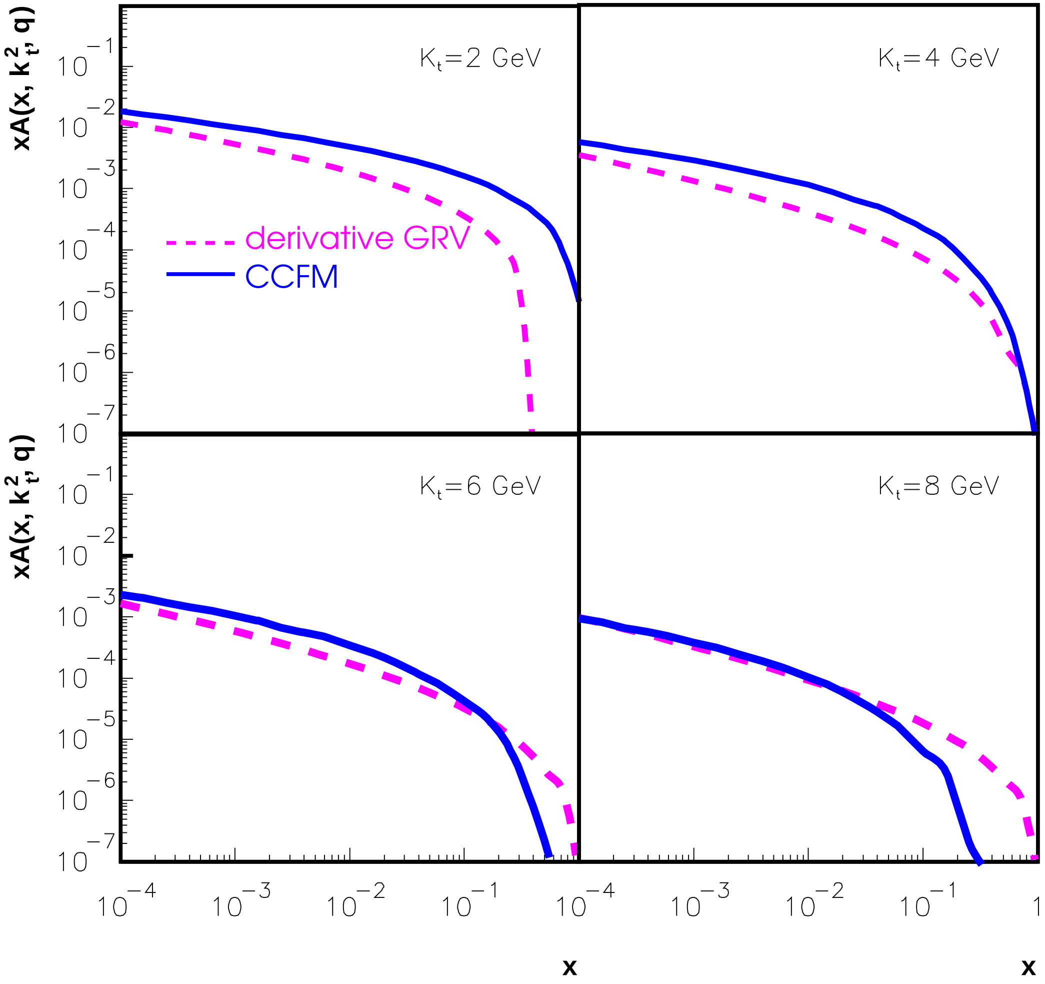
- k_t-factorization: $E_{\text{gluon}} \sim k_t$
- 

- $E_{\text{gluon}} \sim 10^{-2} \cdot 920$ GeV compared to $k_t \sim \mathcal{O}(5$ GeV$)$
- Tevatron similar energies...
- be aware of small x effects !!!

Un-integrated Gluon Density of Photon

together with M. Hansson

- test machinery with one-loop (DGLAP)
- use gluon in photon from GRV as input
  use normalization at input scale
- apply CCFM evolution (sing. terms only)
  with parameters obtained from proton ($Q_0 = 1.4$ GeV)

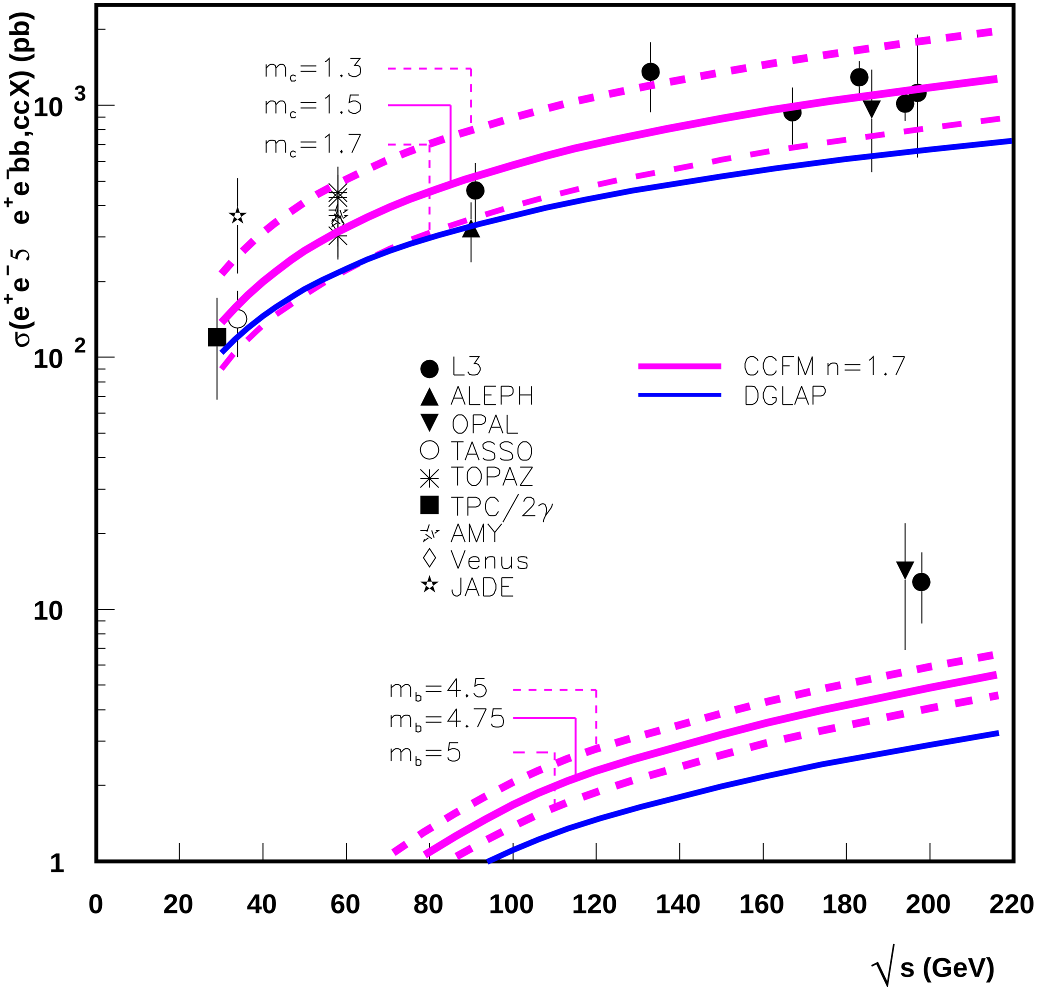


First un-integrated gluon density of real photon with full CCFM evolution

$\gamma\gamma \to b\bar{b}$

together with M. Hansson

- use matrix elements in k_t -factorization
  - $\gamma\gamma \to b\bar{b}$
  - $\gamma g \to b\bar{b}$
  - $gg \to b\bar{b}$
  - universality...



- **compare k_t -factorization & CCFM with NLO:**
- **using norm. from pdf**
  - **CCFM similar to NLO**
- **determine norm. for gluon from charm ($n = 1.7$ for res.γ)**
  - **CCFM larger than NLO**

  BUT still low for $\gamma\gamma \to b\bar{b}$

Conclusions

- k_t - factorization very successful
  - ☞ unintegrated gluon density from CCFM
  - ☞ fitted to F_2 from HERA
  - ☞ similar to NLO for charm and bottom at HERA and LEP
  - ☞ works also for charm and bottom in $p\bar{p}$
  - ☞ NLO better now ... (or better measurements ?) ....
  - ☞ main effect from off-shell ME and uPDF ....
- k_t - factorization useful for estimate of higher order corrections
- k_t - factorization well suited for simulation of had. final state
- well suited for MC implementation (CASCADE)