



*Sudakov logs and power corrections
for selected event shapes*

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Outline

Introduction

- Event shapes in e^+e^- annihilation
- Resummation of Sudakov logarithms
- Dressed gluon exponentiation

The C parameter

- The Sudakov exponent
- Towards phenomenology

A family of event shapes

- The a -thrust
- Scaling of power corrections

Perspective





Event shapes in e^+e^- annihilation

Picturing the final state of high-energy collisions

- ▶ Event shape distributions (and their moments) probe QCD from the perturbative to the non-perturbative regime.

finite order \longrightarrow resummation \longrightarrow power corrections

- ▶ They provide a detailed picture of the final state of high energy collisions.

energy flow \longleftrightarrow hadronization \longleftrightarrow mass effects

- ▶ A large amount of data is available (LEP, HERA ...)

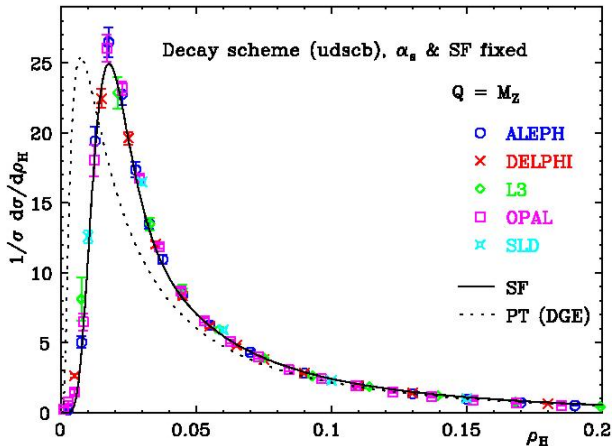
better theory \longleftrightarrow more analysis ?





A striking example

A fit of LEP data for the heavy jet mass distribution (Gardi, Rathsman).





Event shapes in e^+e^- annihilation

Examples

- ▶ Thrust: $T = \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{Q}$; $t = 1 - T$.
 → \vec{n}_T is used to define several other shape variables.
- ▶ C-parameter: $C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot q)(p_j \cdot q)}$.
 → does not require maximization procedures.
- ▶ a -thrust: $\tau_a = \frac{1}{Q} \sum_i (p_{\perp})_i e^{-|\eta_i|(1-a)}$.
 → recently introduced (Berger, Sterman)





Resummation of Sudakov logarithms

Infrared and collinear emission dominates the two-jet limit

- ▶ Large *double* logarithms of the variable vanishing in the two-jet limit ($L = \log t$; $L = \log C$; ...) enhance finite orders \rightarrow need to resum.
- ▶ A pattern of exponentiation emerges

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \rightarrow \exp \left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots \right]$$

- ▶ In general the Laplace transform exponentiates

$$\int_0^\infty d\tau e^{-\nu\tau} \frac{1}{\sigma} \frac{d\sigma}{d\tau} = \exp \left[\int_0^1 \frac{du}{u} (e^{-uv} - 1) \left(B(\alpha_s(uQ^2)) + \int_{u^2 Q^2}^{uQ^2} \frac{dq^2}{q^2} A(\alpha_s(q^2)) \right) \right].$$





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Resummation of Sudakov logarithms

Power corrections and shape functions

- ▶ The exponent is **ill-defined** because of the **Landau pole**
 regularization \rightarrow **ambiguity** \rightarrow **power corrections**
- ▶ Focus on **small** τ , **large** ν , set IR factorization scale μ , and expand in powers of ν/Q (neglecting ν/Q^2).

$$\begin{aligned}
 S_{\text{NP}}(\nu/Q, \mu) &= \int_0^{\mu^2} \frac{dq^2}{q^2} A(\alpha_s(q^2)) \int_{q^2/Q^2}^{q/Q} \frac{du}{u} (e^{-u\nu} - 1) \\
 &= \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\nu}{Q}\right)^n \lambda_n(\mu^2),
 \end{aligned}$$

- ▶ The parameters $\lambda_n(\mu^2)$ encode **non-perturbative** information.



Dressed gluon exponentiation

It is possible to combine **renormalon** methods and **Sudakov resummation** to construct models of power corrections. One method is **dressed gluon exponentiation** (Gardi).

- ▶ **Step 1:** compute **characteristic function** $\mathcal{F}(k^2)$ of the dispersive method in the **Sudakov limit** (resum “bubble graphs”).



- ▶ **Step 2:** use dressed gluon distribution as kernel of exponentiation.

$$\ln \left(\frac{d\bar{\sigma}}{d\nu} \Big|_{DGE} \right) = \int_0^\infty d\tau \frac{d\sigma}{d\tau} \Big|_{SDG} (1 - e^{-\nu\tau}) .$$

- ▶ **Step 3:** Borel representation of the exponent suggests pattern of power corrections.



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Dressed gluon exponentiation

Features of DGE

- ▶ NLL Sudakov resummation reproduced by using “gluon bremsstrahlung” definition of running coupling. All subleading logs computed in the “large n_f ” limit.
- ▶ Factorial growth of subleading logs detected: power accuracy requires going beyond logarithms.
- ▶ Definite prescription for resummed PT at power accuracy.
- ▶ Phenomenology of thrust, jet masses; models of power corrections in the Sudakov region for DIS, Drell-Yan, fragmentation.





The C parameter

Virtual gluon emission at one loop

Renormalon effects in the **inclusive approximation** can be from the shape distribution computed with a **massive gluon**.

- ▶ Define the event shape with a gluon virtuality $\xi = k^2/Q^2$.

$$\mathbf{c}(x_1, x_2, \xi) \equiv \frac{C}{6} = \frac{(1-x_1)(1-x_2)(1-x_3+2\xi) - \xi^2}{x_1 x_2 x_3} .$$

- ▶ One-loop **matrix element** for virtual gluon emission

$$\mathcal{M}(x_1, x_2, \xi) = \frac{(x_1+\xi)^2 + (x_2+\xi)^2}{(1-x_1)(1-x_2)} - \frac{\xi}{(1-x_1)^2} - \frac{\xi}{(1-x_2)^2} .$$

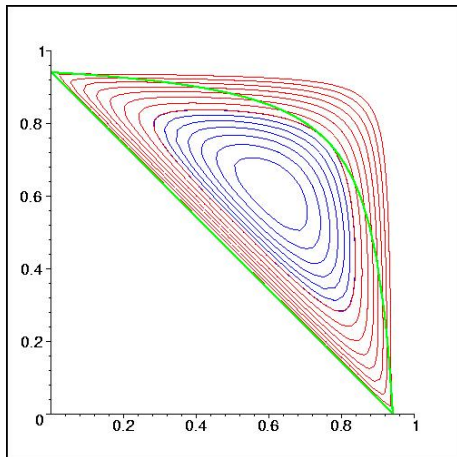
- ▶ **Characteristic function**

$$\mathcal{F}(\xi, c) = \int dx_1 dx_2 \mathcal{M}(x_1, x_2, \xi) \delta(\mathbf{c}(x_1, x_2, \xi) - c) .$$





Phase space with a 'massive' gluon



Phase space (green) and fixed c contours for $\xi = 0.06$

- ▶ Collinear limit:

$$x_{1,2} = 1 - \xi$$

- ▶ Soft limit:

$$x_1 = x_2 = 1 - \sqrt{\xi}$$

- ▶ $c_{\text{soft}} = \xi/x_3$

- ▶ Soft region

$$\frac{\xi}{1+\xi} < c < \frac{\sqrt{\xi}}{2}$$

- ▶ Hard region

$$\frac{\sqrt{\xi}}{2} < c < c_{\text{max}}$$



Constructing the Sudakov exponent

$\mathcal{F}(\xi, c)$ can be computed in closed form in terms of elliptic integrals (Gardi, LM). To perform DGE one must then

- ▶ Extract terms responsible for Sudakov logs.
- ▶ Integrate $\mathcal{F}(\xi, c)$ with the Borel representation of an appropriate running coupling.

$$\frac{1}{\sigma} \frac{d\sigma}{dc} \Big|_{\text{SDG}} = \frac{C_F}{2\beta_0} \int_0^\infty du (Q^2/\Lambda^2)^{-u} B(c, u).$$

- ▶ Exponentiate in Laplace space.

$$\frac{1}{\sigma} \frac{d\sigma}{dc} \Big|_{\text{DGE}} = \int_{k-i\infty}^{k+i\infty} \frac{d\nu}{2\pi i} e^{\nu c} \exp [S(\nu, Q^2)],$$

using the single gluon result as kernel

$$S(\nu, Q^2) = \int_0^\infty dc \frac{1}{\sigma} \frac{d\sigma}{dc} \Big|_{\text{SDG}} (e^{-\nu c} - 1).$$





Exponentiated renormalons

Results are summarized by the exponentiated Borel function.

$$S(\nu, Q^2) = \frac{C_F}{2\beta_0} \int_0^\infty du (Q^2/\Lambda^2)^{-u} B_c(\nu, u).$$

- ▶ For the C -parameter

$$B_c(\nu, u) = 2e^{5u/3} \frac{\sin \pi u}{\pi u} \left[\Gamma(-2u) (\nu^{2u} - 1) 2^{1-2u} \frac{\sqrt{\pi} \Gamma(u)}{\Gamma(\frac{1}{2} + u)} - \Gamma(-u) (\nu^u - 1) \left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u} \right) \right].$$

- ▶ Compare with the thrust

$$B_t(\nu, u) = 2e^{5u/3} \frac{\sin \pi u}{\pi u} \left[\Gamma(-2u) (\nu^{2u} - 1) \frac{2}{u} - \Gamma(-u) (\nu^u - 1) \left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u} \right) \right].$$



Towards phenomenology

Thrust and C -parameter have common features and differences.

- ▶ Sudakov logarithms **coincide** up to NLL (Catani, Webber), but **differ** beyond, due to soft emission.
NOTE: numerical growth of coefficients **milder** for C .
- ▶ Similar **pattern** of power corrections (**odd** powers of ν/Q).
However **smaller residues** of renormalon poles for C .
- ▶ **Origin** of differences: scale of soft emission is $2Qc$ for $c = C/6$, while it is Qt for $t = 1 - T$.
- ▶ **Consequences**: resummed **perturbative** prediction, and approximation of shape function by **shift** expected to **work better** for c than for t .





The a -thrust

- ▶ Definition: $\tau_a = \frac{1}{Q} \sum_i (p_{\perp})_i e^{-|\eta_i|(1-a)}$.

Also: $\tau_a = \frac{1}{Q} \sum_i \omega_i (\sin \theta_i)^a (1 - |\cos \theta_i|)^{1-a}$,

- ▶ Some properties

- ▶ $\tau_0 = t$; $\tau_1 = B$.
- ▶ $a \leq 2$ for IR safety.
- ▶ $a \leq 1$ for feasibility of resummation.

- ▶ For **negative** a , high rapidity particles (*w.r.t.* the thrust axis) are weighted less: **better** collinear behavior.
- ▶ At **one loop**, with the thrust axis given by particle i ,

$$\tau_a = \frac{(1-x_i)^{1-a/2}}{x_i} \left[(1-x_j)^{1-a/2} (1-x_k)^{a/2} + (j \leftrightarrow k) \right] .$$



Resummation for the a -thrust

- ▶ Sudakov logs at one loop have **simple scaling** with a .

$$\left. \frac{d\sigma}{d\tau_a} \right|_{\log}^{(1)} = \frac{2}{2-a} \frac{2}{\tau_a} C_F \frac{\alpha_s}{\pi} \ln\left(\frac{1}{\tau_a}\right) = \frac{2}{2-a} \left. \frac{d\sigma}{dt} \right|_{\log}^{(1)}.$$

- ▶ Resummation (Berger, Kucs, Sterman) is **intricate**.

$$\ln[\tilde{\sigma}_{LL}(\nu, a)] = 2 \int_0^1 \frac{du}{u} \left[\int_{u^2 Q^2}^{u Q^2} \frac{dp_T^2}{p_T^2} A(\alpha_s(p_T)) \left(e^{-u^{1-a} \nu \left(\frac{p_T}{Q}\right)^a} - 1 \right) \right].$$

- ▶ General a -dependence of Sudakov logs is **nontrivial**.

$$g_1(x, a) = -\frac{4}{\beta_0} \frac{1}{1-a} \frac{A^{(1)}}{x} \left[\left(\frac{1}{2-a} - x \right) \ln(1 - (2-a)x) - (1-x) \ln(1-x) \right].$$



Scaling of power corrections

An analysis of power corrections for the a -thrust has been carried out using **resummation** (Berger, Sterman) and using **DGE** (Berger, LM). Remarkably

- ▶ A **simple scaling** holds. DGE yields

$$B_a^{\text{soft}}(\nu, u) = \frac{1}{1-a} \left[2 e^{5u/3} \frac{\sin \pi u}{\pi u} \Gamma(-2u) (\nu^{2u} - 1) \frac{2}{u} \right]$$

- ▶ **Collinear** contribution shows an **intricate** structure of **fractional** power corrections in DGE, but they are suppressed by either ν/Q^{2-a} or ν^b/Q^2 with $0 < b < 1$.
- ▶ In the **shape function** language

$$\ln \left[S_{\text{NP}}^{(a)}(\nu/Q, \mu) \right] = \frac{1}{1-a} \ln \left[S_{\text{NP}}^{(0)}(\nu/Q, \mu) \right] ,$$

in principle, a **testable prediction**.





Perspective

- ▶ Event shape distributions map the transition between **perturbative** and **non-perturbative** QCD (also in DIS!)
- ▶ Theoretical advances lead to **testable** QCD-motivated **models** of power corrections (**shape functions**).
- ▶ **DGE** combines Sudakov resummation with renormalon calculus.
 - ▶ All available **perturbative** information is **retained**.
 - ▶ A well-defined **matching** between PT and NP is **provided**.
- ▶ Different **models** for shape functions (different approximations) yield testable **predictions**: C -parameter, α -thrust ...
- ▶ Detailed **phenomenology** awaits destiny of **data** ...

