Theoretical uncertainties in event shape predictions

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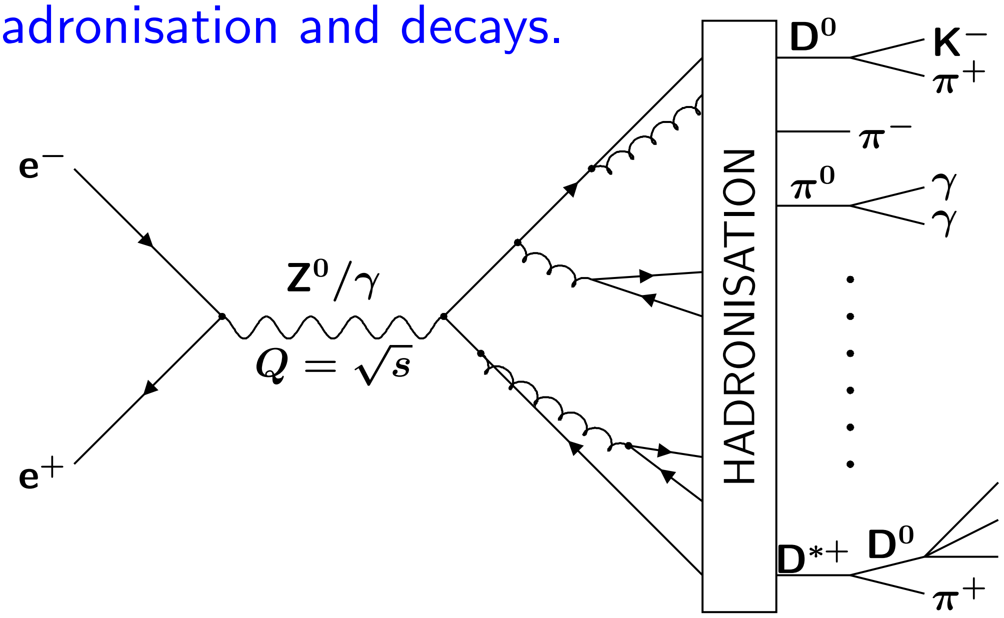
Introduction to event shapes

- Aim to test perturbative QCD and measure α_s in hadronic final states, at large energy scales Q (e.g. $Q = \sqrt{s}$ in e^+e^- annihilation).

- *BUT* experiments can only observe the event after hadronisation and decays.



- Can make our analysis insensitive to non-perturbative physics, as far as possible, by defining *infra-red safe* quantities, which are invariant under perfectly soft or collinear branchings

- Six *event shapes observables* traditionally used in e^+e^- analysis:

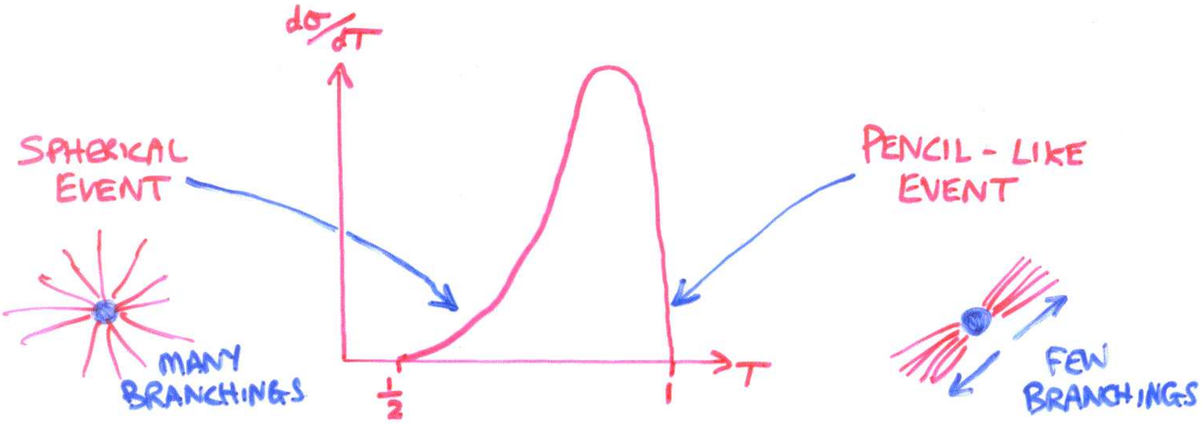
Thrust, T
Heavy jet mass, M_H
C-parameter, C
Total jet broadening, B_T
Wide jet broadening, B_W
Durham y-cut, y_{23}

Each describes a different aspect of the overall event topology.

Introduction to event shapes (contd...)

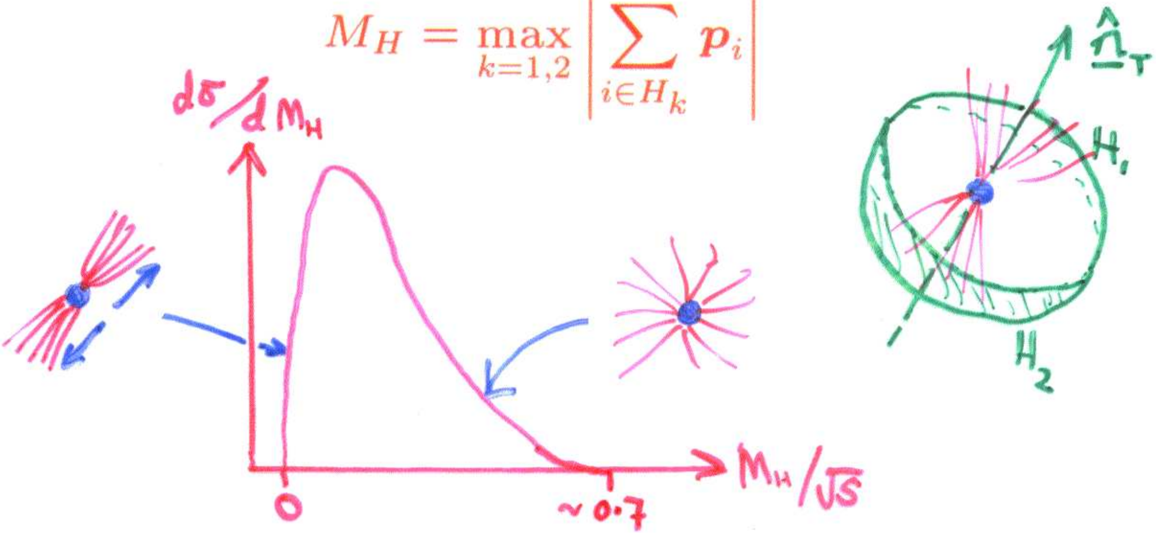
- Example 1: Thrust (T):
  An axis $\hat{\boldsymbol{n}}_T$, called the *thrust axis*, is chosen so as to maximize the sum of absolute momentum components for *all observed particles* projected along that axis.

$$T = \max_{\hat{\boldsymbol{n}}} \left(\frac{\sum_i |\boldsymbol{p}_i \cdot \hat{\boldsymbol{n}}|}{\sum_i |\boldsymbol{p}_i|} \right)$$



- Example 2: Heavy Jet Mass (M_H):
  The event is split into two hemispheres H_1 and H_2, by a plane normal to the thrust axis. We calculate the total invariant mass for particles in each hemisphere, and define M_H to be the larger of the two.

$$M_H = \max_{k=1,2} \left| \sum_{i \in H_k} \boldsymbol{p}_i \right|$$



Experimental analysis at LEP

1. Select events $e^+e^- \rightarrow Z^0/\gamma \rightarrow$ hadrons
2. Measure statistical distributions of event shape observables
3. Fit pQCD predictions to data, with α_s as a free parameter
   (N.B. need to correct "parton" level theory predictions for non-perturbative effects)

Four sources of uncertainty in α_s

- Data statistics
- Experimental systematics
- Non-perturbative corrections
  (e.g. MC hadronisation models)
- Perturbative theory uncertainty

Perturbative theory uncertainty is by far the largest, after combining all LEP measurements
⇒ needs thorough investigation.

$$\alpha_s(M_Z) = 0.1202 \pm 0.0003\ \text{(stat.)} \pm 0.0009\ \text{(exp.)} \pm 0.0013\ \text{(hadr.)} \pm 0.0047\ \text{(theo.)}$$

(LEP preliminary)

Event shapes: theory predictions

1. $\mathcal{O}(\alpha_s^2)$ matrix elements

With current QCD technology we can calculate exact matrix elements up to $\mathcal{O}(\alpha_\mathrm{s}^2)$.

Matrix elements are coded into the EVENT2 Monte Carlo $\Rightarrow$ can determine the cumulative distribution $R(y)$ for an observable y:

$$R(y) = \int_0^y \mathrm{d}y \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}y} = 1 + \mathcal{A}(y)\alpha_s + \mathcal{B}(y)\alpha_s^2$$

($\mathcal{A}(y)$ and $\mathcal{B}(y)$ tabulated numerically for each event shape $y \equiv 1-T, M_H, \ldots$)

Theoretical ambiguity:

Need to choose a renormalisation scale μ for the matrix elements. Conventionally $\mu = Q$, but in general

$$\begin{aligned} R(y) &= 1 + \mathcal{A}(y)\alpha_s(\mu) \\ &\quad + \left[\mathcal{B}(y) + \frac{\beta_0}{\pi}\ln(\mu/Q)\mathcal{A}(y)\right]\alpha_\mathrm{s}^2(\mu) + \ldots \end{aligned}$$

where

$$\alpha_\mathrm{s}(\mu) = \alpha_\mathrm{s}(Q) - \frac{\beta_0}{\pi}\ln(\mu/Q)\alpha_\mathrm{s}^2(Q) + \ldots$$

μ-dependence completely cancels out when series known to all orders.

Traditionally use variation $\frac{1}{2}\sqrt{s} < \mu < 2\sqrt{s}$ as measure of unknown higher order terms.

Event shapes: theory predictions (contd...)

2. NLLA resummations

Power series in α_s are useful provided

- α_s is small (so O.K. at high energy)
- The missing coefficients $\mathcal{C}(y)$,$\mathcal{D}(y)$,. . . do not grow faster than $1/\alpha_s^n$.

Unfortunately as $y \to 0$ (2-jet limit), coefficients are enhanced by powers of $L = \ln(1/y)$
$\Rightarrow$ $\mathcal{O}(\alpha_\mathrm{s}^2)$ not a good approximation.

Need an alternative series expansion, to *resum* the logarithms L to all orders in α_s:

$$\ln R(y) = Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots$$

The functions g_1 and g_2 are known analytically for several event shapes, and can now be computed numerically for others.

Theoretical ambiguity:

Instead of resumming the logarithms $L = \ln(1/y)$, we could re-scale our observables by a factor x_L and resum $\widetilde{L} = \ln[1/(x_L y)]$.

$x_L = 1$ gives the most natural relationship between L and the 'physical' log $\ln k_\mathrm{T}$ of a radiated gluon.

Variation around $x_L = 1$ estimates the theoretical uncertainty due to missing terms in the resummation.

Event shapes: theory predictions (contd...)

3. Combined $\mathcal{O}(\alpha_s^2)$ + NLLA calculations

We have two formulae for the distribution $R(y)$:

- $\mathcal{O}(\alpha_s^2)$ perturbation series, good at large y
- NLLA resummation, good at small y

Need to combine them ⇒ general expression $\forall y$.

Write $\ln R$ as a 2-variable expansion in L and α_s:

$\ln R(y) =$	$A(y)\alpha_s$	$B(y)\alpha_s^2$	$C(y)\alpha_s^3$	$\dots$
$Lg_1(\alpha_s L)$	$G_{12}\alpha_s L^2$	$G_{23}\alpha_s^2 L^3$	$G_{34}\alpha_s^3 L^4$	$\dots$
$g_2(\alpha_s L)$	$G_{11}\alpha_s L$	$G_{22}\alpha_s^2 L^2$	$G_{33}\alpha_s^3 L^3$	$\dots$
$\alpha_s g_3(\alpha_s L)$	$-$	$G_{21}\alpha_s^2 L$	$G_{32}\alpha_s^3 L^2$	$\dots$
$\vdots$	$-$	$-$	$\vdots$	$\ddots$

"Log R matching scheme":

$$\begin{aligned}\ln R(y) &= \text{NLLA resummation } \textit{[first 2 rows]}\\ &+ \text{Exact } \mathcal{O}(\alpha_s^2) \textit{ [first 2 cols]}\\ &- \text{double-counted terms } \textit{[in green]}\end{aligned}$$

Theoretical ambiguity:

Instead of matching expressions for $\ln R(y)$, we could match expressions for $R(y)$ ["R matching scheme"].

$\mathcal{O}(\alpha_s^2)$ terms in $\ln R(y)$ may be $\mathcal{O}(\alpha_s^3)$ in $R(y)$, etc.
⇒ different result

Event shapes: theory predictions (contd...)

4. Applying kinematic contraints

Each event shape variable must lie within certain physical bounds, $0 \leq y < y_{\max}$.

But theory predictions are incomplete
$\Rightarrow$ may give probability $\neq 0$ for unphysical values

Need a trick to restore physical bounds on the distribution artificially. Modify the logarithm L in the NLLA prediction:

$$L = \ln\left(\frac{1}{y}\right) \quad \rightarrow \quad \widetilde{L} \equiv \frac{1}{p} \ln\left[\frac{1}{y^p} - \frac{1}{y_{\max}^p} + 1\right]$$

Then $\widetilde{L} = 0$ when $y = y_{\max}$

$\Rightarrow$
- cumulative distribution $R(y) = 1$
- differential distribution $R'(y) = 0$

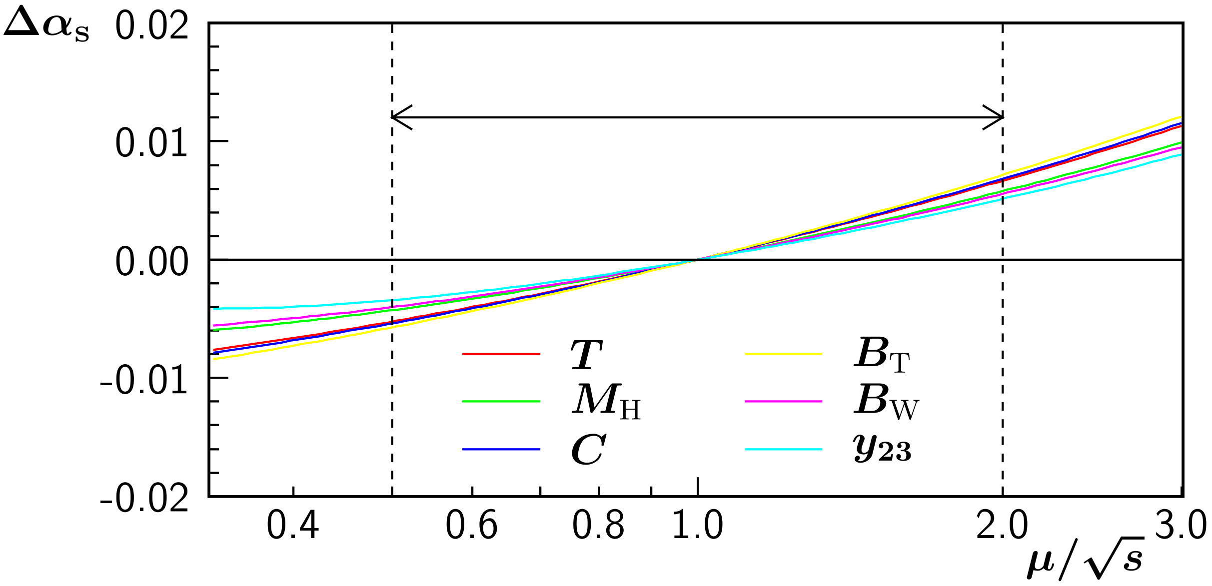
Theoretical ambiguities:

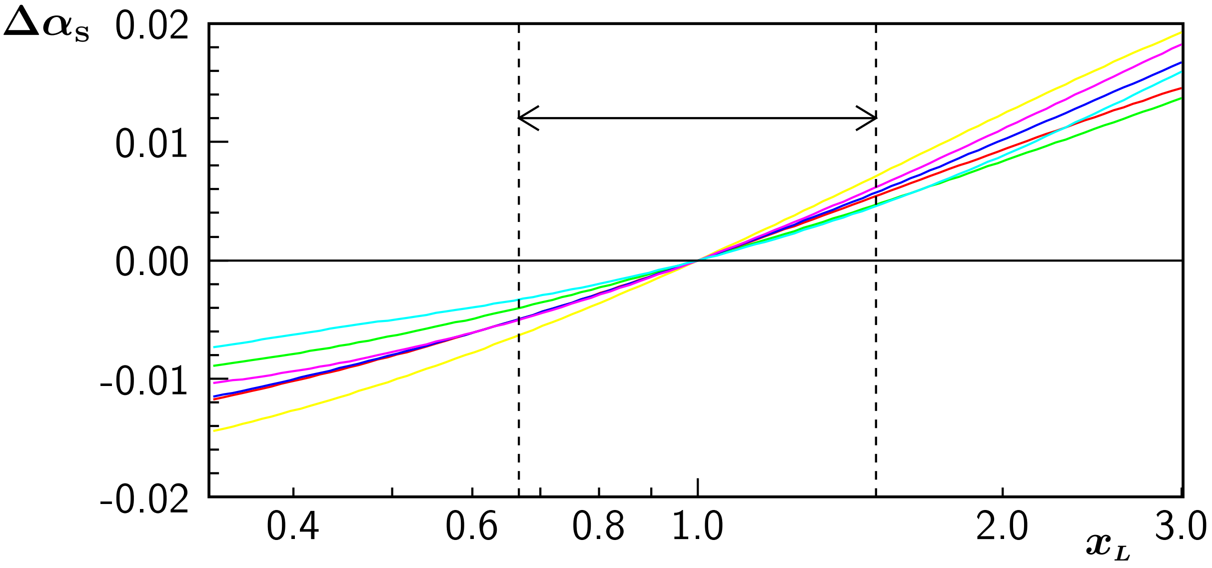
- Choice of parameter p, controlling sharpness of artificial contraint (conventionally $p = 1$)
- Choice of kinematic limit $y_{\max}$ (e.g. 4-parton limit or ∞-parton limit?)

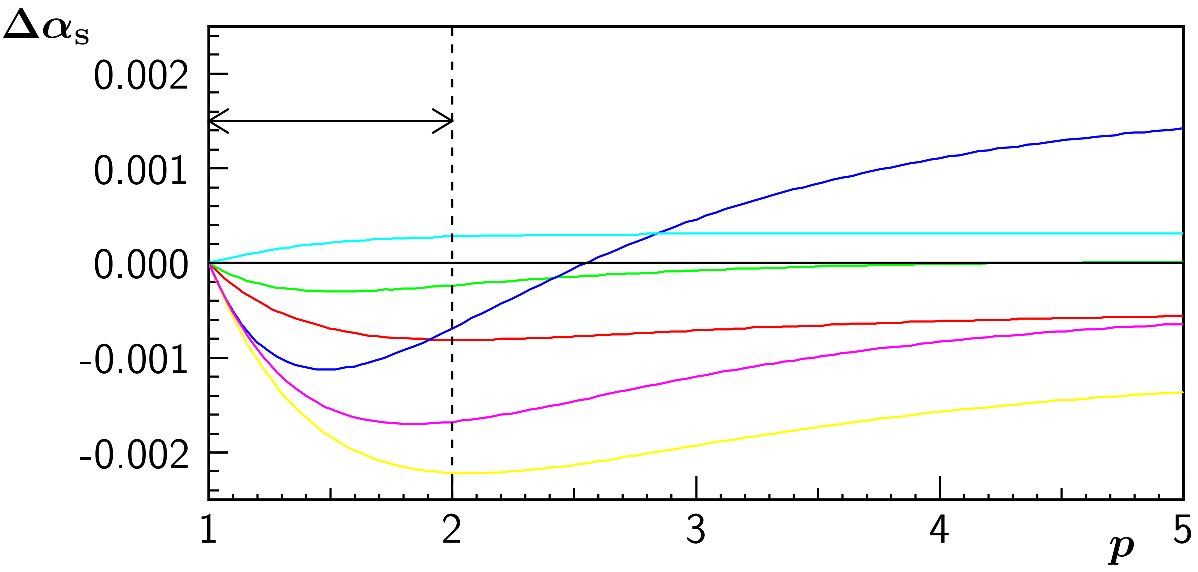
Theoretical uncertainties in α_s fits

Each variable parameter in the theory corresponds to a variation in the fitted α_s

$\Rightarrow$ can define a theoretical "uncertainty"







How to combine theory uncertainties?

- Each variation in the theory (μ, x_L, p, $y_{\max}$, matching scheme) probes a different subset of the missing higher orders.

  ⇒ traditional μ-variation is inadequate.

- The variations are *not* independent "sources" of uncertainty

  ⇒ cannot just "add errors in quadrature".

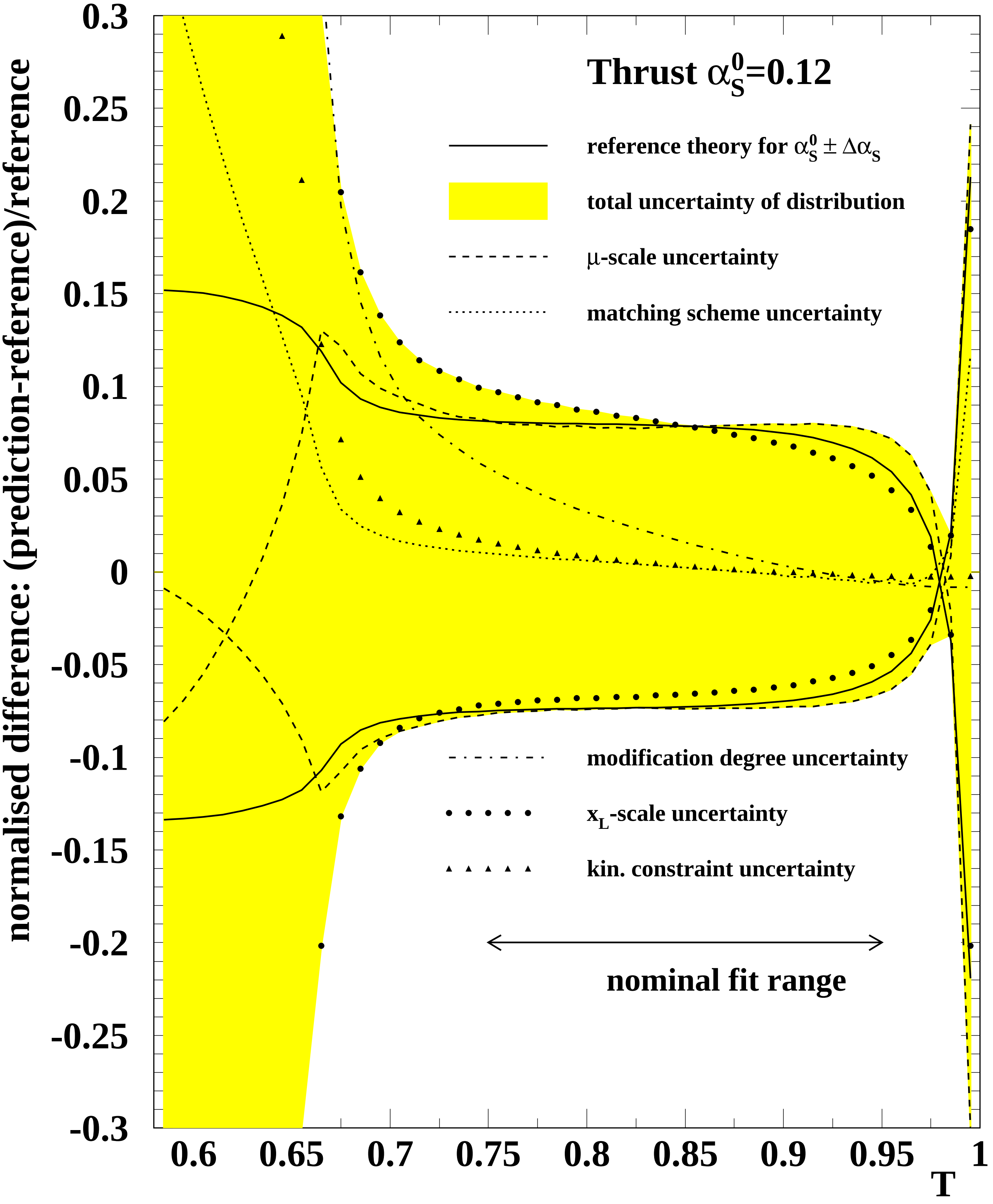
- Parameters can be varied simultaneously, and may affect different regions of the distribution

  ⇒ taking "maximum/average deviation" is too simplistic.

Need to combine uncertainties at the distribution level, to get the combined uncertainty in α_{s} . . .

The uncertainty band

- Overall uncertainty of distribution
  = envelope of uncertainties due to individual parameter/scheme variations [α_s fixed]
- Then vary α_s until *standard* prediction touches edge of band [μ, x_L, p, y_max fixed]



Range of variation for x_L

- Range $\frac{1}{2}\sqrt{s} < \mu < 2\sqrt{s}$ is rather arbitrary. Can we be more objective for x_L variation?

- Several methods proposed to set a range for x_L:
  - Euler's constant $\gamma_E = 0.5772$ appears naturally in the theory, suggesting $e^{-\gamma_E} < x_L < e^{\gamma_E}$.
  - Choose x_L such that certain matching coefficients are zero (e.g. $G_{21} = 0$).
  - Expand NLLA prediction in powers of α_s and compare with true $\mathcal{O}(\alpha_s^2)$ prediction. Vary x_L to reproduce the difference.

$$\int_{\text{fit range}} R'^{[x_L = x_L^{\text{max}}]}_{\text{matched}}(y) - R'^{[x_L=1]}_{\text{matched}}(y)$$
$$\sim \int_{\text{fit range}} R'^{[x_L=1]}_{\text{NLLA}}(y) - R'_{\mathcal{O}(\alpha_s^2)}(y)$$

  Unfortunately methods only agree to an order of magnitude.

- Instead choose x_L variation so that *on average* it has the same effect as the μ variation.

$$\begin{array}{ll} 2/3 < x_L < 3/2 & (T, M_{\mathrm{H}}, C, B_{\mathrm{T}}, B_{\mathrm{W}}) \\ 4/9 < x_L < 9/4 & (y_{23}) \end{array}$$

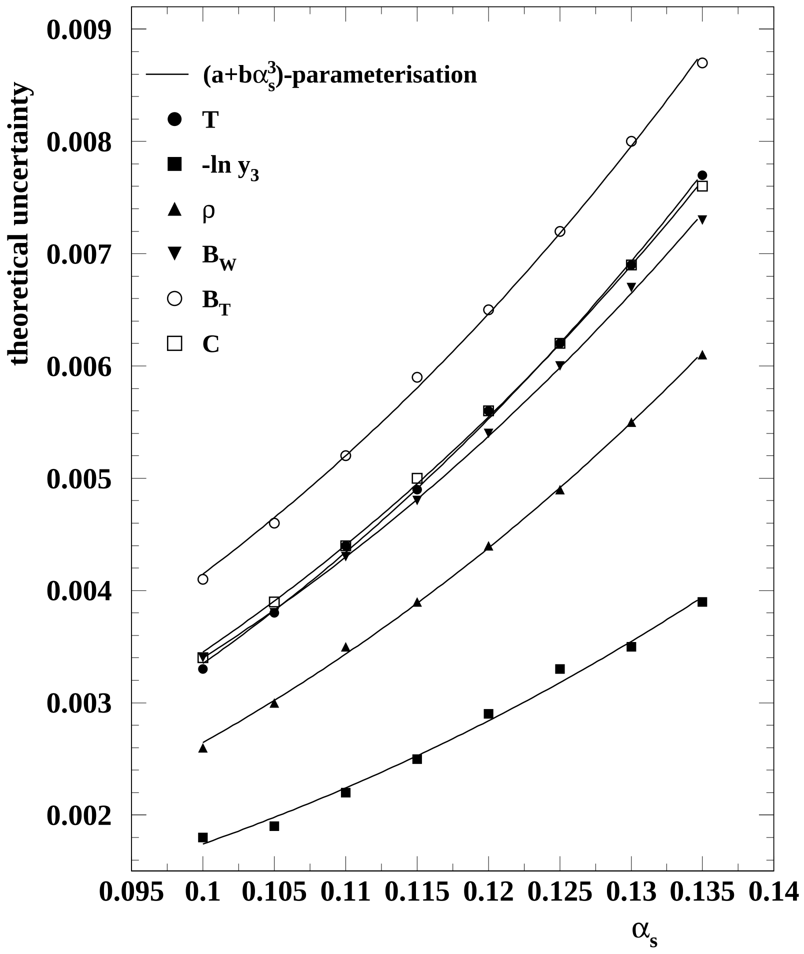
  No new information on absolute size of theory uncertainty. But better info on relative uncertainties for different observables.

Results

- Breakdown of uncertainties in α_s, for typical LEP results at $\sqrt{s} = 91$ GeV (based on $\alpha_s = 0.12$):

	Observable					
	T	M_H	C	B_T	B_W	y_{23}
μ	$+0.0055$ -0.0055	$+0.0028$ -0.0039	$+0.0052$ -0.0053	$+0.0062$ -0.0059	$+0.0018$ -0.0033	$+0.0027$ -0.0008
x_L	$+0.0048$ -0.0047	$+0.0041$ -0.0042	$+0.0044$ -0.0046	$+0.0045$ -0.0059	$+0.0053$ -0.0053	$+0.0017$ -0.0029
p	-0.0001	-0.0001	-0.0003	$+0.0003$	-0.0004	$+0.0002$
$y_{\max}$	$+0.0001$	-0.0001	$+0.0001$	$+0.0001$	-0.0001	-0.0001
Matching	$+0.0004$	$+0.0005$	$+0.0014$	$+0.0014$	$+0.0018$	-0.0007
Combined	$+0.0057$ -0.0055	$+0.0044$ -0.0045	$+0.0058$ -0.0054	$+0.0068$ -0.0062	$+0.0055$ -0.0054	$+0.0028$ -0.0030

- Uncertainty varies as $\sigma = a + b\alpha_s^3$ for other energies
  $\Rightarrow$ slightly smaller theory errors at LEP2



Conclusions

- A more robust method has been developed to assess the theoretical uncertainties in α_s measurements from e^+e^- event shapes.
- Method has been applied to the LEP α_s combination.
  (final publication expected later in 2004)
- Same "uncertainty band" algorithm could be applied to many other analyses involving fits to a theoretical distribution.
- Some arbitrariness remains in the range of variation for μ and x_L.
- Full details in JHEP 12 (2003) 007 or hep-ph/0312016
  (R.W.L. Jones, MTF, G.P. Salam, H. Stenzel, D. Wicke)