Dijet rates with symmetric E_t cuts

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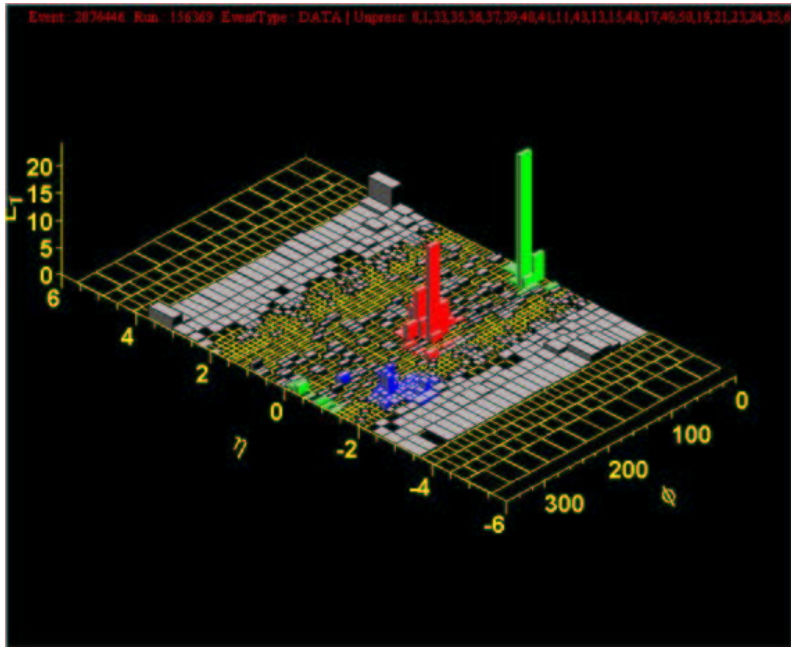
Work done in collaboration with:
Mrinal Dasgupta (CERN)

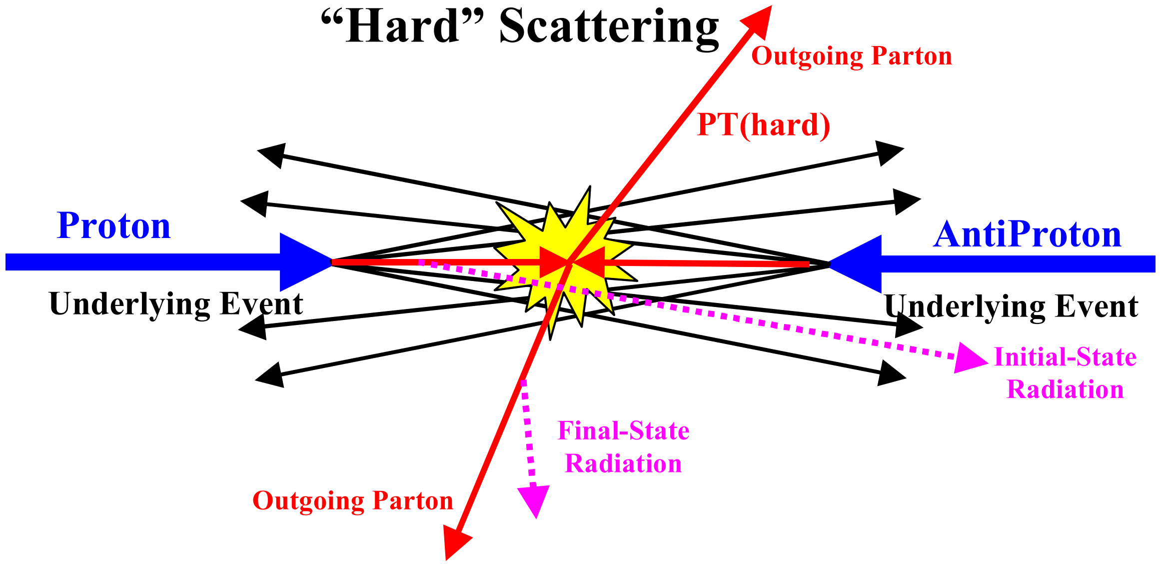
- Part 1: Motivations
  - Dijet events in hard processes
  - IR problems in NLO calculations
- Part 2: Resummation for IR sensitive observables
  - Transverse energy difference
- Part 3: Outlook



DIS 2004 Workshop, 15 April 2003

Hard events in QCD and jets

Jet events ⇔ signals for a hard scattering in QCD



☺ Cluster final state particles into jets ⇒ reflect partonic energy flow

↪Direct access to partonic cross sections

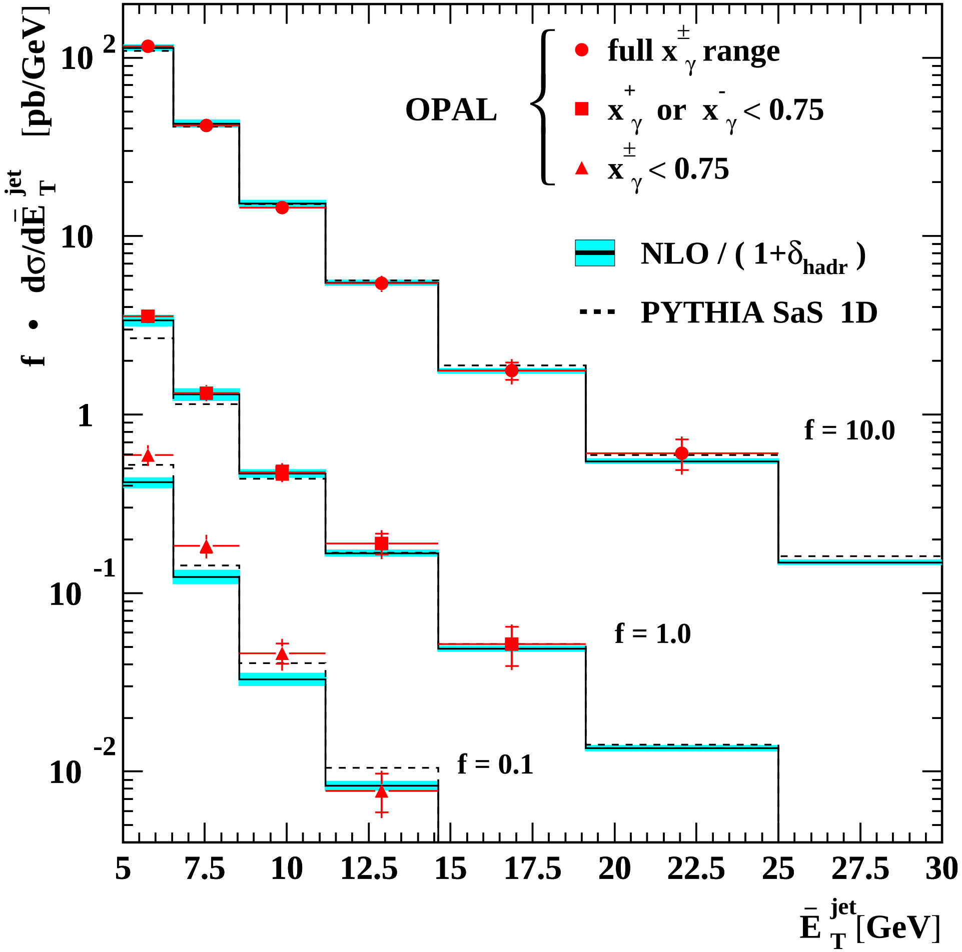
$$d\sigma_{Q\to j_1 j_2} = d\sigma_{Q\to cd} + \mathcal{O}\left(1/Q^2\right)$$

Dijet rates as a QCD probe

Dijet rate ⇒ probability that an event has two (or more) jets

Test of fixed order QCD predictions and QCD inspired MC's

[OPAL EPJC 31 (2003) 307]



Dijet rates as a QCD probe

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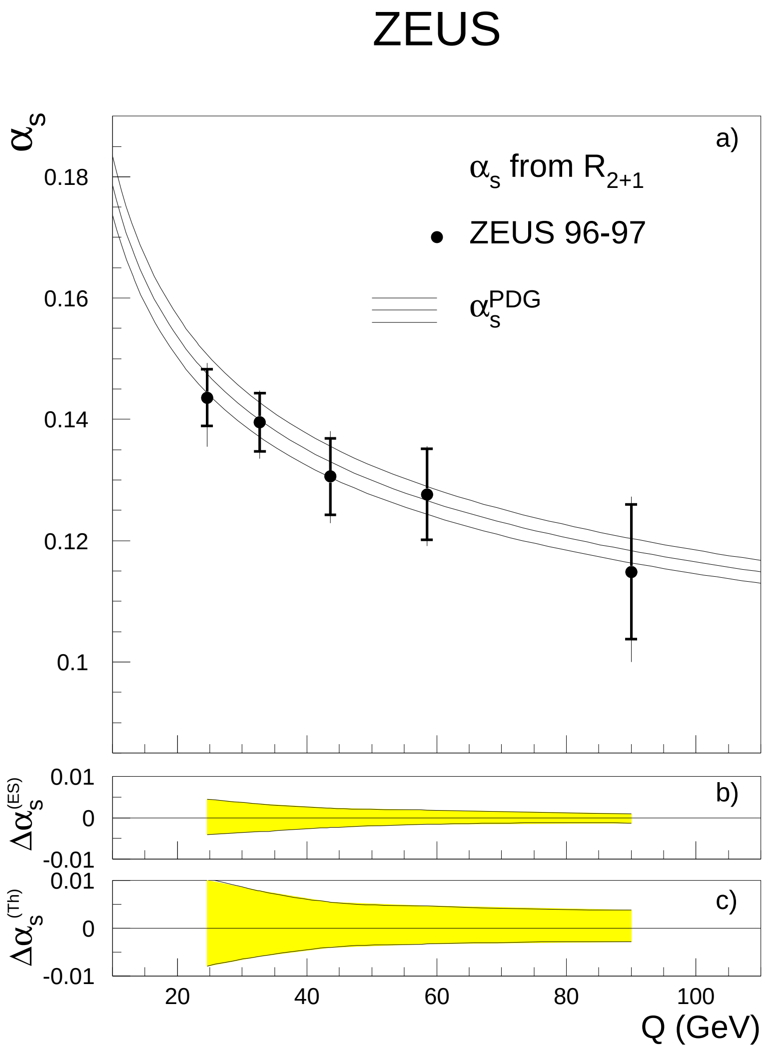
- Measurements of the coupling constant and its renormalisation group running

  [ZEUS PLB 507 (2001) 70]



Dijet rates as a QCD probe

Dijet rate ⇒ probability that an event has two (or more) jets

- Test of fixed order QCD predictions and QCD inspired MC's

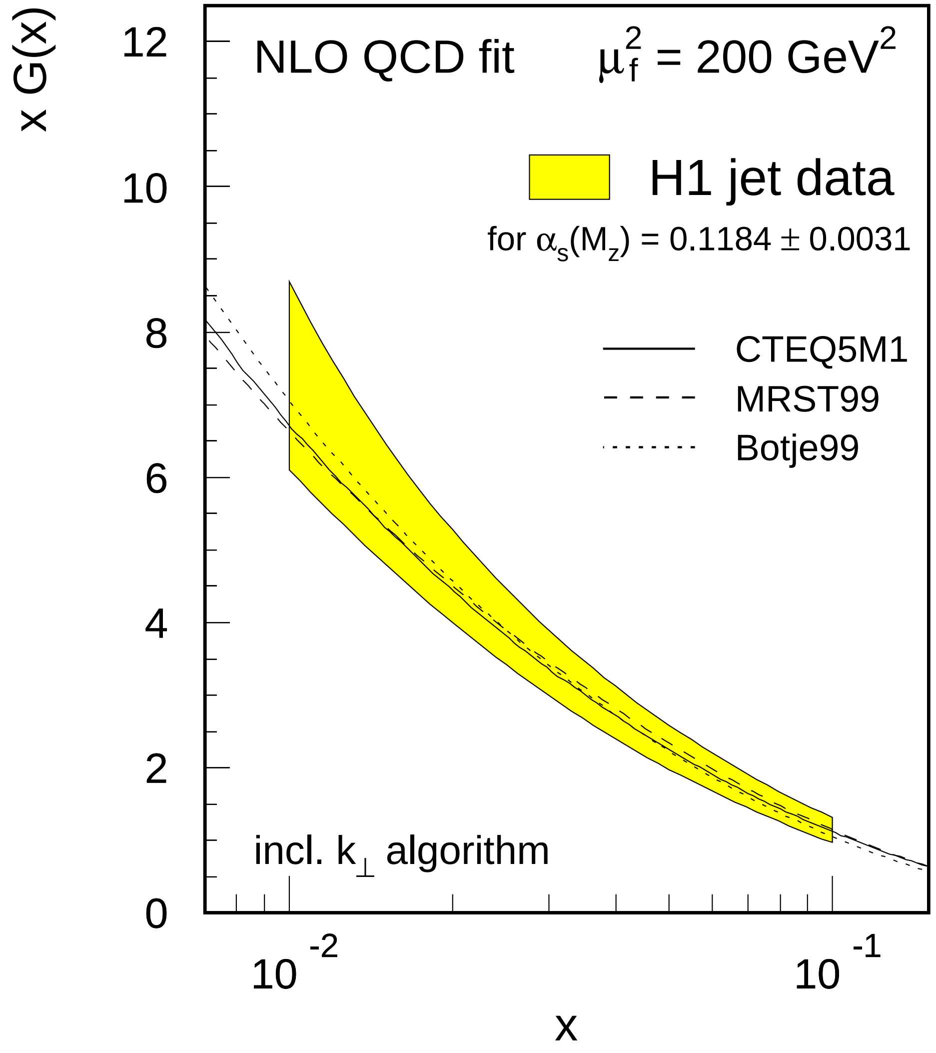
  [OPAL EPJC 31 (2003) 307]

- Measurements of the coupling constant and its renormalisation group running

  [ZEUS PLB 507 (2001) 70]

- Constraints on the gluon density in the proton

  [H1 EPJC 19 (2001) 289]



Problems in fixed order QCD calculations

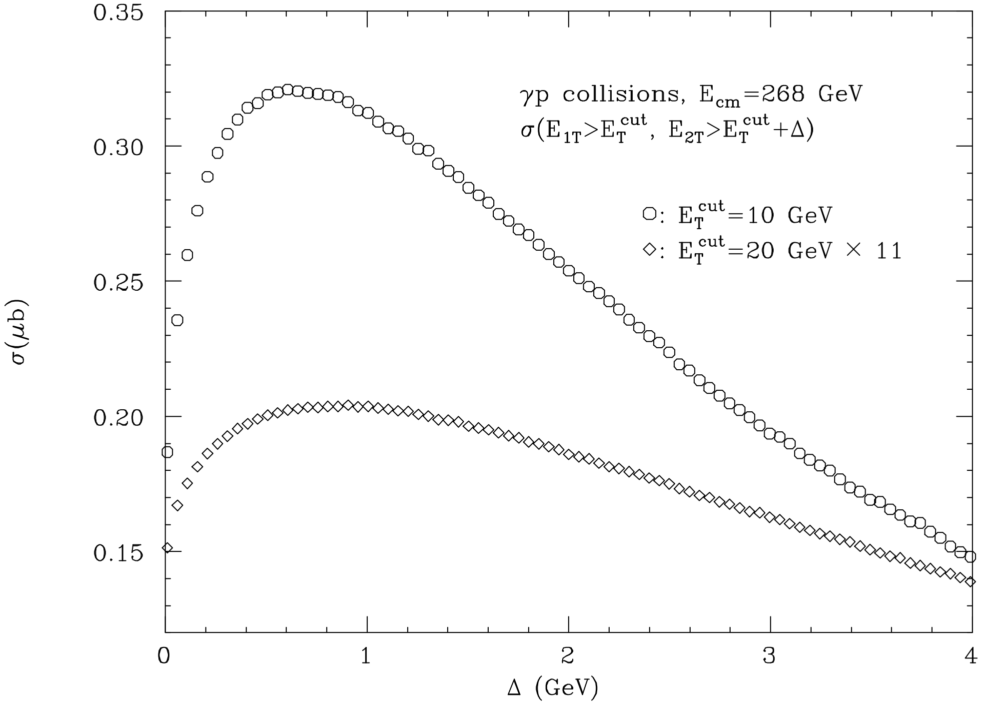
Dijet total rate: two highest E_t jets with $E_{t1} > E_{t2} > E_{\min}$

Symmetric cuts on jet E_t's

[Frixione, Ridolfi NPB 507 (1997) 315]

$$\sigma(\Delta) = \sigma(E_{t1} > E_{\min} + \Delta, E_{t2} > E_{\min})$$

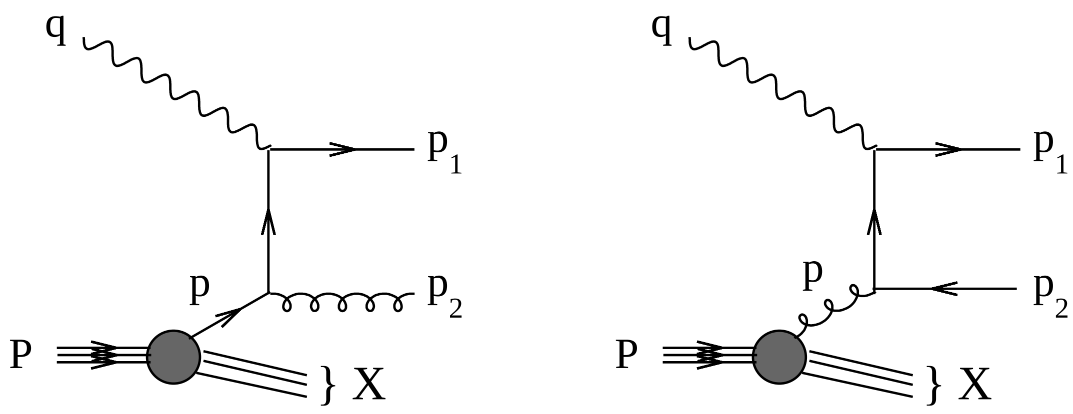
$$\sigma_{\rm FO}(\Delta) = \underbrace{\sigma_0(\Delta)}_{\rm LO} + \alpha_s \cdot \underbrace{\sigma_1(\Delta)}_{\rm NLO} + \ldots$$



The fixed order total rate $\sigma_{\rm FO}(0)$ is finite

The slope $d\sigma_{\rm FO}/d\Delta$ is infinite at $\Delta = 0$

Dijet total rate in DIS



Process kinematics

$$Q^2 = -q^2$$

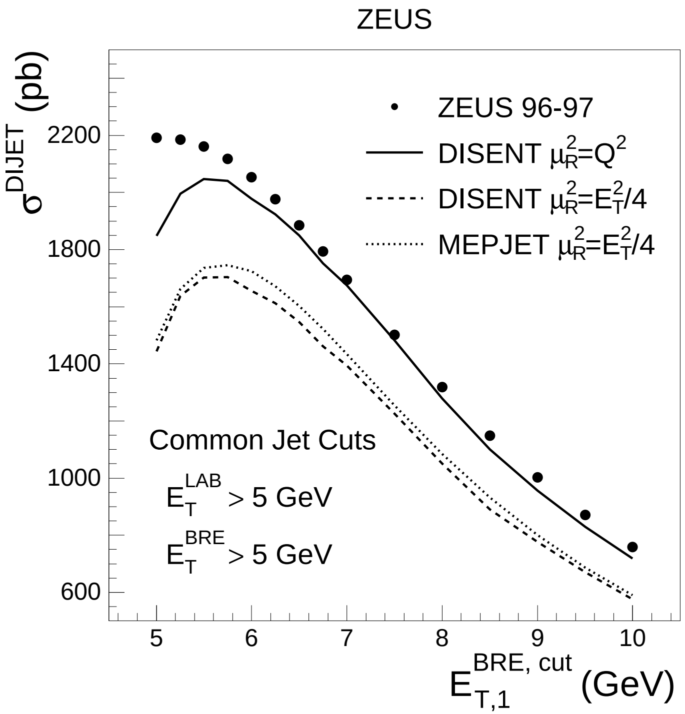
$$x = \frac{Q^2}{2(P \cdot q)}$$

Slope of the dijet rate ⇔ differential E_t distribution of the highest E_t jet

$$\sigma'(\Delta) \equiv \frac{d\sigma}{d\Delta} = -\frac{d\sigma}{dE_{t1}}(E_{t1} = E_{\min} + \Delta)$$

Physical behaviour of the slope $\sigma'(\Delta)$

- always negative definite and bounded
- stays finite for $\Delta \to 0$ ($\sigma'(\Delta) \sim \Delta$?)



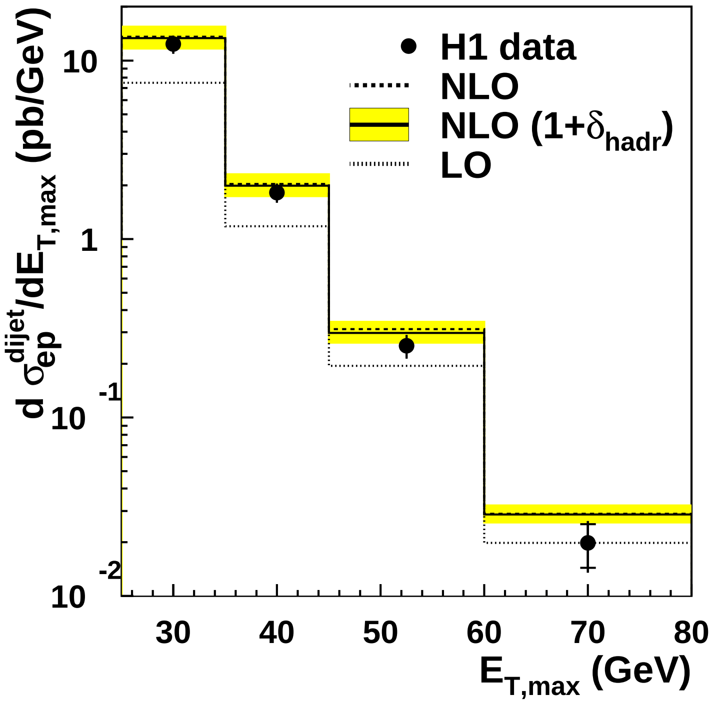
The usual solution: asymmetric E_t cuts

Problems avoided if $\Delta > \Delta_0$

[H1 EPJC 25(2002)13]

$$E_{t1} > 25\text{GeV}$$

$$E_{t2} > 10\text{GeV}$$

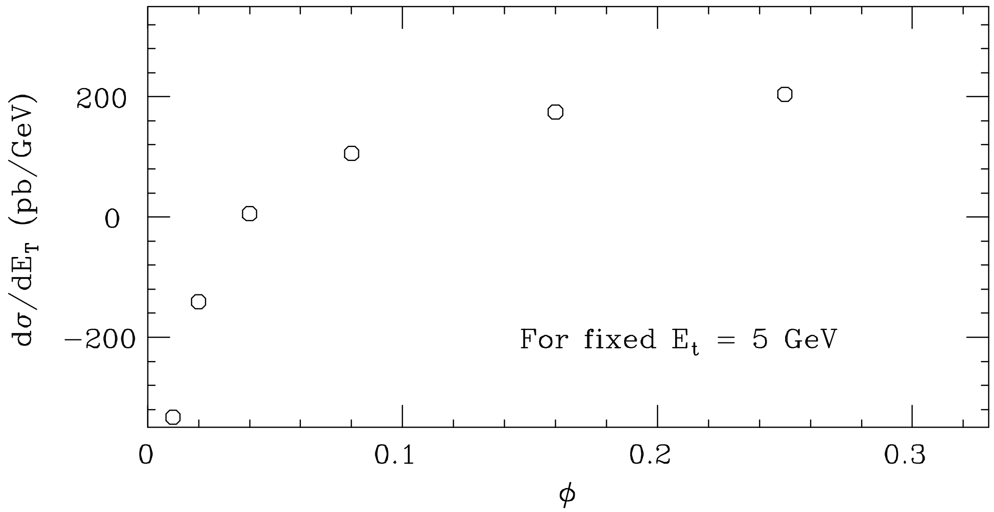


Dangerous regions have to be avoided!

[Bertora hep-ph/0306167]

$$(E_{t1} + E_{t2})/2 > E_{\text{min}}$$

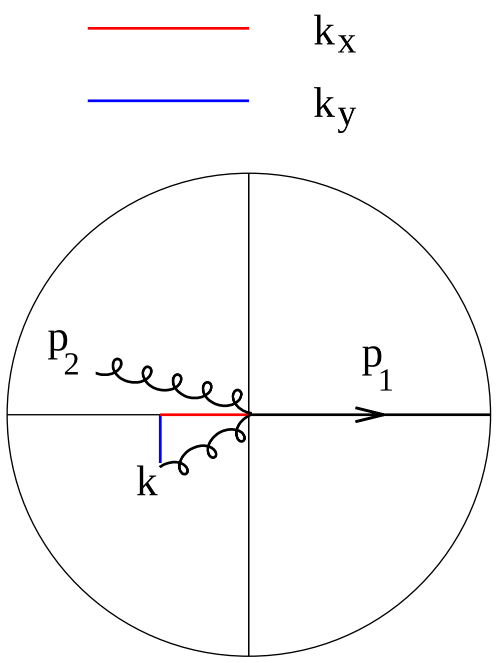
$$\phi = \frac{|E_{t1} - E_{t2}|}{E_{t1} + E_{t2}} < \frac{1}{4}$$



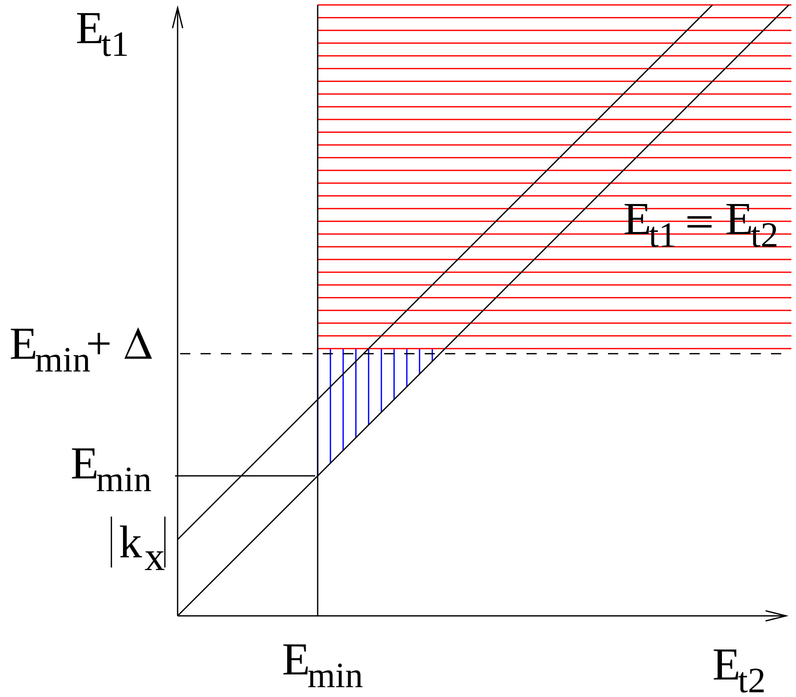
One soft gluon emission

Emission of a soft gluon not recombined with any of the two jets

➥ Transverse energy difference $E_{t1} - E_{t2} \simeq |k_x|$



Transverse plane



✗ E_{t1} lies along the line $E_{\rm min} + \Delta \Rightarrow |k_x| < \Delta$

$$\sigma'(\Delta) = -\frac{d\sigma}{dE_{t1}}\left(E_{t1} = E_{\rm min} + \Delta\right)$$

✗ Letting $\Delta \to 0$ (symmetric cuts) forces $|k_x| \to 0$

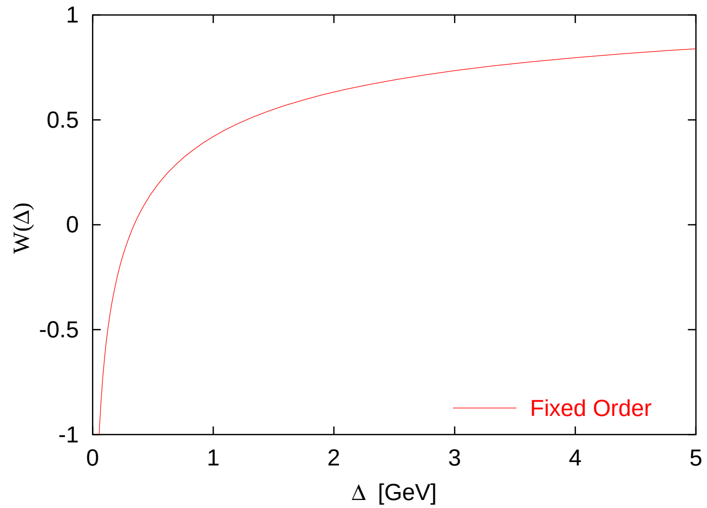
Resummation of soft gluon effects

- Suppression of soft radiation $\Rightarrow$ large logarithms in fixed order calculations

  IR singularities from one gluon soft and collinear to incoming parton p

$$\sigma'_{\rm NLO}(\Delta) = \sigma'_{\rm LO}(\Delta) \cdot W_{\rm NLO}(\Delta)$$

$$W_{\rm NLO}(\Delta) \simeq 1 - \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{\Delta}$$



Resummation of soft gluon effects

Suppression of soft radiation ⇒ large logarithms in fixed order calculations

IR singularities from one gluon soft ($\omega \to 0$) and collinear to the beam ($\theta \to 0$)

$$\sigma'_{\rm NLO}(\Delta) = \sigma'_{\rm LO}(\Delta) \cdot W_{\rm NLO}(\Delta)$$

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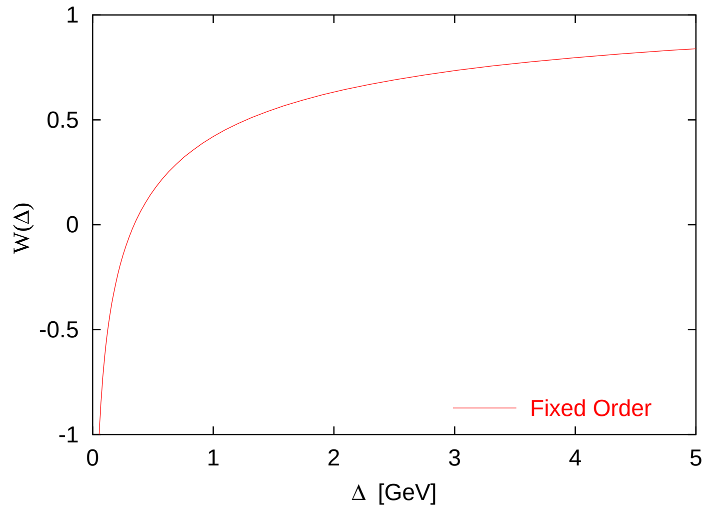
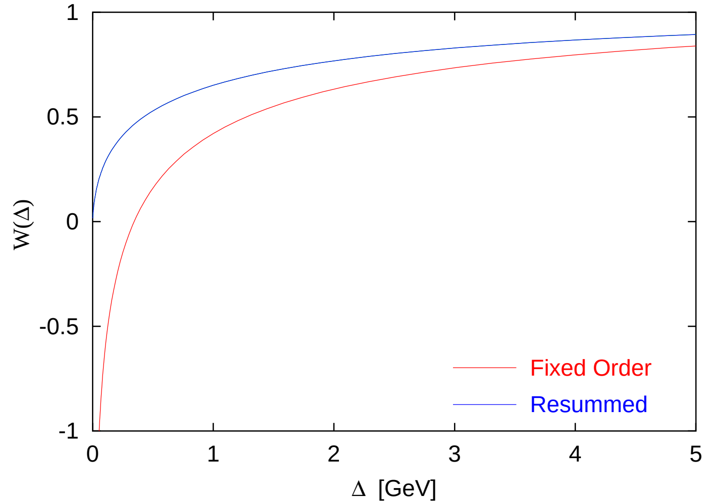


All order resummation of soft/collinear effects ⇒ regular behaviour for $\sigma'(\Delta)$

$$\sigma'(\Delta) = \sigma'_{\rm LO}(\Delta) \cdot W(\Delta)$$

$$W(\Delta) \simeq \frac{2}{\pi} \int_0^\infty \frac{db}{b} \sin(b\Delta) e^{-\frac{\alpha_s C_F}{\pi} \ln^2 bQ}$$



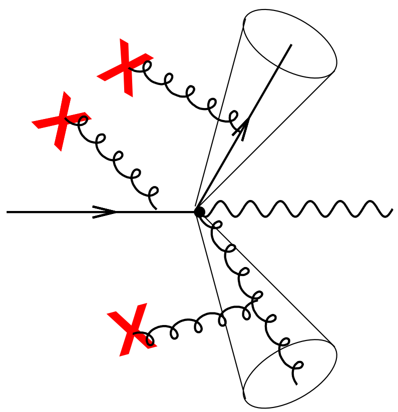
Full NLL resummation

$U(k_x) = \text{Prob}\left(\sum_{i\notin \text{jets}} k_{ix} = k_x\right) \Leftrightarrow k_x$ distribution of out-of-jet soft particles

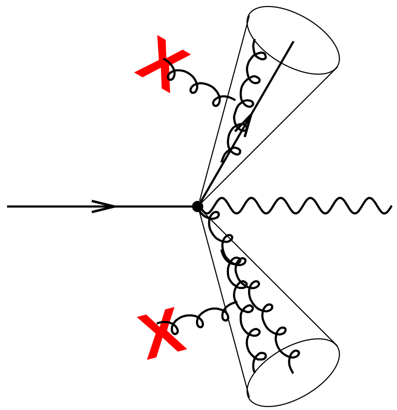
$$W(\Delta) = \int_{-\infty}^{\infty} dk_x\, U(k_x)\, \Theta(\Delta - |k_x|) = \frac{2}{\pi}\int_0^{\infty} \frac{db}{b} \sin(b\Delta)\, \Sigma(b)$$

NLL resummation in the three-jet limit for $\Sigma(b)$

$$\Sigma(b) = e^{\underbrace{L\, g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}}} = \underbrace{\frac{f_{p/P}(Q/b)}{f_{p/P}(Q)} \cdot e^{-R(b)}}_{\text{global}} \cdot \underbrace{S(b)}_{\text{non-global}} \qquad L = \ln b$$



global



non−global

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⚠ Initial state hard collinear $\Rightarrow$ factorisation scale modification $f_{p/P}(Q) \to f_{p/P}(Q/b)$

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Independent global emission $\Rightarrow$ exponentiation of single gluon emission $w(k)$

$$R(b) = \int [dk] w(k) \Theta(|k_x| - 1/b) = R_{\text{coll}}(b) + R_{\text{s}}(b) + R_{\text{alg}}(b)$$

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❶ collinear splitting (soft & hard)$\Rightarrow$ DY-like exponent $R_{\text{coll}}(b)$

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$$R_{\text{coll}}(b) = \int_{b_0^2/b^2}^{Q^2} \frac{dk^2}{k^2}\left[\underbrace{A_p(\alpha_s(k))}_{\text{LL}} \ln\frac{Q^2}{k^2} + \underbrace{B_p(\alpha_s(k))}_{\text{NLL}}\right] \qquad b_0 = 2e^{-\gamma_E}$$

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❷ large angle soft radiation $\Rightarrow$ geometry dependence in $R_s(b)$
↪ same as 3-jet event-shapes

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❸ explicit dependence on the jet algorithm only NLL in $R_{\text{alg}}(b)$

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Correlated non-global emission $\Rightarrow$ correction factor $S(b) = 1 + \alpha_s^2 \ln^2 b + \ldots$

➥Dependent on the jet algorithm

[Dasgupta, Salam PLB 512 (2001) 323]

General features of the result

Explicit results for the cone algorithm with the massive E recombination scheme
➥ $p_{\rm jet} = \sum_{i \in \rm jet} p_i$ and $E_{t,\rm jet} = |\vec{p}_{t,\rm jet}|$ [AB, Dasgupta hep-ph/0312108]

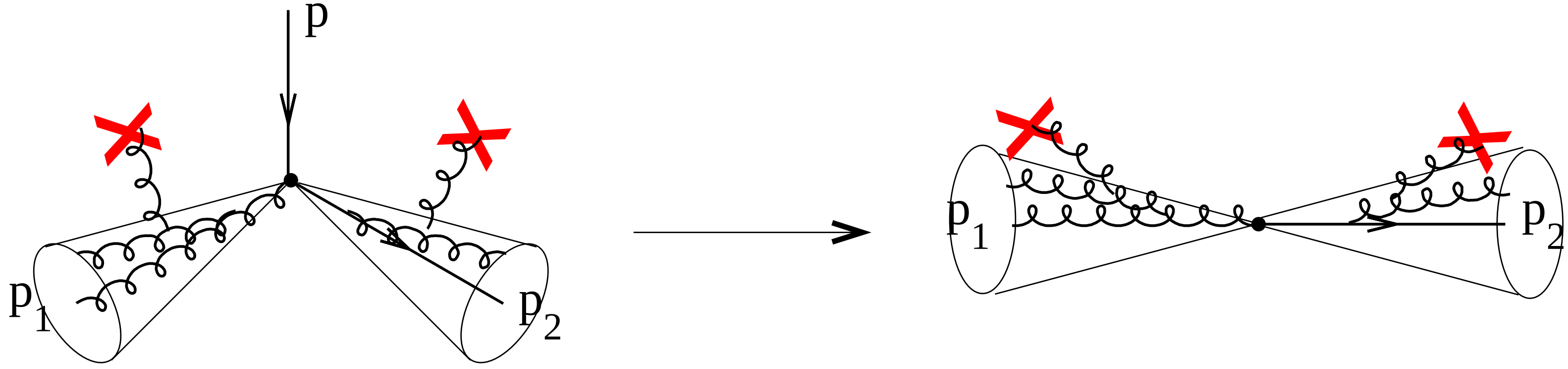
Jets ⇔ particles that flow in a cone of opening angle δ around p_1 and p_2

Non-global effects mainly from the cone boundary
➥ for small cone openings $S(b)$ is geometry independent

$$S(b) = \exp\left(-(C_1 + C_2) C_A \frac{\pi^2}{3} \cdot K(t) \cdot t^2\right) \qquad t = \int_{1/b}^{Q} \frac{dk}{k} \frac{\alpha_s(k)}{2\pi}$$

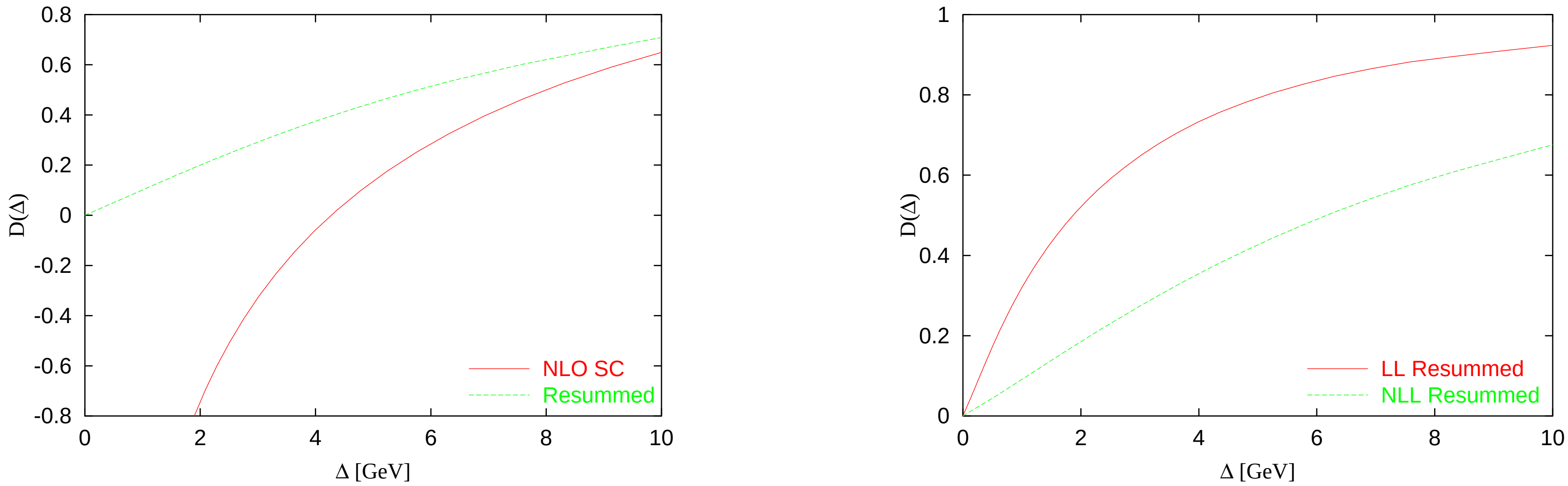
$K(t)$ computed by Dasgupta & Salam with a MC procedure

[Dasgupta, Salam PLB 512 (2001) 323]



Resummed results for the transverse energy difference

- DIS events with $Q = 20\text{GeV}$ and $x = 0.01$
- Dijet selected by cone algorithm with $\delta = 0.2$, $|\eta| < 1$ and $E_{\min} = 10\text{GeV}$
- Plots for the slope ratio $D(\Delta) = \sigma'(\Delta)/\sigma'_{\rm LO}(\Delta)$



- Resummation cures the divergence of fixed order calculations
- Linear behaviour for $\Delta \ll Q$ both at LL and at NLL level
- Non-global logs have a negligible impact on the answer (less than 5 %)

Matching with fixed order $\Rightarrow$ total rate $\sigma(0) = -\int_0^{\Delta_{\max}} d\Delta' \, \sigma'(\Delta')$

Alternative ways to obtain the total dijet rate

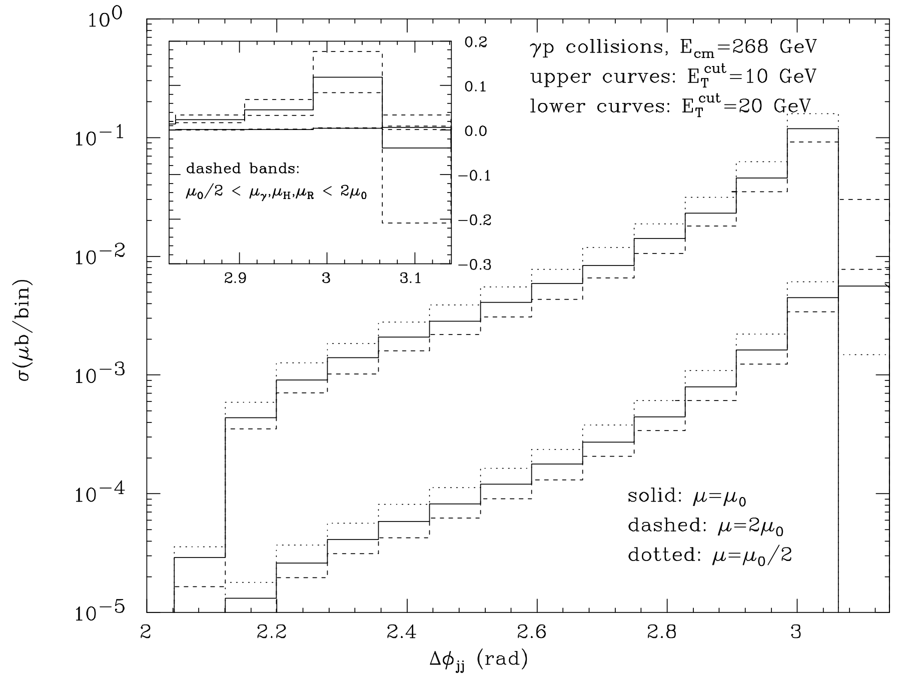
- Consider an observable V that vanishes in the (1+2)-jet limit

  - $V \to 0 \Leftrightarrow$ two jets with equal E_t's

  - After resummation and matching

$$\sigma(0) = -\int_0^{V_{\max}} dV' \frac{d\sigma}{dV'}$$

  - Example: azimuthal angle $V = \pi - \Delta\phi_{jj}$



- Transverse energy difference $E_{t1} - E_{t2}$ in the massless E_0 scheme

$$\left\{ \begin{array}{lcl} E_{ij} & = & E_i + E_j \\ \vec{p}_{ij} & = & \frac{\vec{p}_i+\vec{p}_j}{|\vec{p}_i+\vec{p}_j|} \cdot (E_i + E_j) \end{array} \right. \Rightarrow p_{ij}^2 = 0$$

  - $E_{t1} - E_{t2}$ sensitive to radiation inside the jets $\Rightarrow$ global observable
  - $\sigma'(\Delta)$ computed by the automated resummation program CAESAR
    ➥see Gavin Salam's talk

[AB, Salam, Zanderighi hep-ph/0304148]

Conclusions and outlook

- **Resummation** of soft gluon effects **cures** the pathological behaviour for dijet rates with symmetric cuts

- General features of the resummations of transverse energy difference
  - leading logarithms independent of the jet algorithm
  - soft large angle emissions: same as 3-jet event-shapes
  - algorithm dependence form of non-global logs
  - small hadronisation corrections ⇒ test PT QCD dynamics in 3-jet environment

Outlook. . .

- Study general features of jet invariant mass distributions
  - impact of threshold resummation
  - dependence on the jet algorithm

- Select a process and include matching to fixed order