The wide angles lepton pair production in heavy ion collisions. The role of multiphoton exchanges

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Abstract

The explicit expression for contribution to the matrix element of process of lepton pair creation at high-energy heavy ions collisions with two exchanged photons of each nuclei with pair components is obtained. The relevant contribution to cross section do not depend on the total center of mass energy but contains some enhancement-the square of logarithm of the ratio of values of transversal to the beam axes componenta of lepton and ion momenta. The contribution of amplitudes with more than two photos exchanged don't contain such an enhancement factor. The matrix elements with one exchanged photon with one of ions and one,two,tree exchanged photons with another ion for large angles of pair componenta emission also reviewed. We give the arguments in favor of validity of the same situation in QCD.

1 Introduction

In paper [6] an excellent demonstration of eikonal form of amplitude of type $3 \to 3$ was done considering the electron world line in the field of two colliding relativistic nuclei. It was shown that electron acquire a phase at subsequent interactions with each nuclei light-front field. Process of pair creation at heavy ions collisions implies the fields of nuclei to be considered at the same point of lepton pair creation. It's the reason of naive crossing relation of these amplitudes violation [2].

In our paper [1] we had mentioned the amplitude $M_{(2)}^{(2)}$, which was shown is irrelevant in leading and next-to leading logarithmical approximations. Nevertheless the knowledge of such kind of contributions becomes important for similar processes in QCD with multigluon exchanges between the colour constituents of each of colliding hadrons and the quark-antiquark pair created. One of interesting questions is the gluon density of nuclei,widely explored in phenomenological approaches to vector mesons production channels.

It's the motivation of our paper-further investigation of such a problem in frames QED.

In conclusion we analyze the behavior of the amplitudes $M_{(1,2,3)}^{(1)}$ in the wide angles kinematics of lepton pair creation.

2 $Z_1^2 z_2^2 \alpha^4$ contribution to lepton pair creation amplitude

Let now consider the set of Feynman amplitudes describing muon pair production at collisions of nuclei with charges Z_1, Z_2 by mechanism of two photon exchange each nuclei (See Fig.1):

$$Y_1(p_1, Z_1) + Y_2(p_2, Z_2) \to Y_1(p_1', Z_1) + Y_2(p_2', Z_2) + \mu^-(q_-) + \mu^+(q_+). \tag{1}$$

Keeping in mind the ultrarelativistic case considered below

$$s = (p_1+p_2)^2 >> M_1^2 \sim M_2^2 >> -q_1^2 = -(p_1-p_1')^2 \sim -q_2^2 = -(p_2-p_2')^2 >> m^2, \tag{2}$$
$$p_1^2 = (p_1')^2 = M_1^2, p_2^2 = (p_2')^2 = M_2^2, q_\pm^2 = m^2,$$

we introduce the Sudakov parametrization for 4-momenta of problem:

$$q_1 = a_1\tilde{p}_2 + b\tilde{p}_1 + q_{1\perp}; q_2 = a\tilde{p}_2 + b_2\tilde{p}_1 + q_{2\perp}; \tag{3}$$
$$k_1 = \alpha_1\tilde{p}_2 + \beta_1\tilde{p}_1 + k_{1\perp}; k_2 = \alpha_2\tilde{p}_2 + \beta_2\tilde{p}_1 + k_{2\perp};$$
$$q_\pm = \alpha_\pm\tilde{p}_2 + \beta_\pm\tilde{p}_1 + q_{\pm\perp},$$

with light-cone 4 vectors $\tilde{p}_{1,2}$ obeying the conditions

$$\tilde{p}_1^2 = \tilde{p}_2^2 = 0; \tilde{p}_{1,2}q_\perp = 0; 2\tilde{p}_1\tilde{p}_2 = s; q_\perp^2 = -\vec{q}^2 < 0.$$

We will assume virtual photons γ_1 and γ_2 with momenta $k_1, q_1 - k_1$ are emitted by nuclei Y_1 and photons γ_3, γ_4 with momenta $k_2, q_2 - k_2$-by the nuclei Y_2:

$$k_i = \alpha_i\tilde{p}_2 + \beta_i\tilde{p}_1 + k_{i\perp}, d^4k_i = \frac{s}{2}d\alpha_i d\beta_i d^2\vec{k}_i. \tag{4}$$

Kinematics of main contribution to the total cross section corresponds to the following region of variation of Sudakov variables:

$$\alpha_1 << \beta_1 \sim b\,; \beta_+ + \beta_- = b; \tag{5}$$
$$\beta_2 << \alpha_2 \sim a; \alpha_+ + \alpha_- = a;$$
$$a_1 << a, b_2 << b; q_{i\perp} = \vec{q}_i; \vec{q}_1 + \vec{q}_2 = \vec{q}_+ + \vec{q}_-,$$
$$\alpha_\pm = \frac{\vec{q}_\pm^2}{s\beta_\pm}, \vec{q}_\pm^2 >> m^2.$$

Contribution to the matrix element of such set of FD have a form:

$$M = i\frac{(Z_1 Z_2)^2(4\pi\alpha^2)^4}{(2\pi)^8}\int\int\frac{d^4k_1 d^4k_2}{k_1^2k_2^2(q_1-k_1)^2(q_2-k_2)^2} * \tag{6}$$
$$\bar{u}(p_1')O_1^{\mu_1\nu_1}u(p_1)\bar{u}(p_2')O_2^{\rho_1\sigma_1}u(p_2)\bar{u}(q_-)T^{\mu\nu\rho\sigma}v(q_+)g_{\mu\mu_1}g_{\nu\nu_1}g_{\rho\rho_1}g_{\sigma\sigma_1}.$$

Kinematics of main contribution provides proportionality of matrix element to s. To see it explicitly let adopt Gribov representation of virtual photons Green functions as

$$g_{\mu\mu_1}g_{\nu\nu_1}g_{\rho\rho_1}g_{\sigma\sigma_1} = (\frac{2}{s})^4 p_{1\mu}p_{1\nu}p_{1\rho_1}p_{1\sigma_1}p_{2\mu_1}p_{2\nu_1}p_{2\rho_1}p_{2\sigma}. \tag{7}$$

Numerators of the nuclei Green functions can be written as s^2N_1 and s^2N_2 with $N_1 = \bar{u}(p_1')\hat{p}_2u(p_1)/s, \sum|N_1|^2 = 2$ and similar expression for N_2. Denominators of virtual photons Green functions in the kinematics of main contribution depends only on transversal components of corresponding 4-vectors:

$$k_1^2k_2^2(q_1-k_1)^2(q_2-k_2)^2 = \vec{k}_1^2\vec{k}_2^2(\vec{q}_1-\vec{k}_1)^2(\vec{q}_2-\vec{k}_2)^2.$$

There are 24 FDs contributing to $M_{(2)}^{(2)}$. Instead of them it is convenient to consider $24*2*2 = 96$ FDs which take as well the permutations of emission and absorption points of exchanged photons to the nuclei. The result must be divided by $(2!)^2$. This trick provides the convergence of β_2 integrals:

$$\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} d\beta_2[\frac{s}{s\beta_2 - c - i0} + \frac{s}{-s\beta_2 - d - i0}] = 1 \tag{8}$$

and the similar integral on the variable α_1. After all operations we can write the matrix element in the form:

$$M = is2^8\frac{1}{(2!)^2}\pi^2\alpha^4(Z_1Z_2)^2N_1N_2\int\frac{d^2\vec{k}_1d^2\vec{k}_2}{\pi^2}\frac{R}{\vec{k}_1^2\vec{k}_2^2(\vec{q}_1-\vec{k}_1)^2(\vec{q}_2-\vec{k}_2)^2}, \tag{9}$$

with

$$R = \frac{1}{s}\int\frac{d\beta_1 d\alpha_2}{(2\pi i)^2}p_{1\mu}p_{1\nu}p_{2\rho}p_{2\sigma}\bar{u}(q_-)T^{\mu\nu\rho\sigma}v(q_+). \tag{10}$$

It's convenient to classify FD by order of photons absorption by muon world line. Namely we will mark them as $R_{ijkl}, R = \sum R_{ijkl}$ with different integers i, j, k, l from one to four counting from negative muon emission point. Among 24 FD there present sets which are gauge-invariant itself.

Consider first the set of 4 FD $R_{1234}, R_{2134}, R_{1243}, R_{2143}$. The sum of relevant contributions provides the convergence of β_1, α_2 integrations. After simple calculation we obtain:

$$R_{1234} + R_{2134} + R_{1243} + R_{2143} = \frac{1}{s}\bar{u}(q_-)\frac{\beta_1\hat{p}_1(-\hat{q}_{+\perp} + \hat{q}_{2\perp})\hat{p}_2}{\beta_+\vec{q}_-^2 + \beta_-(\vec{q}_- - \vec{q}_1)^2}v(q_+). \tag{11}$$

the similar expression can be obtained for the set $3412; 3421; 4312; 4321$ with result given below.

It can be seen that the set of FD 1342; 1432; 2341; 2431; 3124; 3214; 4123; 4213 give zero contribution to the matrix element. To see it let consider for definiteness the contribution $I_{1(34)2} = R_{1342} + R_{1432}$. Using Sudakov representation for relevant denominators of muon line we have for integral on β_1, α_2:

$$I_{1(34)2} = \int \frac{d\beta_1}{2\pi i} \frac{1}{s\alpha_-(\beta_- - \beta_1) - (\vec{q}_- - \vec{k}_1)^2 + i0} * \frac{1}{-s\alpha_+(\beta_- - \beta_1) - (-\vec{q}_+ + \vec{q}_1 - \vec{k}_1)^2 + i0} * \quad (12)$$

$$\int \frac{d\alpha_2}{2\pi i} [\frac{s(\beta_- - \beta_1)}{s(\beta_- - \beta_1)(\alpha_- - \alpha_2) - c + i0} + \frac{s(\beta_- - \beta_1)}{s(\beta_- - \beta_1)(-\alpha_+ + \alpha_2) - d + i0}].$$

The second integral after closing the integration α_2 countur in lower half plane give $sign(\beta_- - \beta_1)$. For β-integration we obtain zero :

$$I_{1(34)2} = \int \frac{d\beta_1 sign(\beta_1 - \beta_-)}{2\pi i} \frac{1}{s\alpha_-(\beta_- - \beta_1) - c + i0} * \frac{1}{-s\alpha_+(\beta_- - \beta_1) - d + i0} = 0. \quad (13)$$

Consider now R_{1324} contribution.The numenator have a form $N_{1324} = s\hat{p}_1\hat{p}_2(\hat{q}_- - \hat{k}_1)_\perp(\hat{q}_- - \hat{k}_1 - \hat{k}_2)_\perp(\hat{q}_- - \hat{q}_2 - \hat{k}_2)_\perp$. Performing α_2 integration we use the Sudakov form of relevant denominators:

$$(1) = (q_- - k_1 - k_2)^2 + i0 = s(\beta_- - \beta_1)(\alpha_- - \alpha_2) - (\vec{q}_- - \vec{k}_1 - \vec{k}_2)^2 + i0; \quad (14)$$
$$(2) = (-q_+ + q_2 - k_2)^2 + i0 = s(-\beta_+)(\alpha_- - \alpha_2) - (-\vec{q}_+ + \vec{q}_2 - \vec{k}_2)^2 + i0.$$

Nonzero contribution arises only in the case when the poles are situated in different half-planes (we imply $\beta_\pm > 0$) of α_2 plane. It provides some restriction on the region of β_1 integration:

$$\int \frac{sd\alpha_2}{2\pi i} \frac{1}{(1)(2)} = -\frac{\theta(\beta_- - \beta_1)}{(\beta_1 - \beta_-)(-\vec{q}_+ + \vec{q}_2 - \vec{k}_2)^2 - \beta_+(\vec{q}_- - \vec{k}_1 - \vec{k}_2)^2}. \quad (15)$$

The subsequent β_1 integration give the result:

$$R_{1324} = -\bar{u}(q_-)\frac{\beta_- N_{1324}}{2sD_{1324}} v(q_+), \quad (16)$$
$$D_{1324} = \beta_-(\vec{q}_- - \vec{k}_1)^2(-\vec{q}_+ + \vec{q}_2 - \vec{k}_2)^2 + \beta_+\vec{q}_-^2(\vec{q}_- - \vec{k}_1 - \vec{k}_2)^2.$$

Some simplification can be applied using Dirac equation $\bar{u}(q_-)\hat{q}_- = 0$ (we imply the values of all transversal momenta much larger than the lepton mass):

$$\bar{u}(q_-)\beta_-\hat{p}_1\hat{p}_2 = \bar{u}(q_-)\hat{p}_2\hat{q}_{-\perp}. \quad (17)$$

Similar results can be obtained for another contributions of the same type.

The final result have a form:

$$M_{(2)}^{(2)} = \frac{is}{(2!)^2} 2^8\pi^2(Z_1Z_2\alpha)^2 N_1N_2 \int \frac{d^2\vec{k}_1}{\pi}\frac{d^2\vec{k}_2}{\pi} \frac{\bar{u}(q_-)R_{(2)}^{(2)}\frac{\hat{p}_2}{s}v(q_+)}{\vec{k}_1^2\vec{k}_2^2(\vec{q}_1 - \vec{k}_1)^2(\vec{q}_2 - \vec{k}_2)^2}, \quad (18)$$

with

$$
\begin{aligned}
R^{(2)}_{(2)} = &-\frac{[\hat{q}_-(\hat{q}_- - \hat{q}_1)]_\perp}{\beta_+ q_-^2 + \beta_-(q_- - q_1)^2} + \frac{[(\hat{q}_+ - \hat{q}_1)\hat{q}_+]_\perp}{\beta_- q_+^2 + \beta_+(q_+ - q_1)^2} - \\
&\frac{[\hat{q}_-(\hat{q}_- - \hat{k}_1)(\hat{q}_- - \hat{k}_1 - \hat{k}_2)(\hat{q}_- - \hat{q}_1 - \hat{k}_2)]_\perp}{2[\beta_-(q_- - k_1)^2(-q_+ + q_2 - k_2)^2 + \beta_+ q_-^2(q_- - k_1 - k_2)^2]} - \\
&\frac{[\hat{q}_-(\hat{q}_- - \hat{k}_1)(\hat{q}_- - \hat{q}_2 + \hat{k}_2 - \hat{k}_1)(-\hat{q}_+ + \hat{k}_2)]_\perp}{2[\beta_-(q_- - k_1)^2(-q_+ + k_2)^2 + \beta_+ q_-^2(q_- - q_2 + k_2 - k_1)^2]} - \\
&\frac{[\hat{q}_-(\hat{q}_- - \hat{q}_1 + \hat{k}_1)(\hat{q}_- - \hat{q}_1 + \hat{k}_1 - \hat{k}_2)(-\hat{q}_+ + \hat{q}_2 - \hat{k}_2)]_\perp}{2[\beta_-(q_- - q_1 + k_1)^2(-q_+ + q_2 - k_2)^2 + \beta_+ q_-^2(q_- - q_1 + k_1 - k_2)^2]} - \\
&\frac{[\hat{q}_-(\hat{q}_- - \hat{q}_1 + \hat{k}_1)(-\hat{q}_+ + \hat{k}_1 + \hat{k}_2)(-\hat{q}_+ + \hat{k}_2)]_\perp}{2[\beta_-(q_- - q_1 + k_1)^2(-q_+ + k_2)^2 + \beta_+ q_-^2(-q_+ + k_1 + k_2)^2]} - \\
&\frac{[(\hat{q}_- - \hat{q}_2 + \hat{k}_2)(-\hat{q}_+ + \hat{k}_1 + \hat{k}_2)(-\hat{q}_+ + \hat{k}_1)\hat{q}_+]_\perp}{2[\beta_- q_+^2(-q_+ + k_1 + k_2)^2 + \beta_+(-q_+ + k_1)^2(q_- - q_2 + k_2)^2]} - \\
&\frac{[(\hat{q}_- - \hat{k}_2)(\hat{q}_-\hat{q}_1 + \hat{k}_1 - \hat{k}_2)(-\hat{q}_+ + \hat{k}_1)\hat{q}_+]_\perp}{2[\beta_- q_+^2(q_- - q_1 + k_1 - k_2)^2 + \beta_+(-q_+ + k_1)^2(q_- - k_2)^2]} - \\
&\frac{[(\hat{q}_- - \hat{q}_2 + \hat{k}_2)(\hat{q}_- - \hat{q}_2 + \hat{k}_2 - \hat{k}_1)(-\hat{q}_+ + \hat{q}_1 - \hat{k}_1)\hat{q}_+]_\perp}{2[\beta_- q_+^2(q_- - q_2 + k_2 - k_1)^2 + \beta_+(q_- - q_2 + k_2)^2(-q_+ + q_1 - k_1)^2]} - \\
&\frac{[(\hat{q}_- - \hat{k}_2)(\hat{q}_- - \hat{k}_1 - \hat{k}_2)(-\hat{q}_+ + \hat{q}_1 - \hat{k}_1)\hat{q}_+]_\perp}{2[\beta_- q_+^2(q_- - k_1 - k_2)^2 + \beta_+(-q_+ + q_1 - k_1)^2(q_- - k_2)^2]}.
\end{aligned}
\tag{19}
$$

Here we imply all the 4-vectors in numerators to be the transversal vectors $[\hat{q}_- ...]_\perp = \hat{q}_\perp ...)$ and in the denominators all vectors to be euclidean 2-dimentional vectors ($q_-^2 = \vec{q}_-^{\,2}$).

This (rather cumbersome) form permits,nevertheless, to be convinced in explicit gauge-invariance of expression for $R^{(2)}_{(2)}$. Really one can see the properties:

$$
\begin{aligned}
R^{(2)}_{(2)} &= 0, \vec{k}_1 = 0; \\
R^{(2)}_{(2)} &= 0, \vec{k}_2 = 0; \\
R^{(2)}_{(2)} &= 0, \vec{k}_1 = \vec{q}_1, \\
R^{(2)}_{(2)} &= 0, \vec{k}_2 = \vec{q}_2.
\end{aligned}
\tag{20}
$$

The property provides the infrared convergence performing the $d^2\vec{k}_i$ integration.

Applying the replacements of integration variables of the form $\vec{k}_i \to \vec{q}_i - \vec{k}_i$ the expression for $R^{(2)}_{(2)}$ can be simplified as

$$
\begin{aligned}
R^{(2)}_{(2)} = &-\frac{[\hat{q}_-(\hat{q}_- - \hat{q}_1)]_\perp}{\beta_+ q_-^2 + \beta_-(q_- - q_1)^2} + \frac{[(\hat{q}_+ - \hat{q}_1)\hat{q}_+]_\perp}{\beta_- q_+^2 + \beta_+(q_+ - q_1)^2} - \\
&2\frac{[\hat{q}_-(\hat{q}_- - \hat{k}_1)(\hat{q}_- - \hat{k}_1 - \hat{k}_2)(\hat{q}_- - \hat{q}_1 - \hat{k}_2)]_\perp}{\beta_-(q_- - k_1)^2(-q_+ + q_2 - k_2)^2 + \beta_+ q_-^2(q_- - k_1 - k_2)^2} - \\
&2\frac{[(\hat{q}_- - \hat{q}_2 + \hat{k}_2)(-\hat{q}_+ + \hat{k}_1 + \hat{k}_2)(-\hat{q}_+ + \hat{k}_1)\hat{q}_+]_\perp}{\beta_- q_+^2(-q_+ + k_1 + k_2)^2 + \beta_+(-q_+ + k_1)^2(q_- - q_2 + k_2)^2}.
\end{aligned}
\tag{21}
$$

The contribution to the cross section arising from the interference of $M^{(2)}_{(2)}$ with $M^{(i)}_{(k)}$ matrix elements do not depend on s. For the case when the transversal componenta of lepton momentum is large compared with momentum transferred to ions and with lepton mass:

$$\vec{q}_- \approx -\vec{q}_+ = \vec{q}, |\vec{q}| >> |\vec{q}_{1,2}|, \tag{22}$$

the main contribution to the matrix element arises from the region:

$$|\vec{q}_i| << |\vec{k}_i| << |\vec{q}|. \tag{23}$$

The matrix $R^{(2)}_{(2)}$ play a role of cut-off for the region $|\vec{k}_i| > |\vec{q}|$. It can be present in form :

$$R^{(2)}_{(2)} \approx \frac{k_1^\mu (q_1 - k_1)^\nu k_2^\alpha (q_2 - k_2)^\beta}{(\vec{q}^2)^2} R_{\mu\nu\alpha\beta}, \tag{24}$$

with some dimentionless tenzor R, independent on $\vec{k}_i, \vec{q}_i$. A simple calculation give the result:

$$\int \frac{d^2 k_1 d^2 k_2}{\pi^2} \frac{R^{(2)}_{(2)}}{\vec{k}_1^2 \vec{k}_2^2 (\vec{q}_1 - \vec{k}_1)^2 (\vec{q}_2 - \vec{k}_2)^2} = \frac{1}{(\vec{q}^2)^2} \frac{2(1 + \beta_- + \beta_+)(\beta_- - \beta_+)}{(\beta_- + \beta_+)^3} \ln \frac{\vec{q}^2}{\vec{q}_1^2} \ln \frac{\vec{q}^2}{\vec{q}_2^2}. \tag{25}$$

Such an enhancement absent if the number of exchanged photons exceed two. Really the amplitudes $M^{(2)}_{(k)}, M^{(k)}_{(2)}, k > 2$ will contain only first power of large logarithm enhancement, whereas $M^{(m)}_{(k)}, m, k > 2$ do not contain such a factor: the correcponding loop momenta integrals are convergent both in infrared and ultraviolet regions and one can put $|\vec{q}_{1,2}| = 0$.

The similar situation presumably take place for the case of creation two jets at inelastic heavy ions collisions moving at large angles.

In conclusion let consider the wide angles limit of amplitudes $M^{(1)}_{(m)}, m = 1, 2 >$.

For amplitude$M^{(1)}_{(1)}$ we have:

$$M^{(1)}_{(1)} = -is \frac{(8\pi\alpha)^2 N_1 N_2 Z_1 Z_2}{\vec{q}_1^2 \vec{q}_2^2} \frac{1}{s} \bar{u}(q_-) R^{(1)}_{(1)} v(q_+), \tag{26}$$

$$R^{(1)}_{(1)} = \hat{p}_1 \frac{\hat{q}_- - \hat{q}_1 + m}{(q_- - q_1)^2 - m^2} \hat{p}_2 + \hat{p}_2 \frac{\hat{q}_1 - \hat{q}_+ + m}{(q_1 - q_+)^2 - m^2} \hat{p}_1.$$

For wide angles kinematics we have

$$\frac{1}{s} R^{(1)}_{(1)} = \frac{\hat{p}_2}{s} \frac{1}{a^2 (\vec{q}^2)^2} [2\vec{q}\vec{q}_2 [a\hat{q}\hat{q}_1 + 2\beta_- \vec{q}\vec{q}_1] + \vec{q}^2 [a\hat{q}_1 \hat{q}_2 + 2\beta_+ \vec{q}_1 \vec{q}_2]], \tag{27}$$

with $a = \beta_- + \beta_+, \vec{q} = \vec{q}_- \approx -\vec{q}_+$ and $\vec{q}_{1,2}$- the transferred to ions momenta. For matrix element $M^{(1)}_{(2)}$ we have (in agreement with result obtained in the paper [12]):

$$M^{(1)}_{(2)} = is \frac{2^7 \pi^2 \alpha^3 Z_1 Z_2^2 N_1 N_2}{\vec{q}_1^2} \int \frac{d^2 k_2}{\pi} \frac{1}{\vec{k}_2^2 (\vec{q}_2 - \vec{k}_2)^2} \bar{u}(q_-) I^{(1)}_{(2)} \frac{\hat{p}_2}{s} v(q_+), \tag{28}$$

with

$$I^{(1)}_{(2)} = \frac{\hat{q}_{-\perp} (\hat{q}_- - \hat{q}_1)_\perp}{\beta_+ \vec{q}_-^2 + \beta_- (\vec{q}_- - \vec{q}_1)^2} + \frac{(\hat{q}_+ - (\hat{q}_1)_\perp \hat{q}_{+\perp}}{\beta_- \vec{q}_+^2 + \beta_+ (\vec{q}_+ - \vec{q}_1)^2} - \tag{29}$$

$$\frac{(\hat{q}_- - (\hat{k}_2)_\perp (\hat{q}_- - \hat{q}_1 - \hat{k}_2)_\perp}{\beta_- (\vec{q}_- - \vec{q}_1 - \vec{k}_2)^2 + \beta_+ (\vec{q}_- - \vec{k}_2)^2} - \frac{(\hat{q}_+ - (\hat{k}_2 - \hat{q}_1)_\perp (\hat{q}_+ - \hat{k}_2)_\perp}{\beta_+ (\vec{q}_- - \vec{q}_2 + \vec{k}_2)^2 + \beta_- (\vec{q}_+ - \vec{k}_2)^2}.$$

One can see that in the limit of small momenta the quantity I vanish at $\vec{q}_1 \to 0, \vec{k}_2 \to 0; \vec{k}_2 \to \vec{q}_2$

$$I^{(1)}_{(2)} \sim \frac{1}{a\vec{q}^2}[(2\beta_-\vec{q}_-\vec{q}_1 + \hat{q}_-\hat{q}_1)(\frac{4(\vec{q}_-\vec{k}_2)^2}{(\vec{q}^2)^2} - \frac{\vec{k}_2^2}{\vec{q}^2}) - \frac{2\vec{q}_-\vec{k}_2}{\vec{q}^2}(\hat{k}_2\hat{q}_1 + 2\beta_-\vec{k}_2\vec{q}_1)] + (\beta_- \to \beta_+), |\vec{k}_2| >> |\vec{q}_2|. \tag{30}$$

This quantity turns to zero when performing angular averaging. This fact is in agreement with the general result for the sum of contributions os $M^{(1)}_{(k)}$ obtained in [13]. The quantity $M^{(1)}_{(3)}$ as well turns to zero in the limit of wide angles pair emission-it is proportional to $|\vec{q}_2|/|\vec{q}| << 1$.

The general structure of multiexchange matrix element have a form

$$M^{2n_1}_{2n_2} = isN_1N_2Z_1^{2n_1}Z_2^{2n_2}\alpha^{2(n_1+n_2)}\frac{\pi^2}{16}\frac{1}{(2n_1)!(2n_2)!}\bar{u}(q_-)R^{(2n_1)}_{(2n_2)}\frac{\hat{p}_2}{s}v(q_+),$$

$$R^{(2n_1)}_{(2n_2)} = \int \frac{d^2k_1}{\pi}...\frac{d^2k_{2n_1-1}}{\pi}\frac{d^2\kappa_1}{\pi}...\frac{d^2\kappa_{2n_2-1}}{\pi}\frac{1}{\vec{k}_1^2...\vec{k}_{2n_1}^2}\frac{1}{\vec{\kappa}_1^2...\kappa_{2n_2}^2}[\frac{\hat{q}_-(-\hat{q}_1+\hat{q}_-)}{(q_1-q_-)^2} + \frac{\hat{k}^4}{[4]} + \frac{\hat{k}^8}{[8]} + ...\frac{\hat{k}^{4(n_1+n_2)}}{[2(n_1+n_2)]}], \tag{31}$$

$$k_{2n_1} = q_1 - \sum_1^{2n_1-1} k_i,\ \kappa_{2n_2} = q_2 - \sum_1^{2n_2-1}\kappa_i$$

where we use the schematical notions $\hat{k}^{4n}$ denotes the nominator and

$$[4] = \beta_-(\vec{q}_1 - \vec{k} - i)^2(\vec{\kappa}_i - \vec{q}_2)^2 + \beta_+\vec{q}_-^2(\vec{q}_- - \vec{k}_i - \vec{\kappa}_i)^2 = (1)(3)\beta_- + \vec{q}_-^2(2)\beta_+, \tag{32}$$
$$[8] = (1)(3)(5)(7)\beta_\pm + \vec{q}_-^2(2)(4)(6)\beta_\mp,$$

typical multiplier from the denomunator. We see that the nonlinearity of the denominators increases.

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