Update of MRST Parton Distributions

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Parton Uncertainties - Experiment – currently an issue attracting a lot of work. Number of approaches.

Hessian (Error Matrix) approach first used by H1 and ZEUS, recently extended by CTEQ.

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

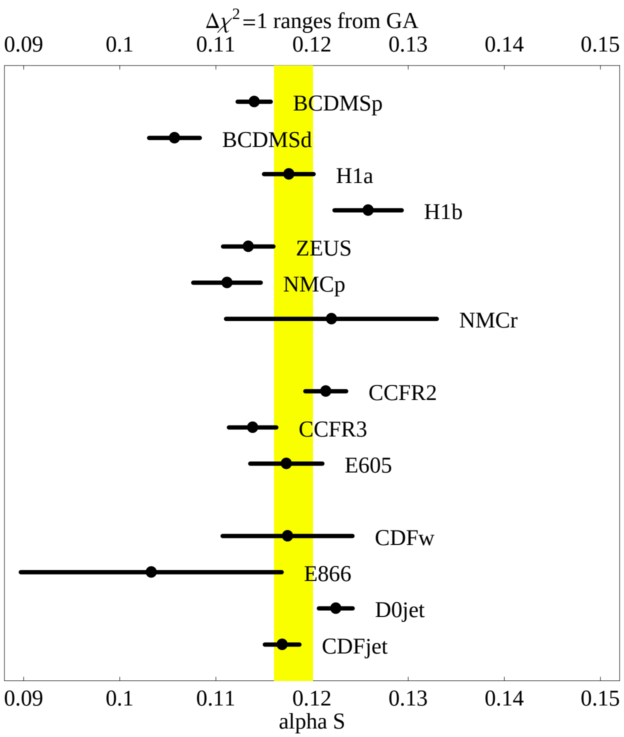
We can then use the standard formula for linear error propagation.

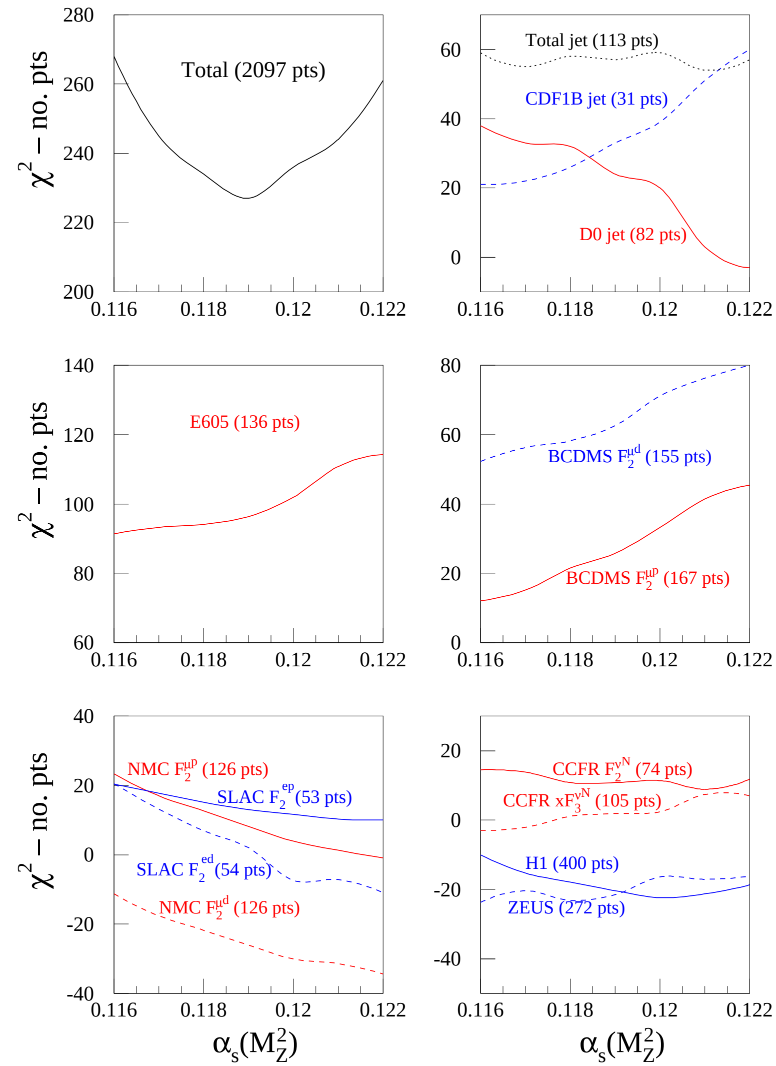
$$(\Delta F)^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F}{\partial a_i}(H)^{-1}_{ij}\frac{\partial F}{\partial a_j},$$

Simple method problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space - particularly with more parameters (more data). $\longrightarrow$ numerical instability.

Solved (helped) by finding and rescaling eigenvectors of H leading to diagonal form $\Delta\chi^2 = \sum_i z_i^2$. First used by CTEQ. Now used in slightly weaker form by MRST and ZEUS.

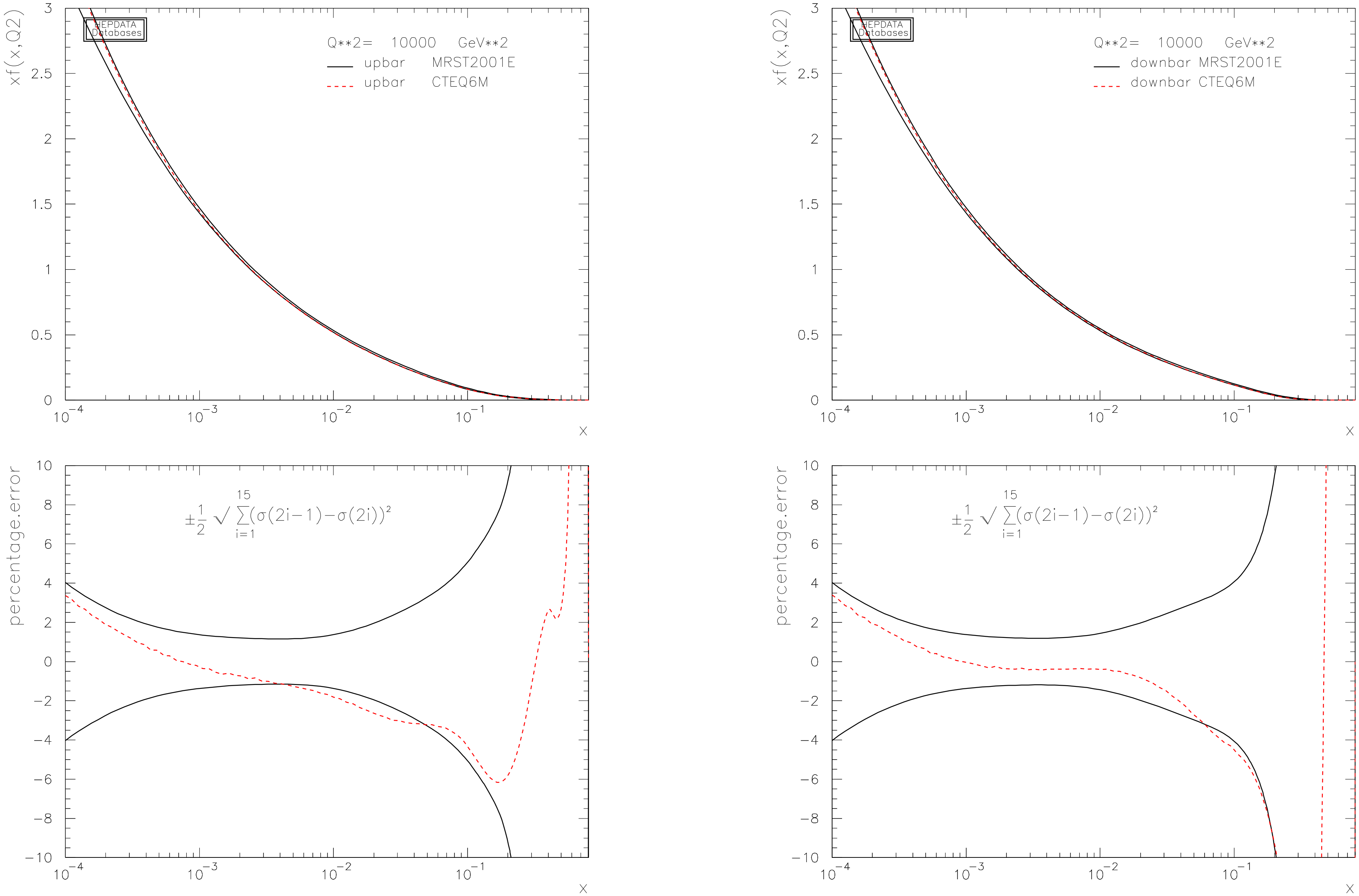
In full **global** fit art in choosing "correct" $\Delta\chi^2$ given complication of errors. Ideally $\Delta\chi^2 = 1$, but unrealistic.





Many approaches use $\Delta\chi^2 \sim 1$. CTEQ choose $\Delta\chi^2 \sim 100$ for 90% confidence limit, i.e. ~ 40 for $1-\sigma$ error. MRST choose $\Delta\chi^2 \sim 20$ for $1-\sigma$ error.

Uncertainty on MRST $\bar{u}$ and $\bar{d}$ distributions, along with CTEQ6. Central rapidity $x = 0.006$ is ideal for MRST uncertainty in W, Z (Higgs?) at the LHC.

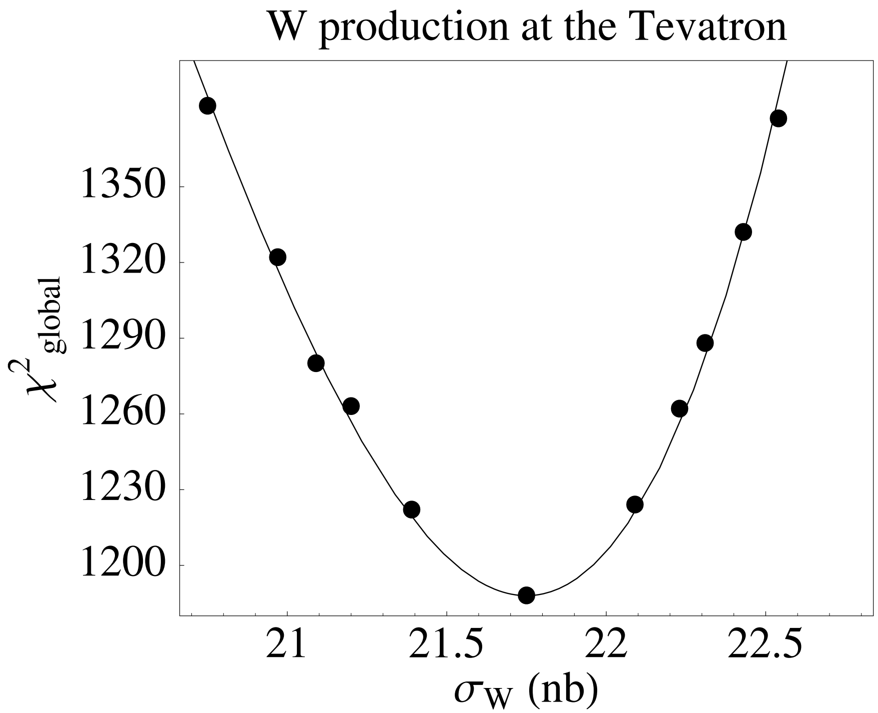


Lagrange Multiplier Method

Can also look at uncertainty on a given physical quantity using Lagrange Multiplier method, first suggested by CTEQ and concentrated on by MRST. Minimize

$$\Psi(\lambda, a) = \chi^2_{global}(a) + \lambda F(a).$$

Gives best fits for particular values of quantity $F(a)$ without relying on Gaussian approx for χ^2. Uncertainty then determined by deciding allowed range of $\Delta\chi^2$.



CTEQ obtain for $\alpha_S = 0.118$

$$\Delta\sigma_W(\mathrm{LHC}) \approx \pm 4\% \quad \Delta\sigma_{\mathrm{W}}(\mathrm{Tev}) \approx \pm 4$$

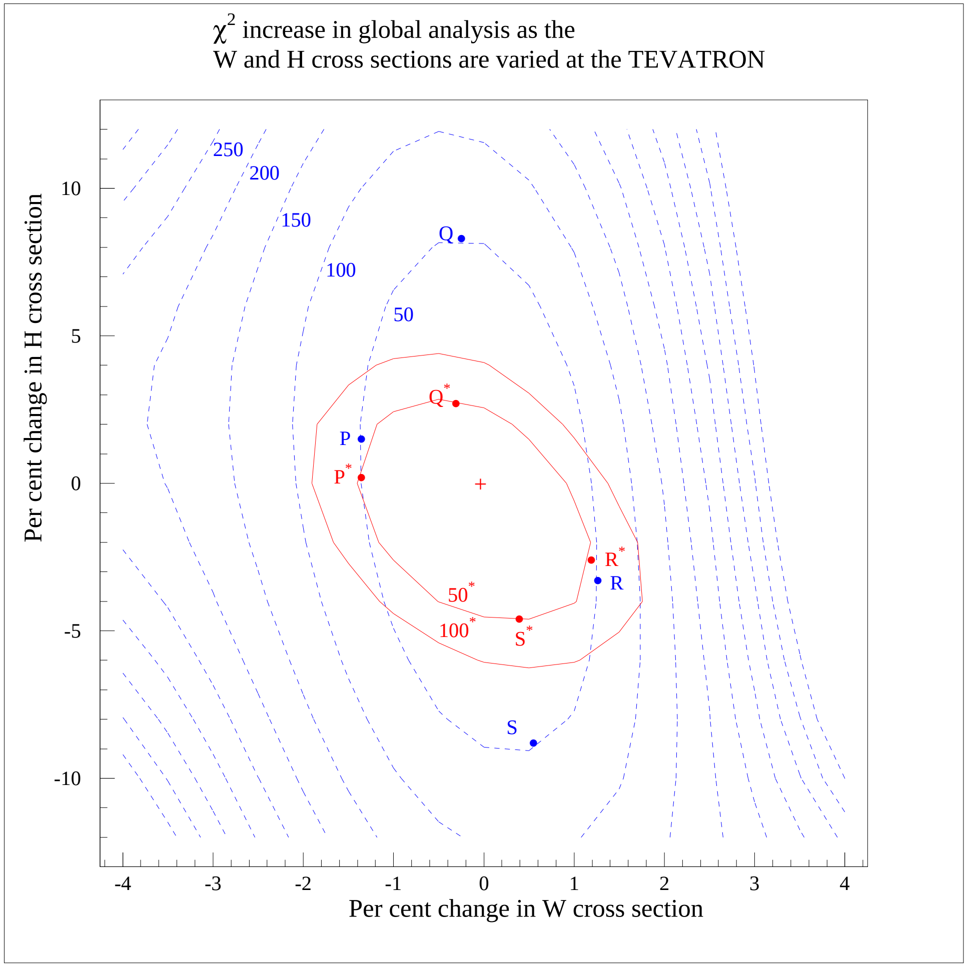
$$\Delta\sigma_H(\mathrm{LHC}) \approx \pm 5\%.$$

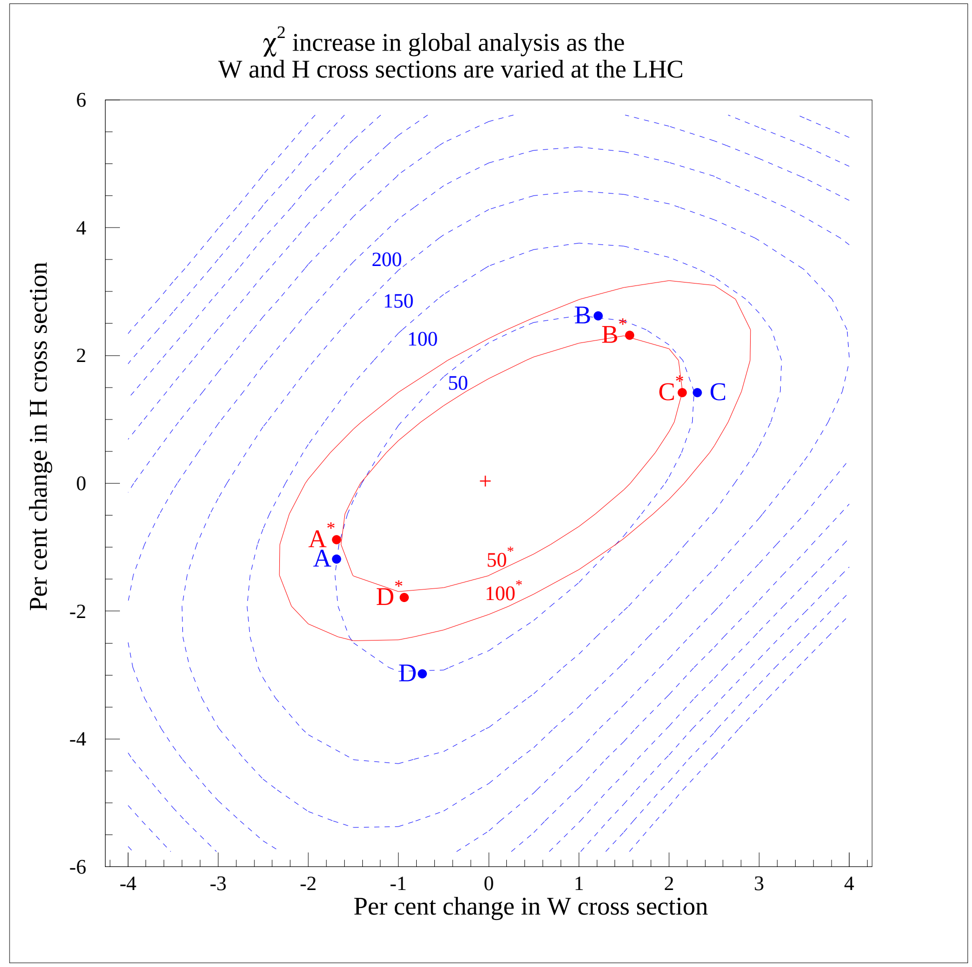
MRST use a wider range of data, and if $\Delta\chi^2 \sim 50$ find for $\alpha_S = 0.119$

$$\Delta\sigma_W(\mathrm{Tev}) \approx \pm 1.2\% \quad \Delta\sigma_{\mathrm{W}}(\mathrm{LHC}) \approx \pm 2\%$$

$$\Delta\sigma_H(\mathrm{Tev}) \approx \pm 4\% \quad \Delta\sigma_{\mathrm{H}}(\mathrm{LHC}) \approx \pm 2\%.$$

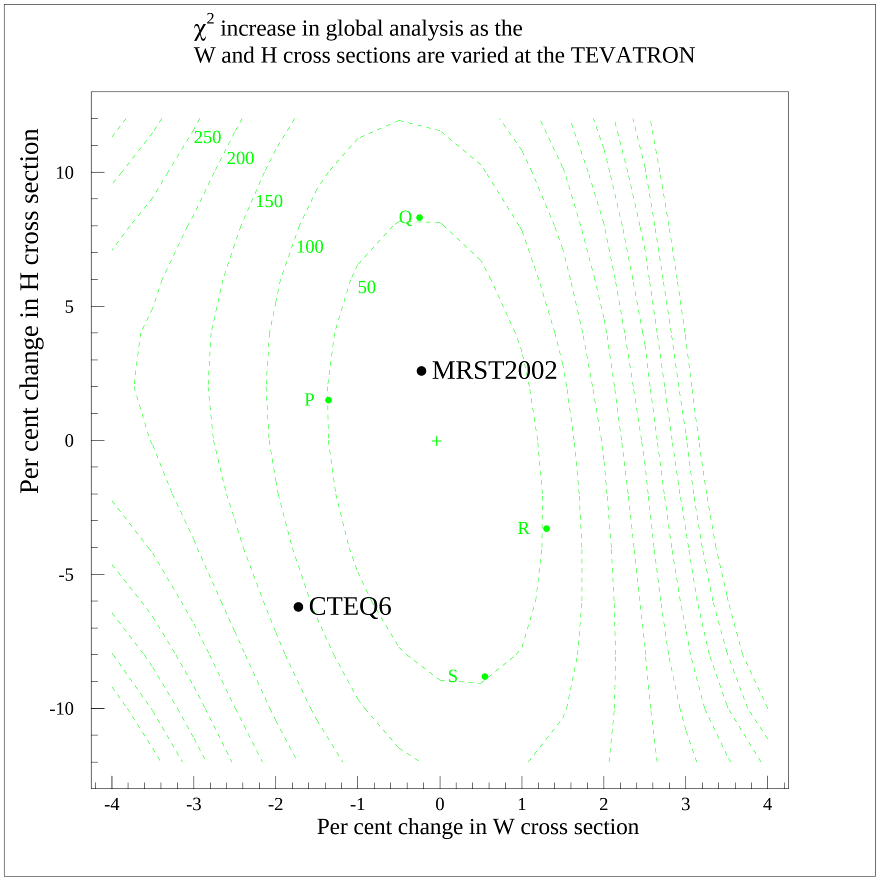
MRST also allow α_S to be free.

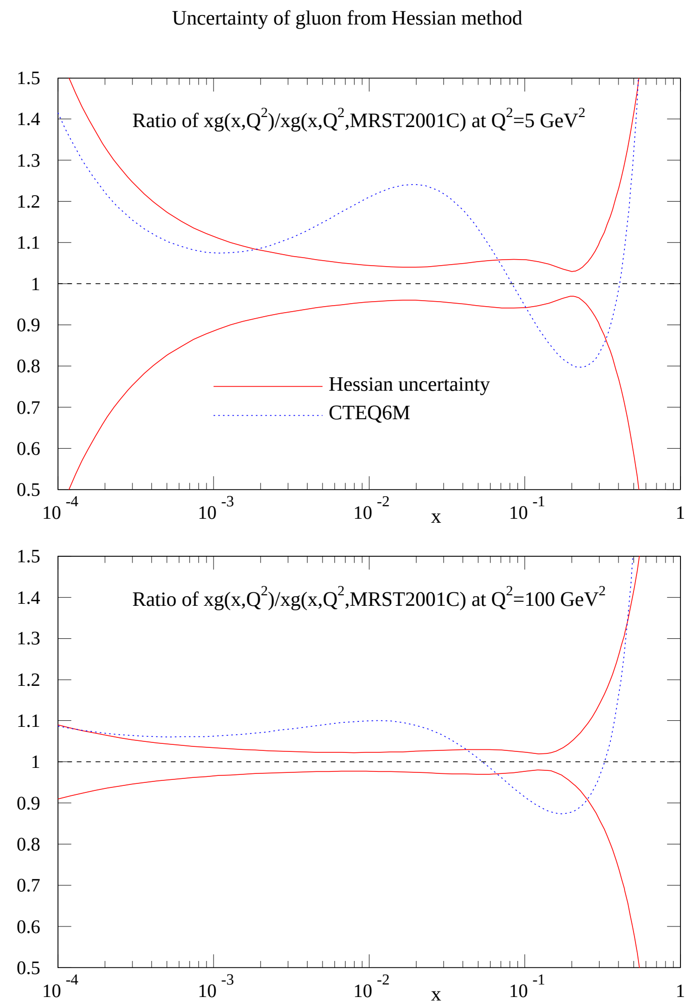




χ^2-plots for W and Higgs $(120\mathrm{GeV})$ production at the Tevatron and LHC α_S free (blue) and fixed (red) at $\alpha_S = 0.119$.

Different approaches lead to similar accuracy of measured quantities, but can lead to different central values. Must consider effect of assumptions made during fit.





Cuts made on data, data sets fit, correctness of NLO QCD, parameterization for input sets, heavy flavour prescription, no isospin violation, strong coupling ......

Many can be as important as experimental errors on data used (or more so).

Results from LHC/LP Study Working Group (Bourilkov).

Table 1: Cross sections for Drell-Yan pairs (e^+e^-) with PYTHIA 6.206, rapidity < 2.5. The errors shown are the PDF uncertainties.

PDF set	Comment	xsec [pb]	PDF uncertainty %
$81 < M < 101$ GeV			
CTEQ6	LHAPDF	1065 ± 46	4.4
MRST2001	LHAPDF	$1091 \pm$ ...	3
Fermi2002	LHAPDF	853 ± 18	2.2

Comparison of $\sigma_W \cdot B_{l\nu}$ for MRST2002 and Alekhin partons.

PDF set	Comment	xsec [nb]	PDF uncertainty
Alekhin	Tevatron	2.73	$\pm$ 0.05 (tot)
MRST2002	Tevatron	2.59	$\pm$ 0.03 (expt)
CTEQ6	Tevatron	2.54	$\pm$ 0.10 (expt)
Alekhin	LHC	215	$\pm$ 6 (tot)
MRST2002	LHC	204	$\pm$ 4 (expt)
CTEQ6	LHC	205	$\pm$ 8 (expt)

In both cases differences (mainly) due to detailed constraint (by data) on quark decomposition.

Problems in the fit.

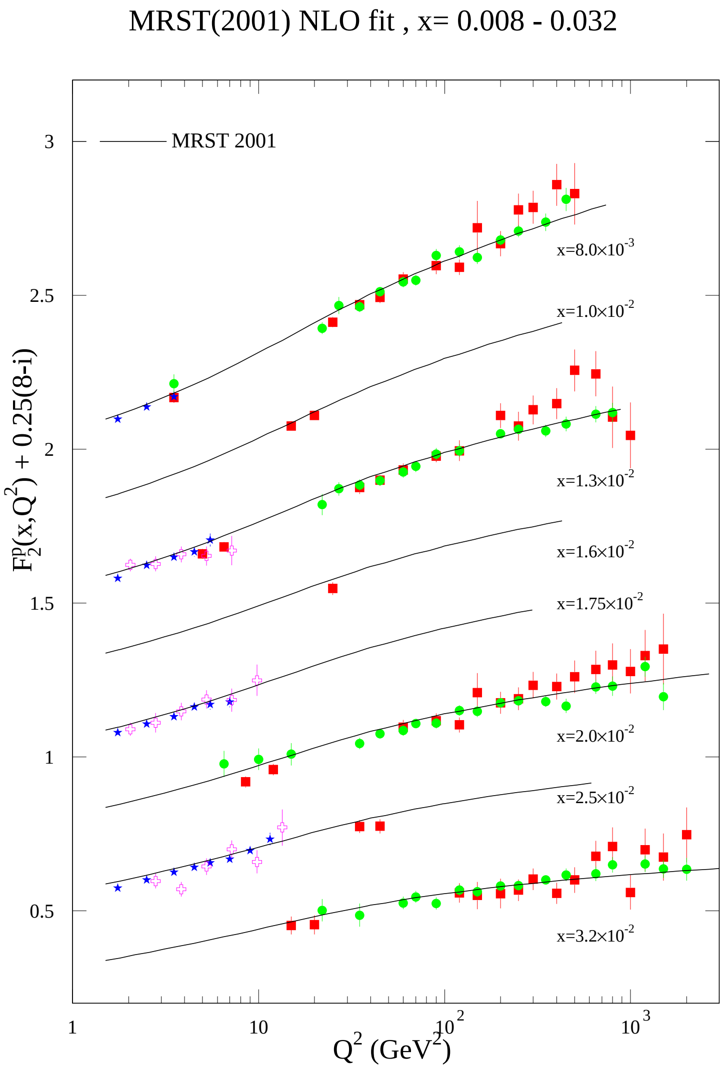
Variations from different approaches partially due to inadequacy of theory .

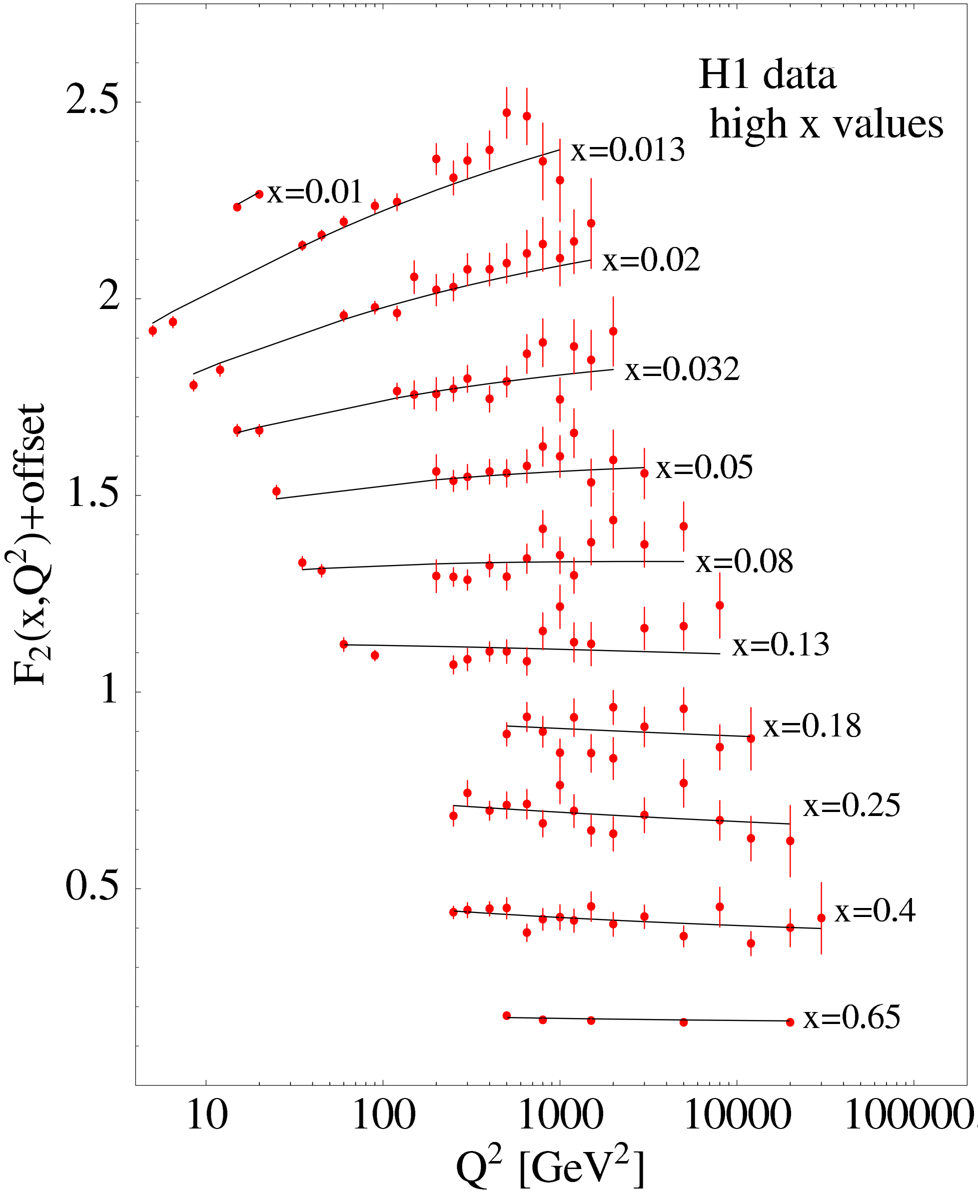
Failings of NLO QCD indicated by some areas where fit quality could be improved.

Good fit to HERA data, but some problems at highest Q^2 at moderate x, i.e. in $dF_2/d\ln Q^2$. $\rightarrow$ possible underestimate of quarks in this region.

Want more gluon in the $x \sim 0.01$ range, and/or larger $\alpha_S(M_Z^2)$.

Possible sign of required $\ln(1/x)$ corrections.





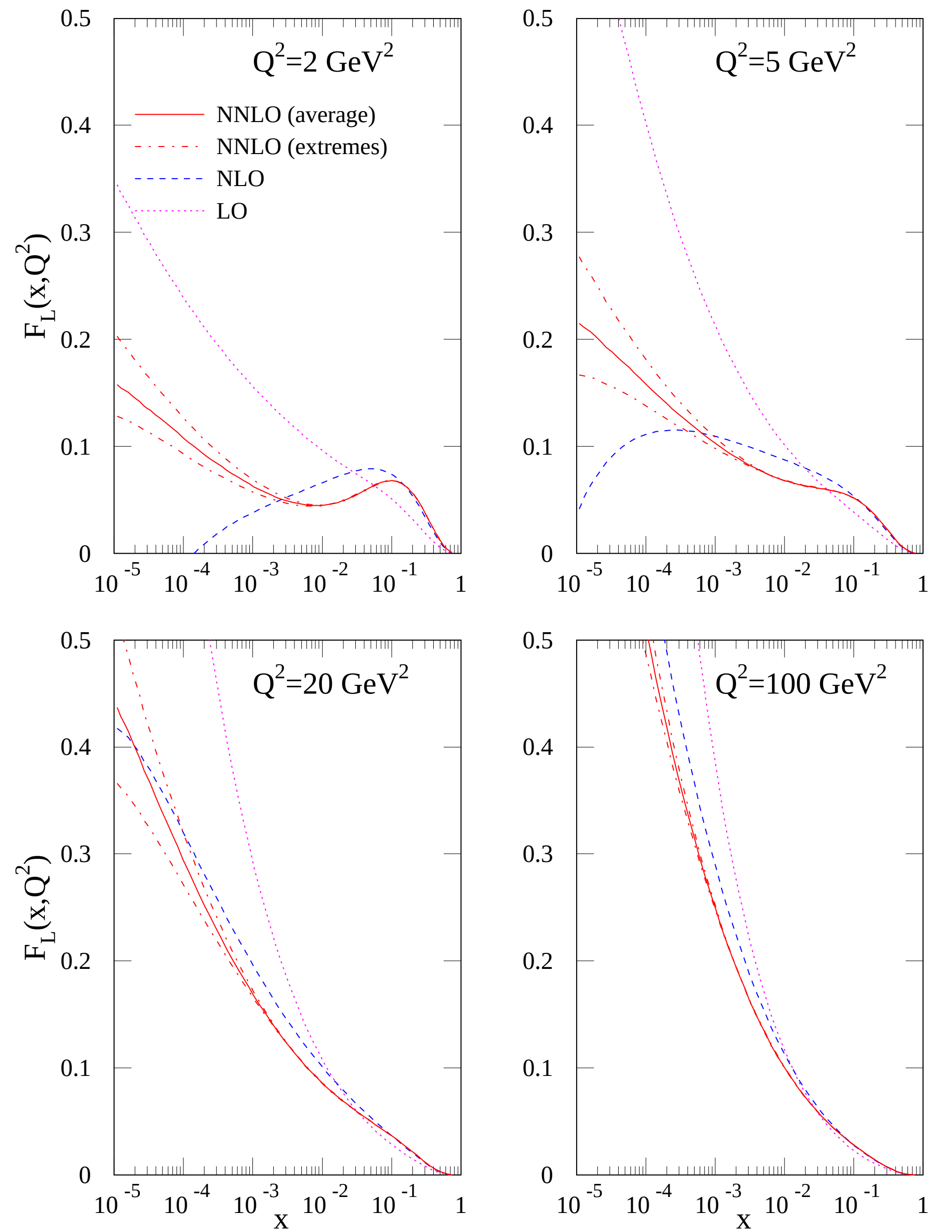
Comparison of MRST(2001) $F_2(x, Q^2)$ with HERA, NMC and E665 data (left) and of CTEQ6 $F_2(x, Q^2)$ and H1 data.

Data require gluon to be negative at low Q^2, e.g. MRST $Q_0^2 = 1\text{GeV}^2$. Needed by all data (e.g Tevatron jets) not just low Q^2 low x data.

$\rightarrow$ $F_L(x,Q^2)$ dangerously small at smallest x, Q^2.

Other groups find similar problems with gluon and/or $F_L(x,Q^2)$ at low x, e.g. ZEUS.

Note also instability in going from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

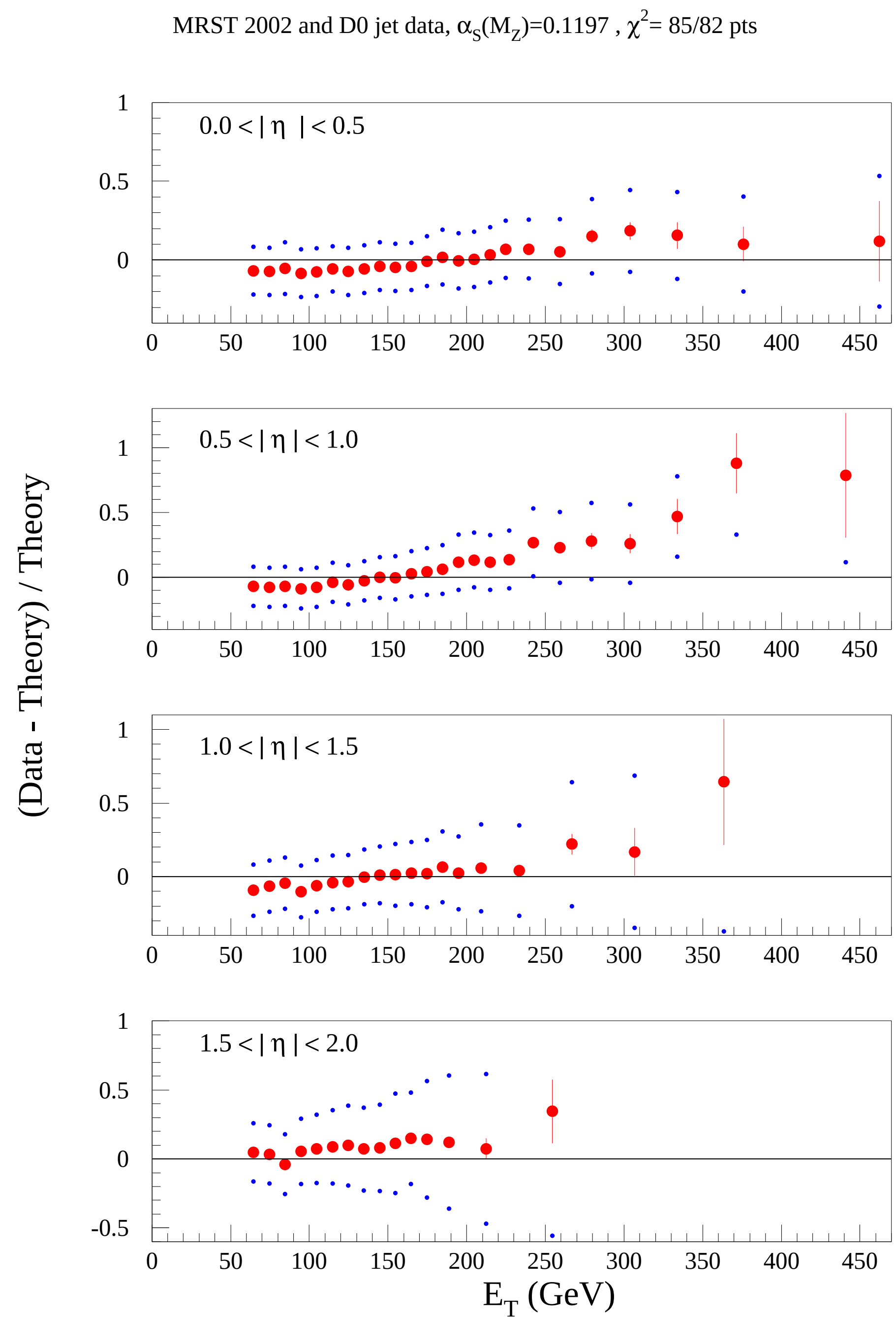


Difficult to reconcile fit to jets and rest of data.

MRST find a reasonable fit to jet data, but need to use the large systematic errors.

Better for CTEQ6 largely due to different cuts on other data. Usually worse for other partons (jets not in fits). General tension between HERA and NMC data and jets.

In general different data compete over the gluon and $\alpha_S(M_Z^2)$.



Theoretical Errors

Hence it is vital to consider theoretical corrections. These include ....

- possibility of isospin violation, $s(x) \neq \bar{s}(x)$, etc.

- higher orders (NNLO)

- QED (comparable to NNLO ? ($\alpha_s^3 \sim \alpha$)

- large x ($\alpha_s^n \ln^{2n-1}(1-x)$)

- low Q^2 (higher twist)

- small x ($\alpha_s^n \ln^{n-1}(1/x)$)

In order to investigate true theoretical error must consider large and small x resummations, and/or use what we already know about e.g. NNLO and QED.

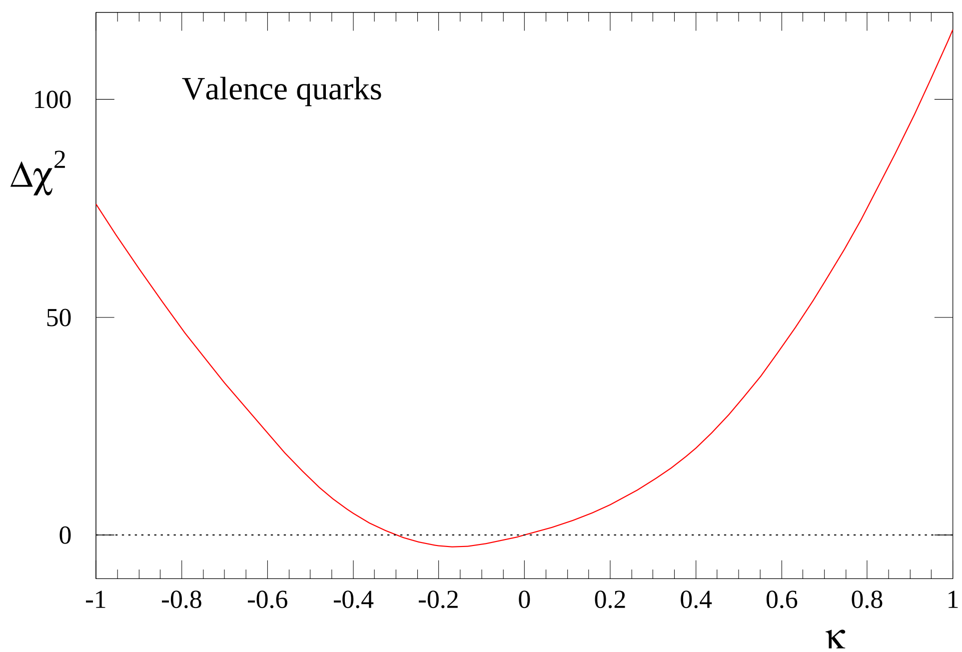
MRST look at effect of **isospin violation**. NuTeV measure (with a $3-\sigma$ discrepancy)

$$R^- = \frac{1}{2} - \sin^2\theta_W + (1 - \frac{7}{3}\sin^2\theta_W)\frac{[\delta U_{\rm v}] - [\delta D_{\rm v}]}{2[V^-]}.$$

$$[\delta U_{\rm v}] = [U^p_{\rm v}] - [D^n_{\rm v}], \qquad [\delta D_{\rm v}] = [D^p_{\rm v}] - [U^n_{\rm v}].$$

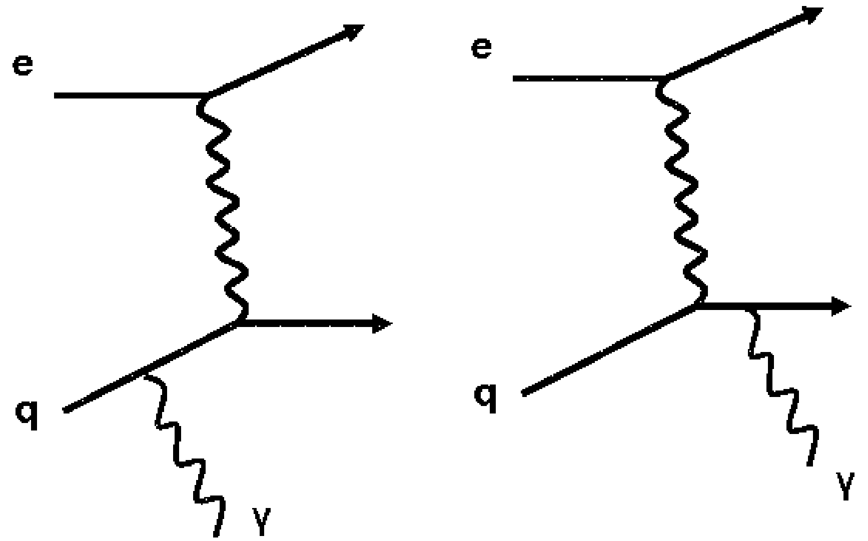
$$u^p_{\rm v}(x) = d^n_{\rm v}(x) + \kappa f(x), \qquad d^p_{\rm v}(x) = u^n_{\rm v}(x) - \kappa f(x).$$

$\kappa = -0.2 \rightarrow$ reduction of the NuTeV anomaly to $\sim 1.5-\sigma$, i.e. $\Delta sin^2\theta_W \sim -0.002$. Larger (more negative) κ allowed.



QED **Effects**.

QED corrections to DIS include



$\Rightarrow$ mass singularity when $\gamma \parallel q$

QED collinear singularities are *universal* and can be absorbed into pdfs, exactly as for QCD collinear singularities, leaving finite (as $m_q \to 0$) $\mathcal{O}(\alpha)$ QED corrections in coefficient functions

QED –improved DGLAP equations.

$$
\begin{aligned}
\frac{\partial q_i(x,\mu^2)}{\partial \log \mu^2} &= \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{ P_{qq}(y)\ q_i(\frac{x}{y},\mu^2) + P_{qg}(y,\alpha_S)\ g(\frac{x}{y},\mu^2)\right\} \\
&+ \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left\{ \tilde{P}_{qq}(y)\ e_i^2 q_i(\frac{x}{y},\mu^2) + P_{q\gamma}(y)\ e_i^2 \gamma(\frac{x}{y},\mu^2)\right\} \\
\frac{\partial g(x,\mu^2)}{\partial \log \mu^2} &= \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{ P_{gq}(y)\ \sum_j q_j(\frac{x}{y},\mu^2) + P_{gg}(y)\ g(\frac{x}{y},\mu^2)\right\} \\
\frac{\partial \gamma(x,\mu^2)}{\partial \log \mu^2} &= \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left\{ P_{\gamma q}(y)\ \sum_j e_j^2\ q_j(\frac{x}{y},\mu^2) + P_{\gamma\gamma}(y)\ \gamma(\frac{x}{y},\mu^2)\right\}
\end{aligned}
$$

at leading order in α_S and α, where

$$
\tilde{P}_{qq} = C_F^{-1} P_{qq}, \qquad P_{\gamma q} = C_F^{-1} P_{gq},
$$

$$
P_{q\gamma} = T_R^{-1} P_{qg}, \qquad P_{\gamma\gamma} = -\tfrac{2}{3}\ \textstyle\sum_i e_i^2\ \delta(1-x)
$$

and momentum is conserved:

$$
\int_0^1 dx\ x\left\{ \sum_i q_i(x,\mu^2) + g(x,\mu^2) + \gamma(x,\mu^2)\right\} = 1.
$$

First quantitative estimate of effect on pdfs by Spiesberger, Phys. Rev. D52, 4936 (1995).

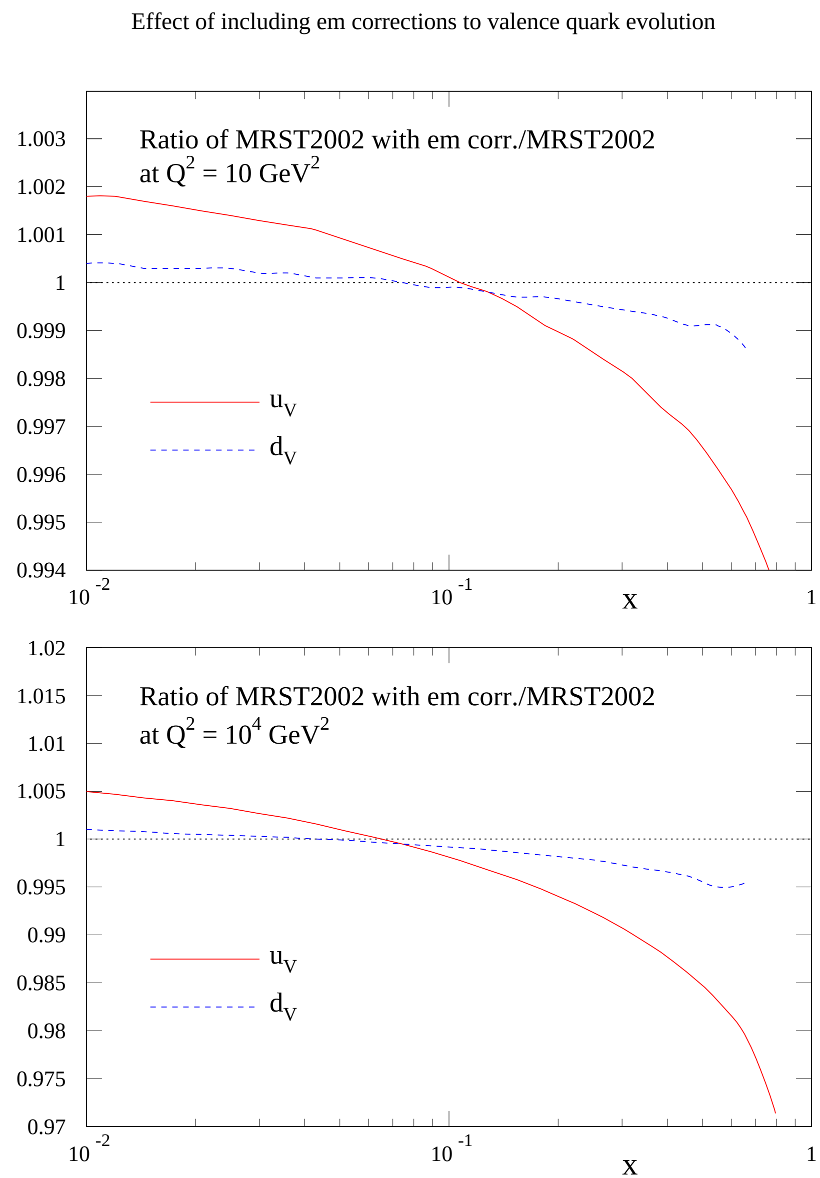
New study by MRST in progress.

Effect on quark distributions is entirely negligible at small x where gluon contribution dominates DGLAP evolution.

At large x, effect only becomes noticeable (percent order) at very large Q^2, where it is equivalent to a slight shift in α_S:

$$\Delta\alpha_S(M_Z^2) \simeq +0.0003$$

Much smaller effects than many sources of uncertainty.



However, QED effects to lead to small isospin violation.

$u_V^p(x)$ quarks radiate more photon than $d_V^n(x)$ quarks.

To rough approximation

$$\gamma(x,Q^2) = \sum_j e_j^2 \frac{\alpha}{2\pi} \ln(Q^2/m_q^2) \int_x^1 \frac{dy}{y} P_{\gamma q}(y) \;\; q_j(\frac{x}{y},Q^2).$$

So more photon momentum in proton than neutron due to high-x up quarks radiating more than high-x down quarks.

Momentum conservation $\rightarrow u_V^p(x) < d_V^n(x)$ at high .

Hence, $[\delta U_{\rm v}] < 0$ as required by NuTeV anomaly.

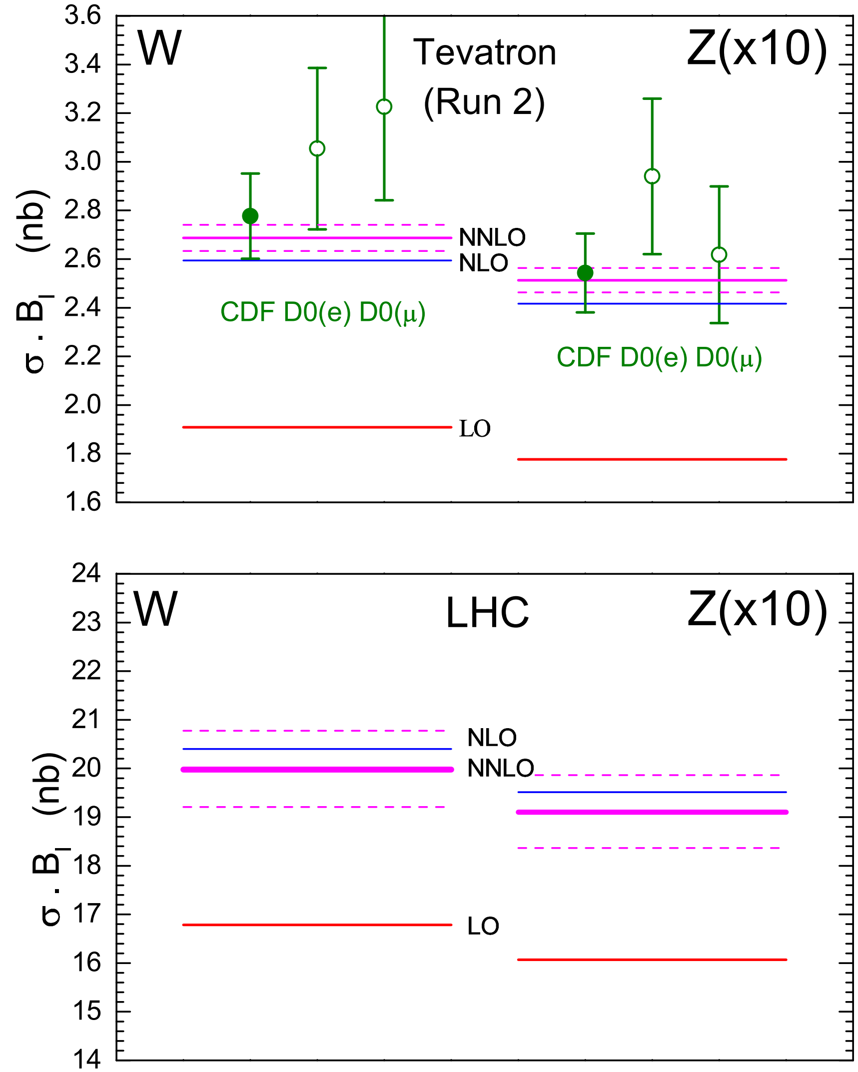
Estimates imply $\sim 3/4$ of isospin violation observed by best fit! Reduces NuTeV anomaly to about $1/2$.

NNLO

Coefficient functions known at NNLO. Singular limits $x \to 1$, $x \to 0$ known for NNLO splitting functions as well as limited moments. Complete soon (Non-Singlet now known! Moch, Vermaseren and Vogt). Approximate NNLO splitting functions devised by van Neerven and Vogt. Improve quality of fit very slightly (MRST). Reduces $\alpha_S \to 0.1155$.

Reasonable stability order by order for (quark-dominated) W and Z cross-sections.

This fairly good convergence is largely guaranteed because the quarks are fit directly to data. Much worse for gluon dominated quantities e.g. $F_L(x, Q^2)$.



partons: MRST2002
NNLO evolution: van Neerven, Vogt approximation to Vermaseren et al. moments
NNLO W,Z corrections: van Neerven et al. with Harlander, Kilgore corrections

Alternative approach.

In order to investigate real quality of fit and regions with problems vary kinematic cuts on data.

Procedure – change W^2_{cut}, Q^2_{cut} and x_{cut}, re-fit and see if quality of fit to remaining data improves and/or input parameters change dramatically. Continue until quality of fit and partons stabilize.

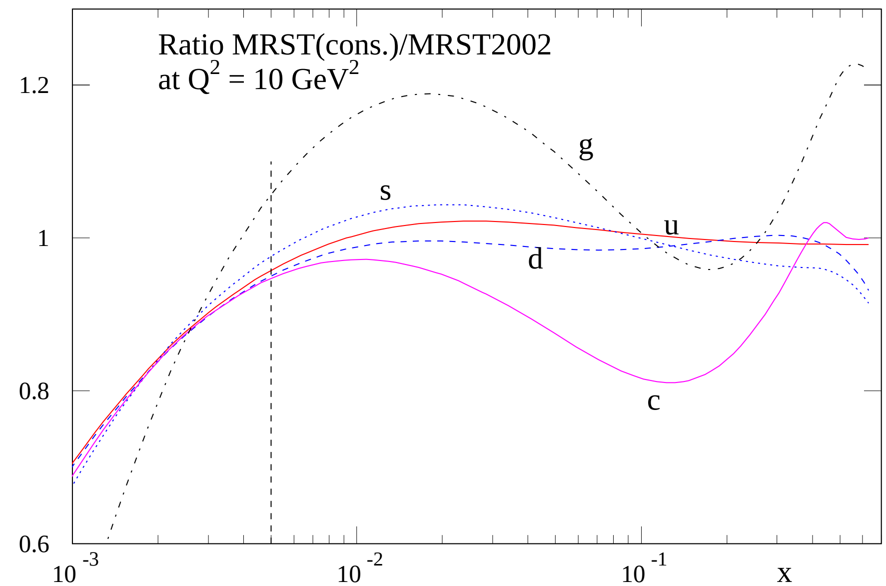
For W^2_{cut} raising from $12.5\mathrm{GeV}^2$ to $15\mathrm{GeV}^2$ sufficient.

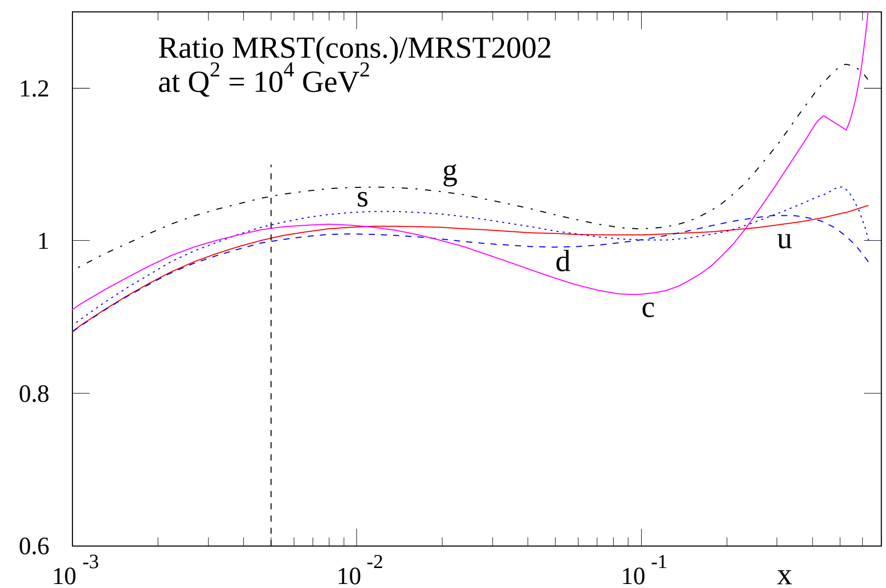
Raising Q^2_{cut} from $2\mathrm{GeV}^2$ in steps there is a slow continuous and significant improvement for higher Q^2 up to $> 10\mathrm{GeV}^2$ (cut 560 data points) – suggests any corrections mainly higher orders not higher twist.

Raising x_{cut} from 0 to 0.005 (cut 271 data points) continuous improvement. At each step moderate x gluon becomes more positive.

$\rightarrow$ MRST2003 conservative partons. Should be most reliable method of parton determination ($\Delta\chi^2 = -70$ for remaining data), but only applicable for restricted range of x, Q^2. $\rightarrow \alpha_S(M_Z^2) = 0.1165 \pm 0.004$.

The ratio of the conservative partons to the default partons at NLO. One can see the dip of the conservative partons below $x_{cut} = 0.005$.

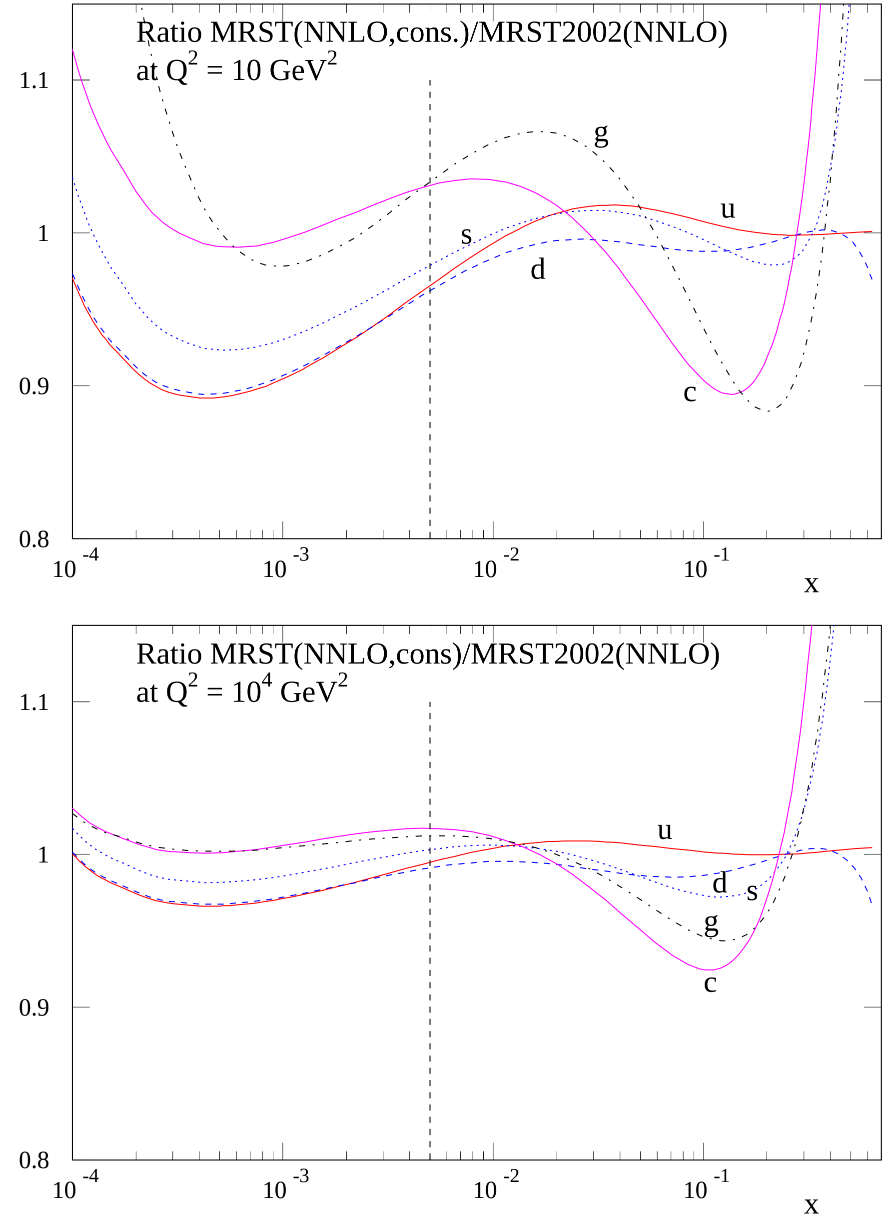




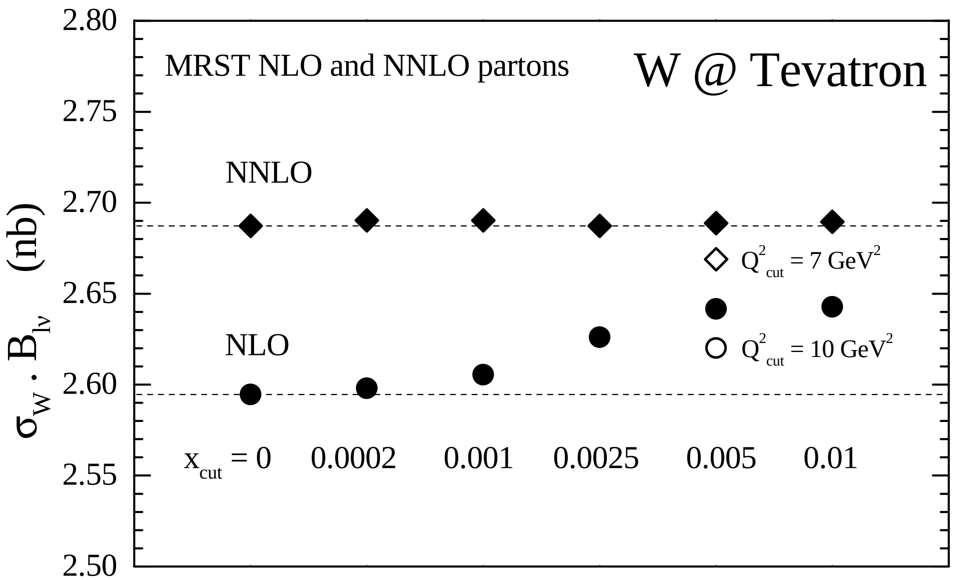
The ratio of the conservative partons to the default partons at NNLO. Now $x_{cut} = 0.005$ and $Q^2_{cut} = 7\text{GeV}^2$. Slight improvement.

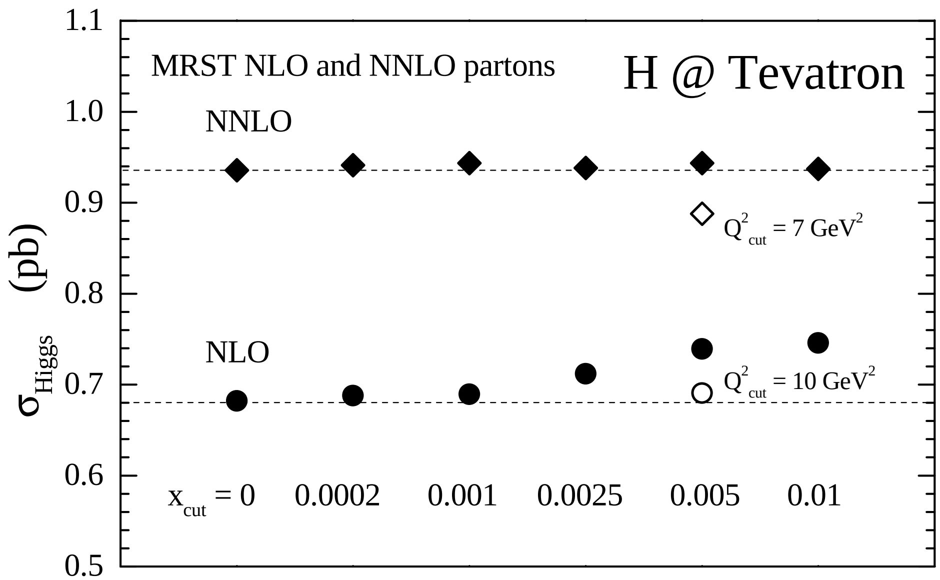
$\Delta\chi^2$ still large.

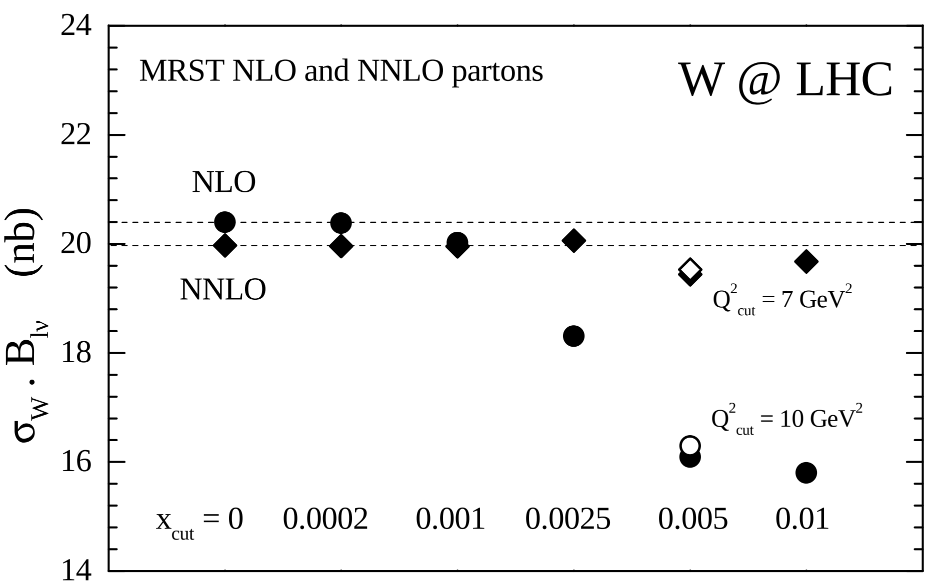
However, now the partons are similar below $x_{cut} = 0.005$. Significant or partially accidental?

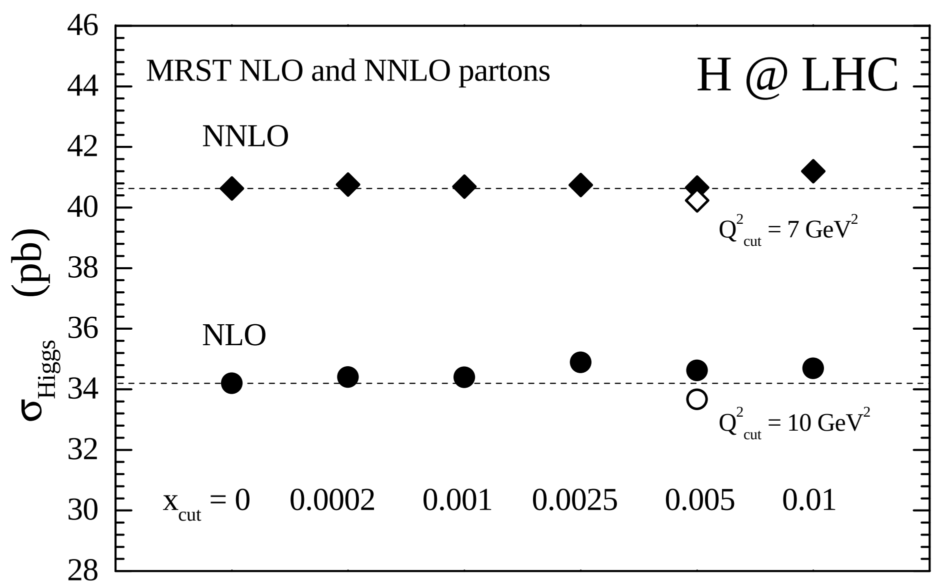


Variation in predictions with cuts. Follows patterns expected. Range of possible theoretical error.







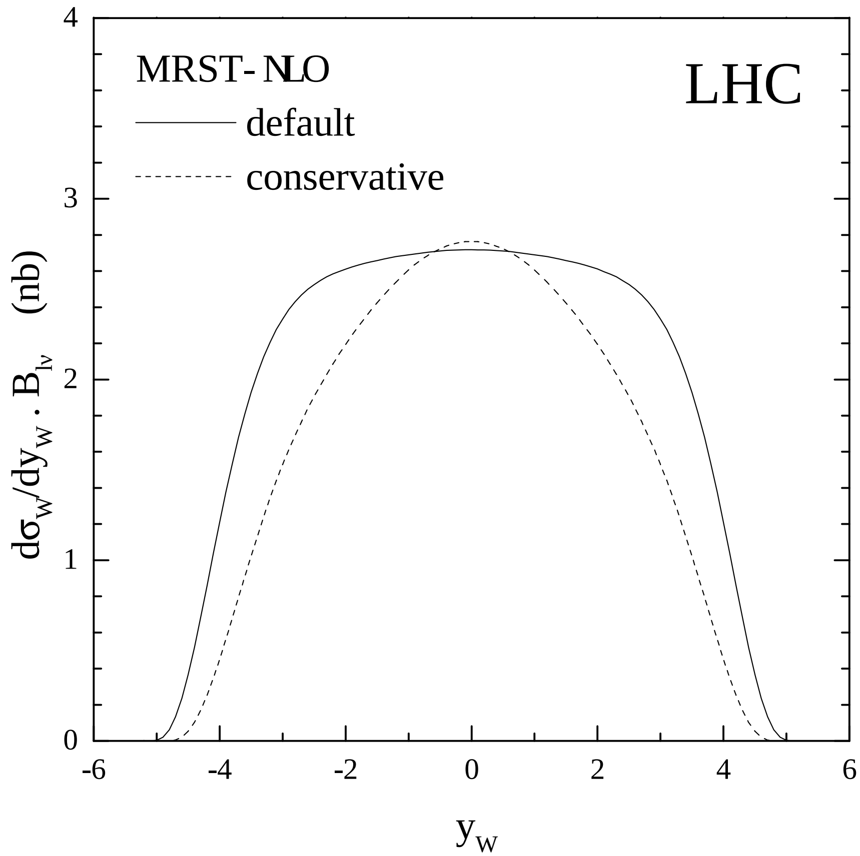


Rapidity Distribution

Comparison of prediction for $(d\sigma_W/dy_W)$ for the standard MRST partons and the conservative set, where only central rapidity from direct fit to data.

The reduction in the total cross-section in the latter case is clearly due to the huge reduction at high y_W and represents the possible type of theoretical uncertainty in this region when working at NLO.

Note a slight increase in cross-section for $y_W = 0$ ($x = 0.006$). Due to increased evolution of quarks here.



Large x and Low Q^2.

Perform fits with the known NNNLO large $\ln(1-x)$ terms included explicitly.

Also parameterize higher twist contributions by

$$F_i^{\rm HT}(x,Q^2) = F_i^{\rm LT}(x,Q^2)\left(1+\frac{D_i(x)}{Q^2}\right)$$

where i spans bins of x (and/or try saturation corrections at low x and Q^2 – see later).

In this type of expansion $\ln(1-x)$-corrections become indistinguishable from $1/W^2$ corrections at low W^2. Alternative $\ln(1-x)$ expansions possible.

No evidence for any higher twist except at low W^2.

x	LO	NLO	NNLO	NNNLO
0–0.0005	-0.07	-0.02	-0.02	-0.03
0.0005–0.005	-0.03	-0.01	0.03	0.03
0.005–0.01	-0.13	-0.09	-0.04	-0.03
0.01–0.06	-0.09	-0.08	-0.04	-0.03
0.06–0.1	-0.02	0.02	0.03	0.04
0.1–0.2	-0.07	-0.03	-0.00	0.01
0.2–0.3	-0.11	-0.09	-0.04	0.00
0.3–0.4	-0.06	-0.13	-0.06	-0.01
0.4–0.5	0.22	0.01	0.07	0.11
0.5–0.6	0.85	0.40	0.41	0.39
0.6–0.7	2.6	1.7	1.6	1.4
0.7–0.8	7.3	5.5	5.1	4.4
0.8–0.9	20.2	16.7	16.1	13.4

Table 2: The values of the higher-twist coefficients D_i, in the chosen bins of x, extracted from the LO, NLO, NNLO and NNNLO (NNLO with the approximate NNNLO non-singlet quark coefficient function) global fits.

Small x – gluon outside conservative range very negative, and $dF_2(x,Q^2)/d\ln Q^2$ incorrect, (NNLO much more stable than NLO). Theory corrections could cure this (quite plausible). Empirical resummation corrections improve global fit, e.g.

$$P_{gg} \to .... + \frac{1}{x}\left[A\bar{\alpha}_S^4\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right) + B\bar{\alpha}_S^5\left(\frac{\ln^4(1/x)}{24} - \frac{\ln^3(1/x)}{6}\right)\right],$$

$$P_{qg} \to .... + \alpha_S\frac{N_f}{3\pi x}\left[C\bar{\alpha}_S^3\left(\frac{\ln^2(1/x)}{2} - \ln(1/x)\right) + D\bar{\alpha}_S^4\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right)\right].$$

At NLO $A = -0.27, \quad B = 4.08, \quad C = 1.09, \quad D = 2.79$.

At NNLO $A = -0.35, \quad B = 5.49, \quad C = 2.81, \quad D = -2.00$.

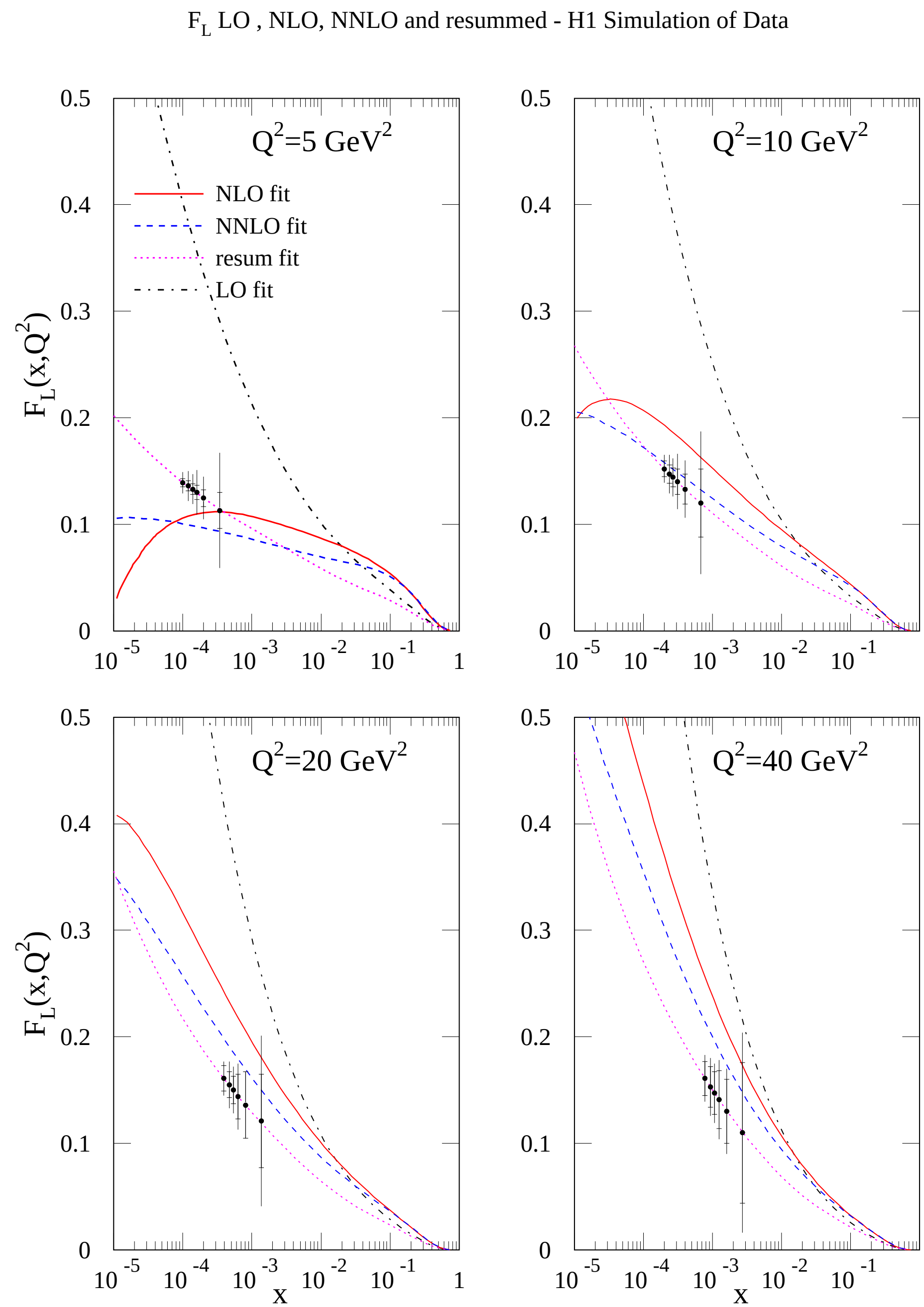
Saturation corrections do not help at NLO or NNLO.

Cuts suggestive of possible/probable theoretical errors for small x and/or small Q^2.

Much explicit work on $\ln(1/x)$-resummation in structure functions. Can suggest improvements to fit and changes in predictions.

Comparison of prediction for $F_L(x,Q^2)$ at LO, NLO and NNLO using MRST partons and also a $\ln(1/x)$-resummed prediction RT.

Accurate and direct measurements of $F_L(x,Q^2)$ and other quantities at low x and/or Q^2 (predicted range and accuracy of $F_L(x,Q^2)$ measurements posible at HERA II shown on picture) would be a great help in determining whether $NNLO$ is sufficient or whether resummed (or other) corrections are necessary, or helpful for maximum precision.



Conclusions

One can determine the parton distributions and predict boson cross-sections, by performing global fits to all up-to-date data over wide range of parameter space. The fit quality using NLO or NNLO QCD is fairly good.

QED corrections small. Various ways of looking at uncertainties due to errors on data. Uncertainties rather small – $\sim 1 - 5\%$ except in certain regions of parameter space.

Uncertainty from input assumptions e.g. cuts on data, data used, ..., comparable and potentially larger. Can shift central values of predictions significantly.

Errors from higher orders/resummation potentially large in some regions of parameter space - most important at high rapidity. Cutting out low x and/or Q^2 allows much improved fit to remaining data, and altered partons. NNLO appears to be much more stable than NLO.

Theory often the dominant source of uncertainty at present. Systematic study needed. Much progress – NNLO, resummations ..., but much still to do. Both for theory and in obtaining useful new data.

MRST fit with shadowing corrections extrapolated to $Q^2 \leq 5\mathrm{GeV}^2$

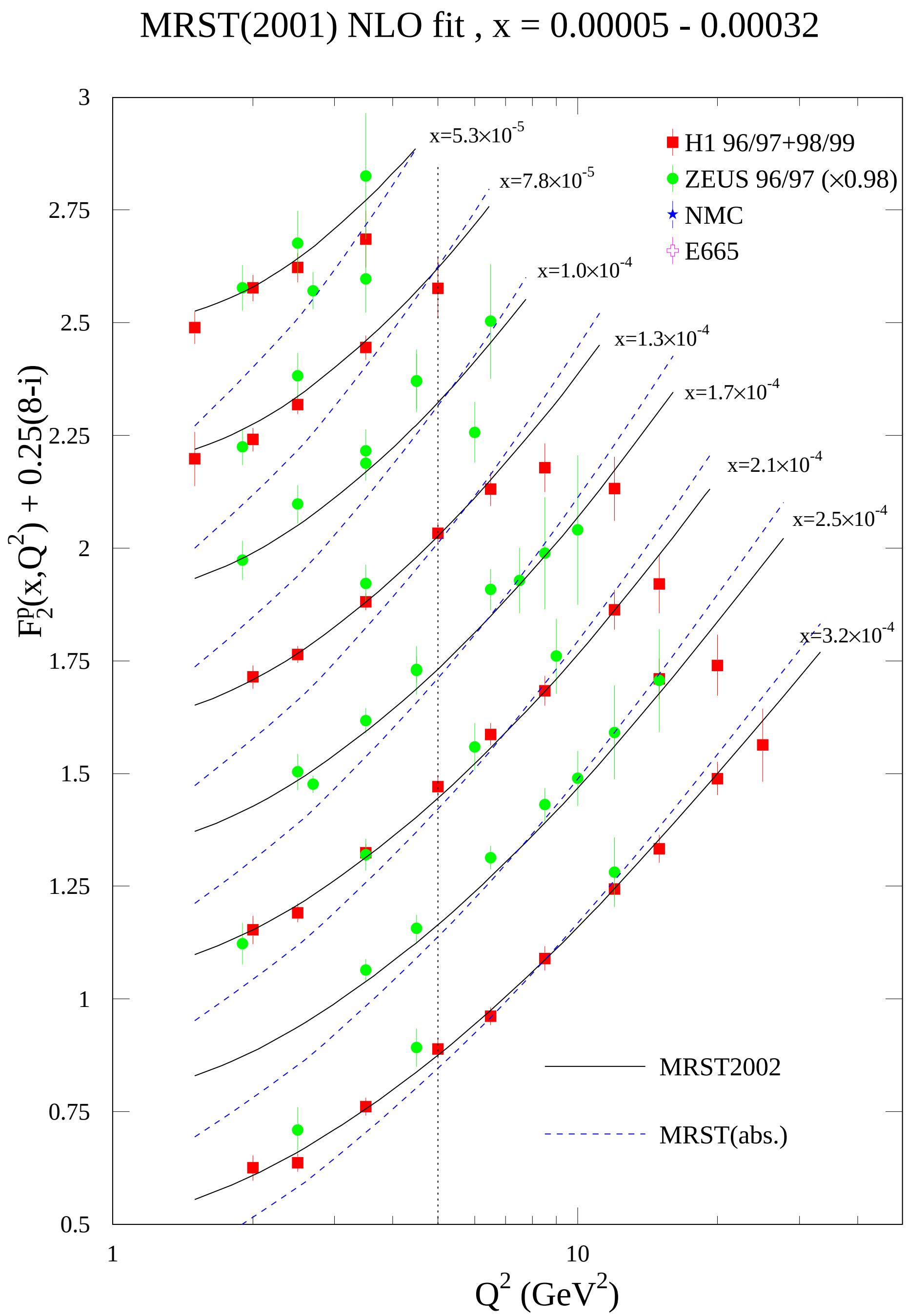


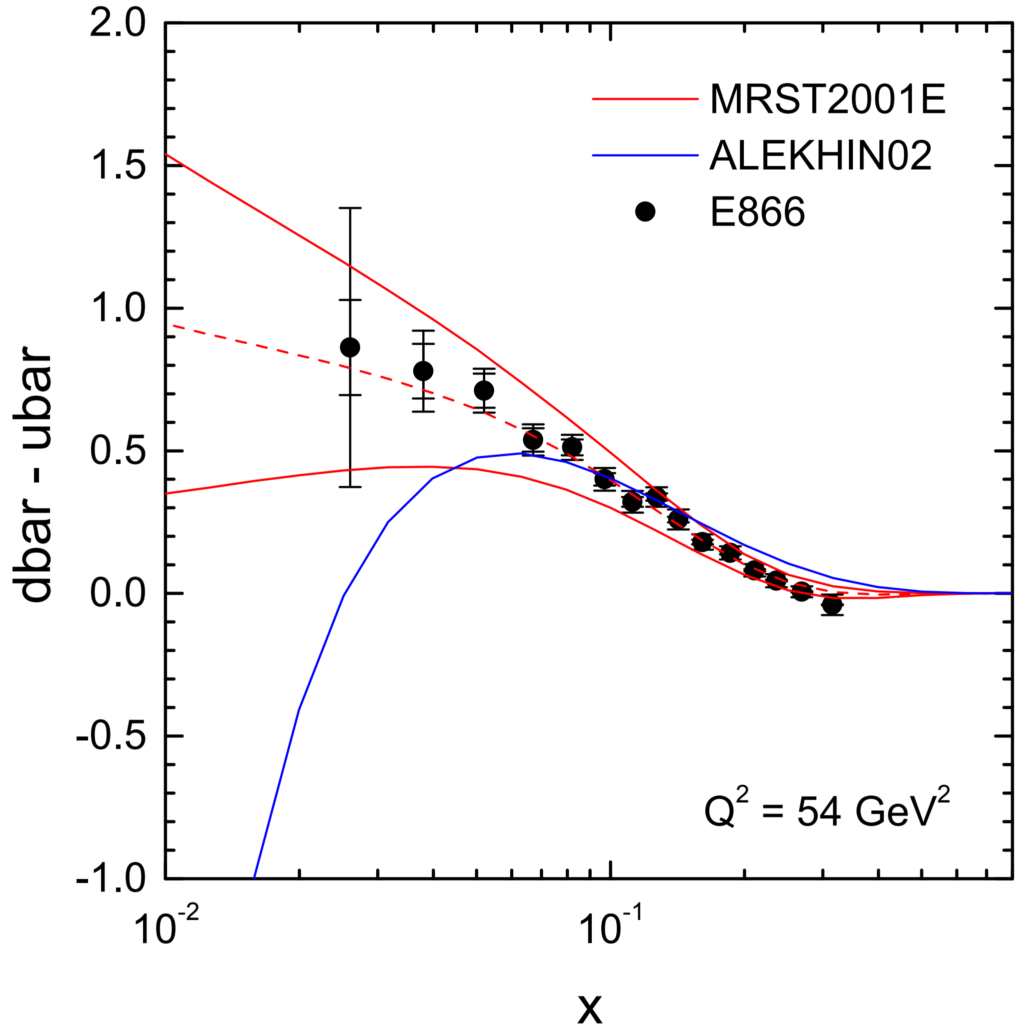
Table 3: Cross sections for Drell-Yan pairs (e^+e^-) with PYTHIA 6.206. The errors shown are the statistical errors of the Monte-Carlo generation.

PDF set	Comment	xsec
$81 < M < 101$ GeV		
CTEQ5L	PYTHIA internal	1516 ± 5 pb
CTEQ5L	PDFLIB	1536 ± 5 pb
CTEQ6	LHAPDF	1564 ± 5 pb
MRST2001	LHAPDF	1591 ± 5 pb
Fermi2002	LHAPDF	1299 ± 4 pb
$M > 1000$ GeV		
CTEQ5L	PYTHIA internal	6.58 ± 0.02 fb
CTEQ5L	PDFLIB	6.68 ± 0.02 fb
CTEQ6	LHAPDF	6.76 ± 0.02 fb
MRST2001	LHAPDF	7.09 ± 0.02 fb
Fermi2002	LHAPDF	7.94 ± 0.03 fb

Note anti-correlation between deviations at high and low mass, i.e. high and low x. Typical result from sum rules and evolution.

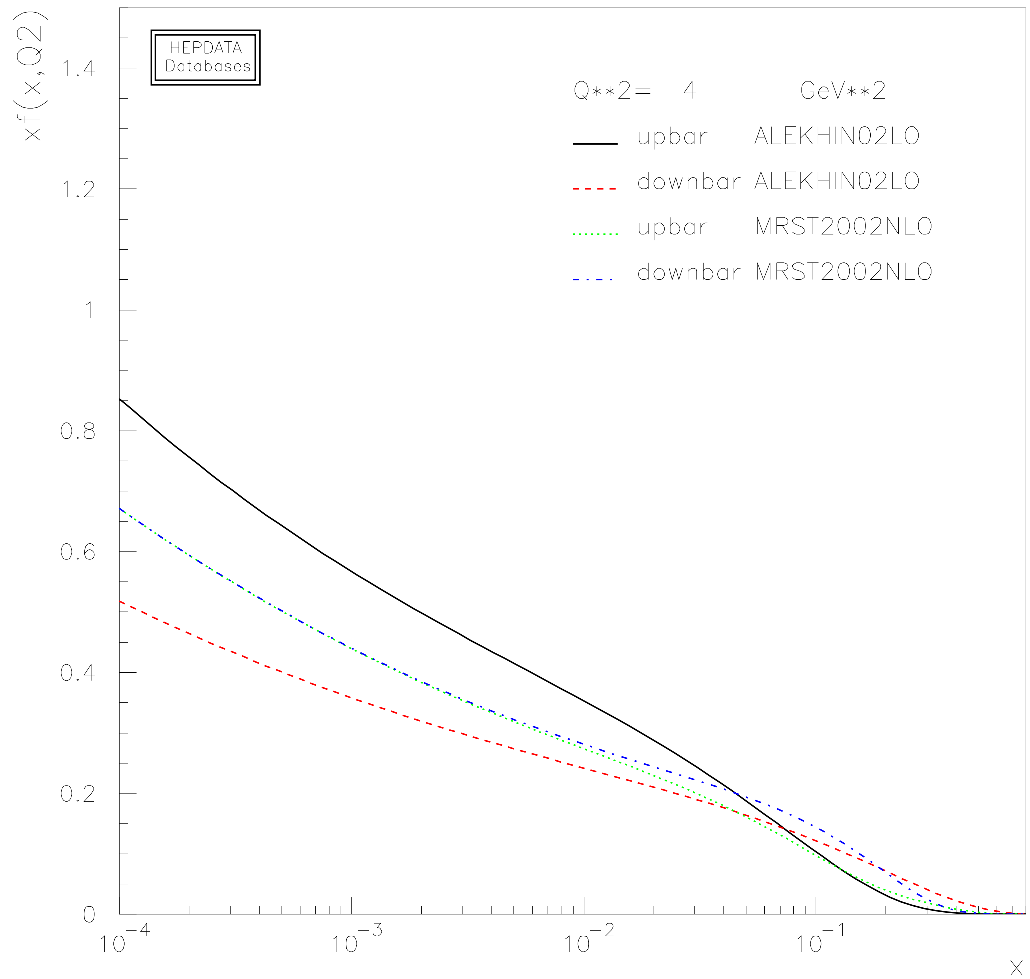
$\bar{d} - \bar{u}$ data and partons

Comparison of the $\bar{d}$ and $\bar{u}$ difference for MRST and Alekhin compared to the E866 data.



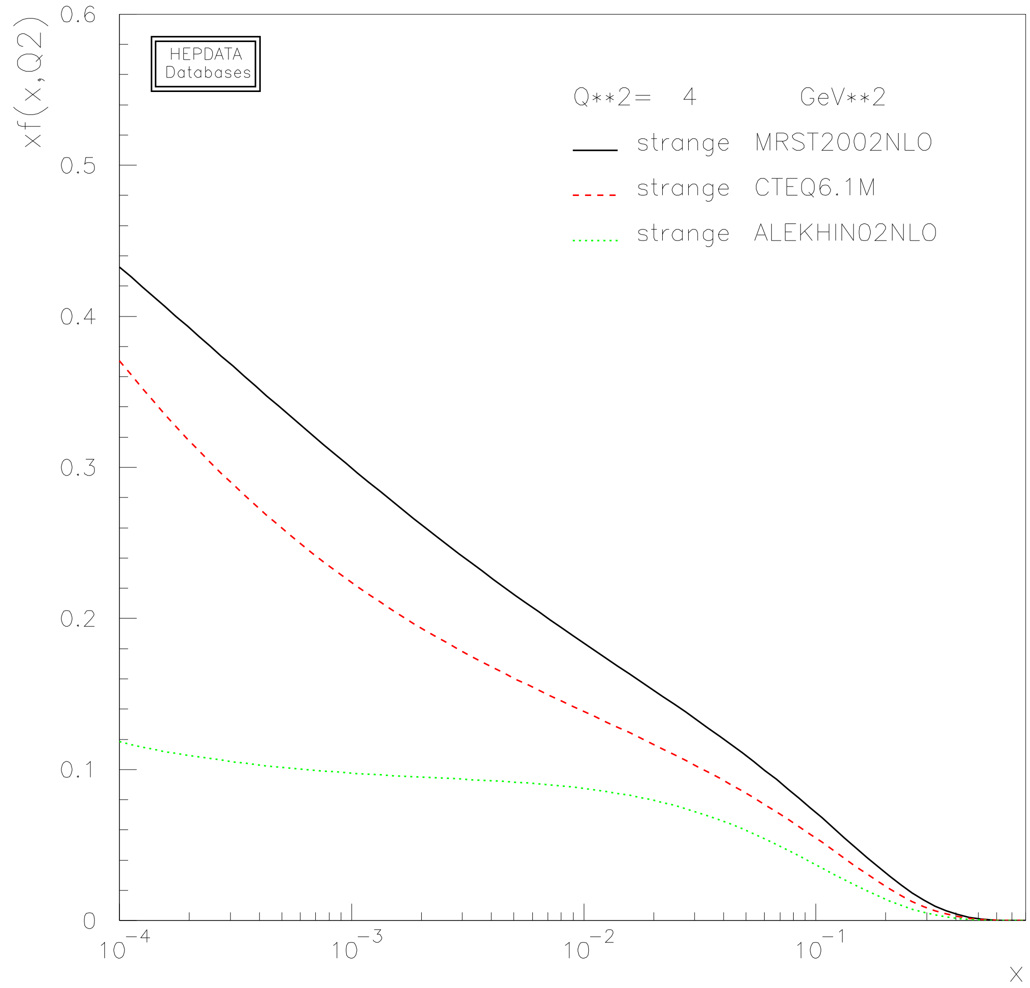
$\bar{d}$ and $\bar{u}$ partons

Comparison of the $\bar{d}$ and $\bar{u}$ partons for MRST and Alekhin.



s partons

Comparison of the s partons for MRST, CTEQ and Alekhin.



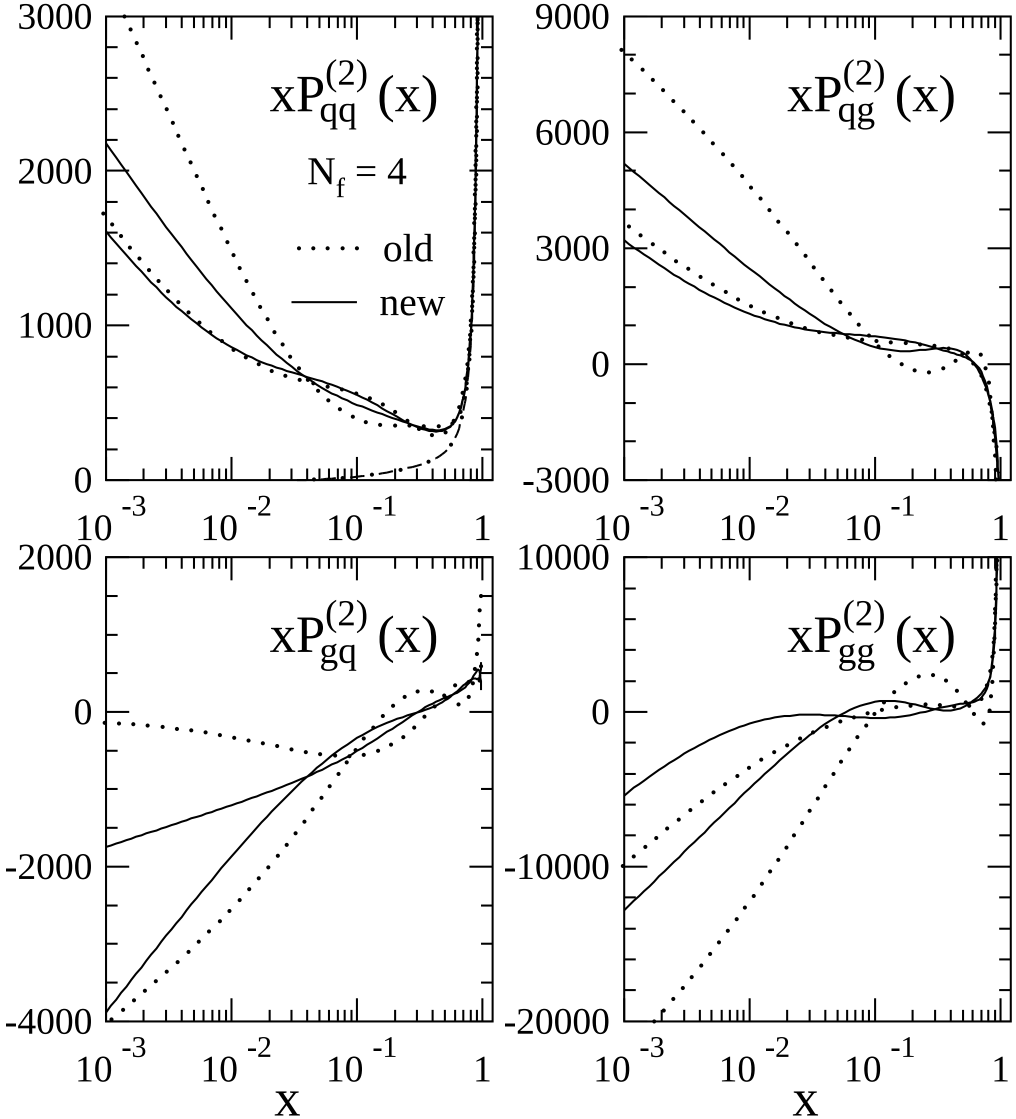
Hence, the estimation of uncertainties due to experimental errors has many different approaches and different types and amount of data actually fit. Overall conclude that uncertainty due to experimental errors only more than few % for quantities determined by high x gluon and very high x down quark.

Values of $\alpha_s(M_Z^2)$ and its error from different NLO QCD fits with different error tolerances. Reasonable agreement in general – but some outliers.

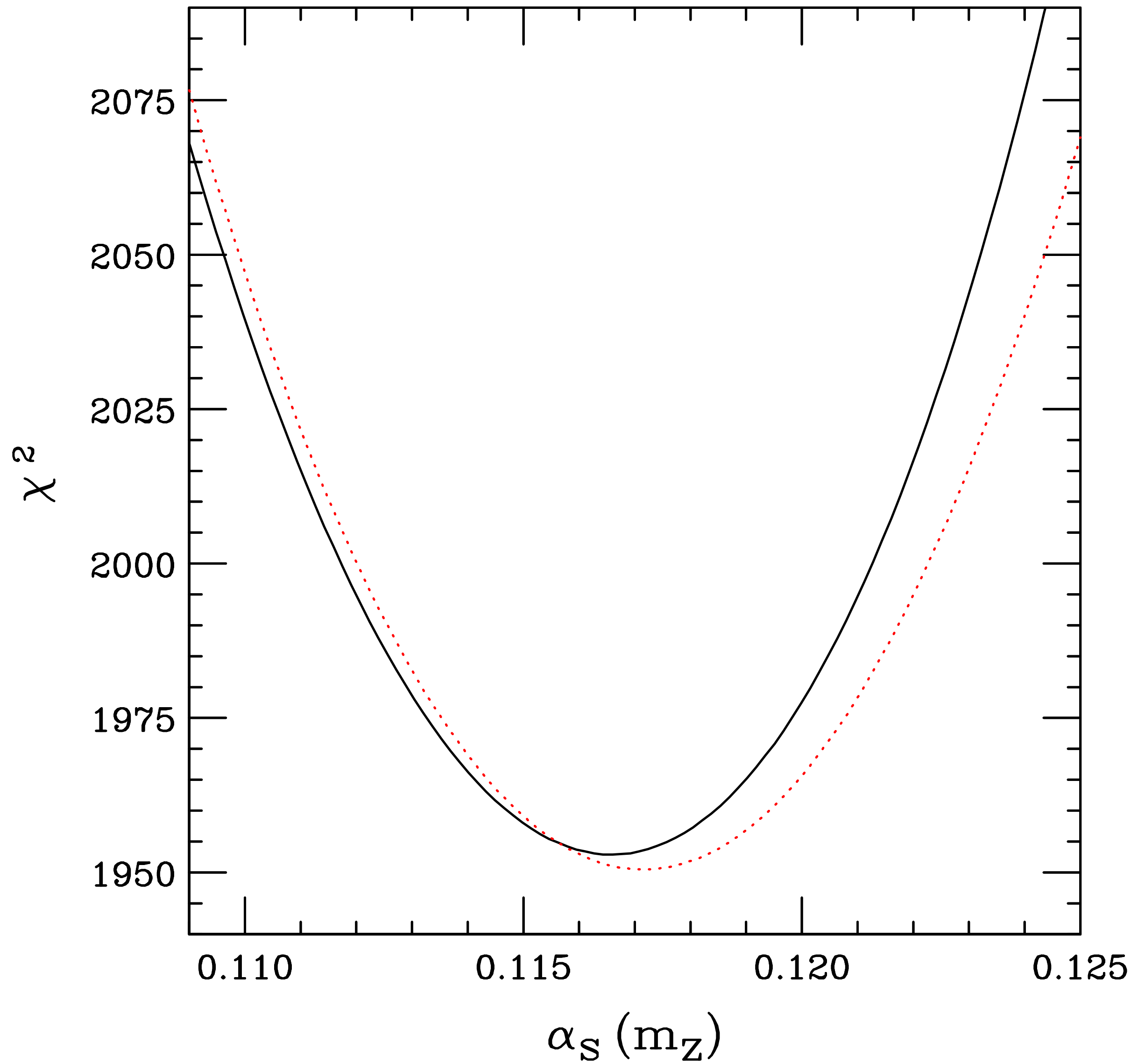
CTEQ6	$\Delta\chi^2 = 100$	$\alpha_s(M_Z^2) = 0.1165 \pm 0.0065(exp)$
ZEUS	$\Delta\chi^2_{eff} = 50$	$\alpha_s(M_Z^2) = 0.1166 \pm 0.0049(exp)$ $\pm 0.0018(model)$ $\pm 0.004(theory)$
MRST01	$\Delta\chi^2 = 20$	$\alpha_s(M_Z^2) = 0.1190 \pm 0.002(exp)$ $\pm 0.003(theory)$
H1	$\Delta\chi^2 = 1$	$\alpha_s(M_Z^2) = 0.115 \pm 0.0017(exp)$ ${}^{+0.0009}_{-0.0005}(model)$ $\pm 0.005(theory)$
Alekhin	$\Delta\chi^2 = 1$	$\alpha_s(M_Z^2) = 0.1171 \pm 0.0015(exp)$ $\pm 0.0033(theory)$
GKK	CL	$\alpha_s(M_Z^2) = 0.112 \pm 0.001(exp)$

Theory errors highly correlated.

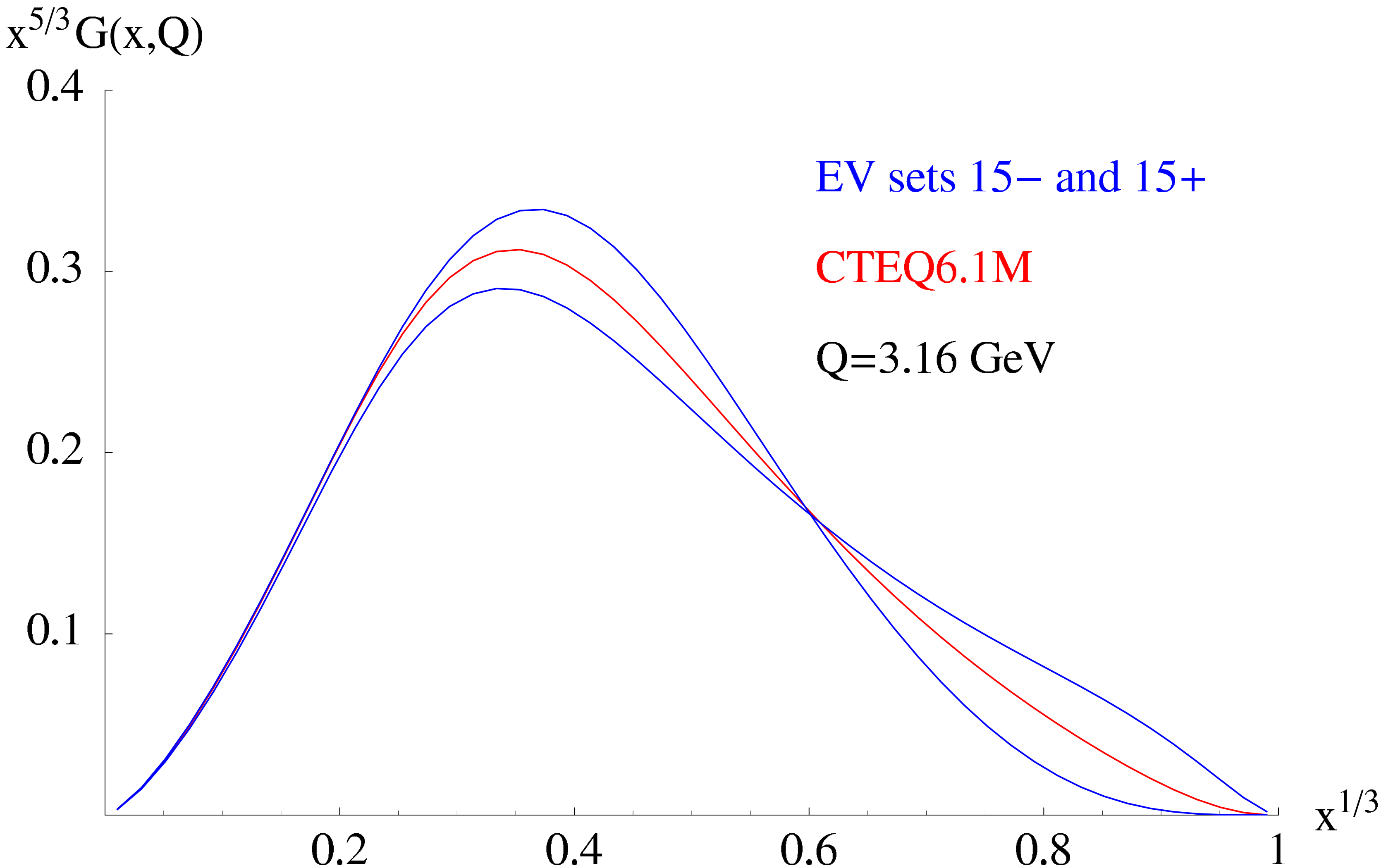
Approximate NNLO splitting functions devised by van Neerven and Vogt.



χ^2 against $\alpha_S(M_Z^2)$ for CTEQ for two choices of definition of NLO coupling.



Variation in CTEQ6 gluon along most sensitive eigenvalue direction.



Variation in CTEQ6 jet predictions for variations in each of the 20 eigenvector directions.