Structure functions on the lattice

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Outline

- Introduction
- Selected results
- Systematics
- Conclusions and outlooks

Introduction

- QCD is the fundamental theory of strong interaction
- Using Λ_{QCD} and m_q as input we should be able to compute all the physical quantities
- Perturbation theory can be used only to compute hard processes (the coupling constant being small)
- Non-perturbative tecniques for reliable computations are badly needed in the soft (non-perturbative) region
- The setup is euclidean field theory (J. Schwinger '58, K. Symanzik '66)

Lattice QCD

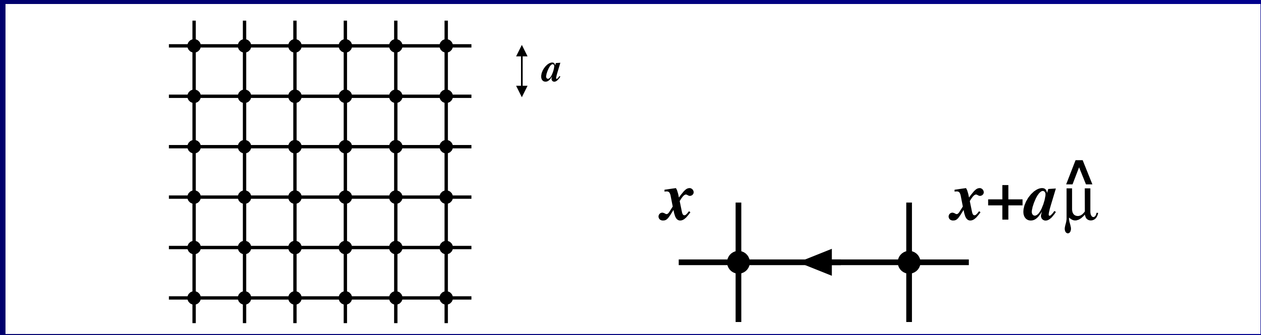
- Introduced many years ago (K. G. Wilson ’74)
  - quark confinement is a property of QCD
  - development of computational tools to determine the basic properties of the quark bound states
- Replace the 4-D space-time continuum through a hypercubic lattice (quarks and gauge fields restricted to lattice points)
- $\mathcal{L}_{\mathcal{QCD}}$ needs to be discretized in a sensible way
- The relation between correlation functions computed on the lattice and physical quantities must be understood

Lattice QCD

Key elements

- Gauge symmetry can be fully preserved
- The details of the discretization become irrelevant in the continuum limit
- Provides a regularization of the UV divergences $\rightarrow$ a lattice theory is mathemathically well defined from the beginning

Lattice QCD



$$x = a(n_0, n_1, n_2, n_3) \qquad n_\mu = Z$$

$$\psi(x) = \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \mathrm{e}^{ipx} \tilde{\psi}(p)$$

$$U(x,\mu) \in SU(3) \qquad U(x,\mu) \longrightarrow \Lambda(x) U(x,\mu) \Lambda^{-1}(x + a\hat{\mu})$$

$$\nabla_\mu \psi(x) = \frac{1}{a}[U(x,\mu)\psi(x+a\hat{\mu}) - \psi(x)]$$

$$\nabla_\mu^* \psi(x) = \frac{1}{a}[\psi(x) - U(x - a\hat{\mu})\psi(x - a\hat{\mu})]$$

Lattice QCD

K. G. Wilson '74

$$D_W = \frac{1}{2}[\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - a\nabla^*_\mu\nabla_\mu] + m_0$$

$$S_F[U, \bar{\psi}, \psi] = a^4 \sum_x \bar{\psi}(x) D_W \psi(x)$$

Lattice QCD

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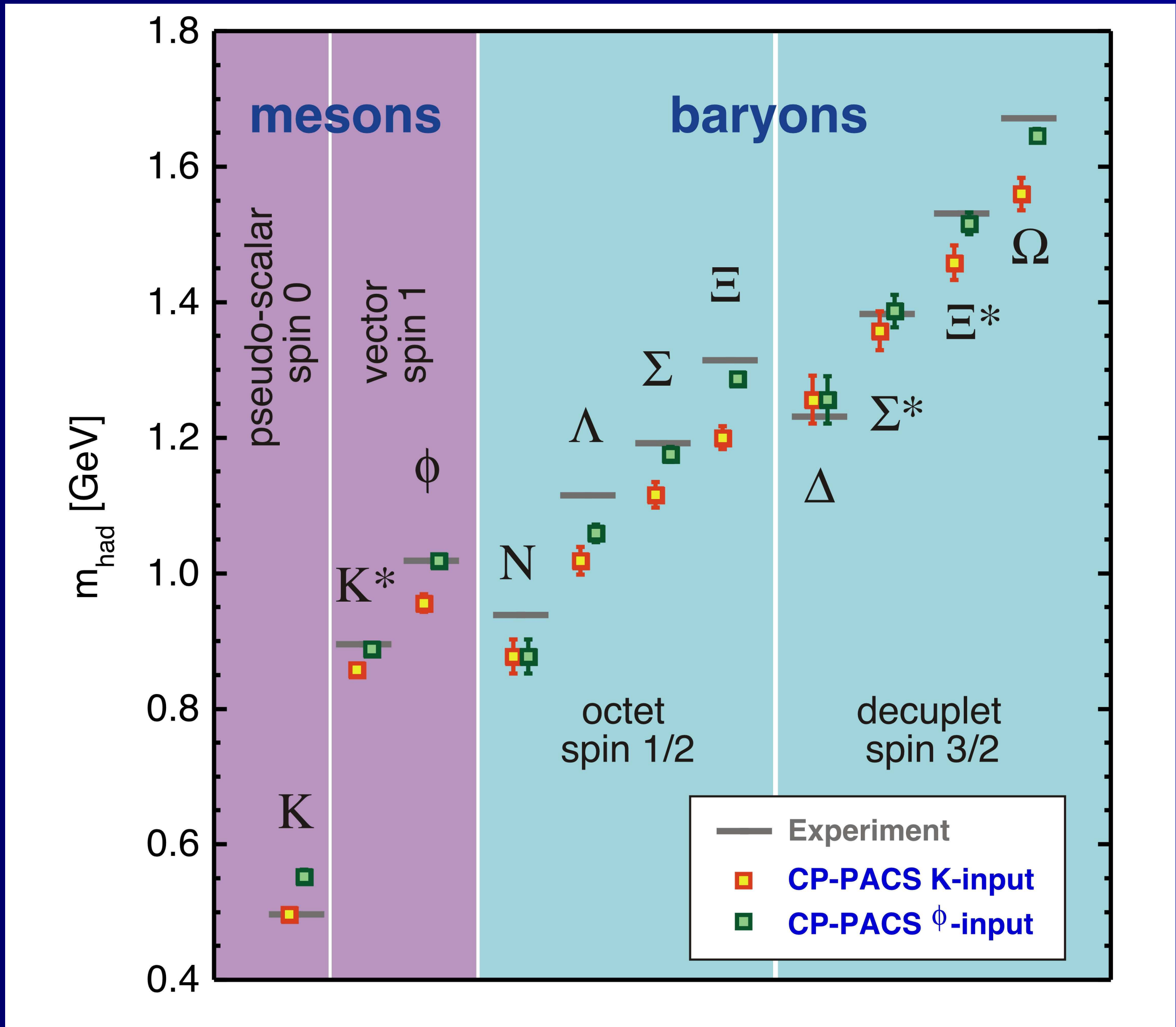
- Numerical simulations have the reputation of being an approximate method
- Simulations of lattice QCD produce results that are exact (on the given lattice) up to statistical errors

Spectrum of QCD (quenched)



Basic approach

- The OPE and the PDF approaches are connected
- The moments of PDF are related to leading twist τ (τ =dim-spin) operators of given spin

$$\langle x^N\rangle(\mu) = M_a^{(N)}(\mu^2 = Q^2) = \int_0^1 dx\, x^N [f_f(x,Q^2) + (-1)^{N+1} f_{\bar{f}}(x,Q^2)]$$

$$\langle P|O^{(i)}_{\mu_1\ldots\mu_N}(0)|P\rangle = M_a^{(N-1)}(\mu) P_{\mu_1}\cdots P_{\mu_n} + \text{terms containing } g_{\mu_i\mu_j}$$

List of operators

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$$\langle x^n \rangle \sim \langle h | \overline{\psi} \gamma_{\{\mu} D_{\mu_1} \cdots D_{\mu_n\}} \psi | h \rangle$$

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- Transversity

$$\langle (\delta x)^n \rangle \sim \langle h | \overline{\psi} \gamma_5 \sigma_{\mu\{\mu_1} D_{\mu_2} \cdots D_{\mu_n\}} \psi | h \rangle$$

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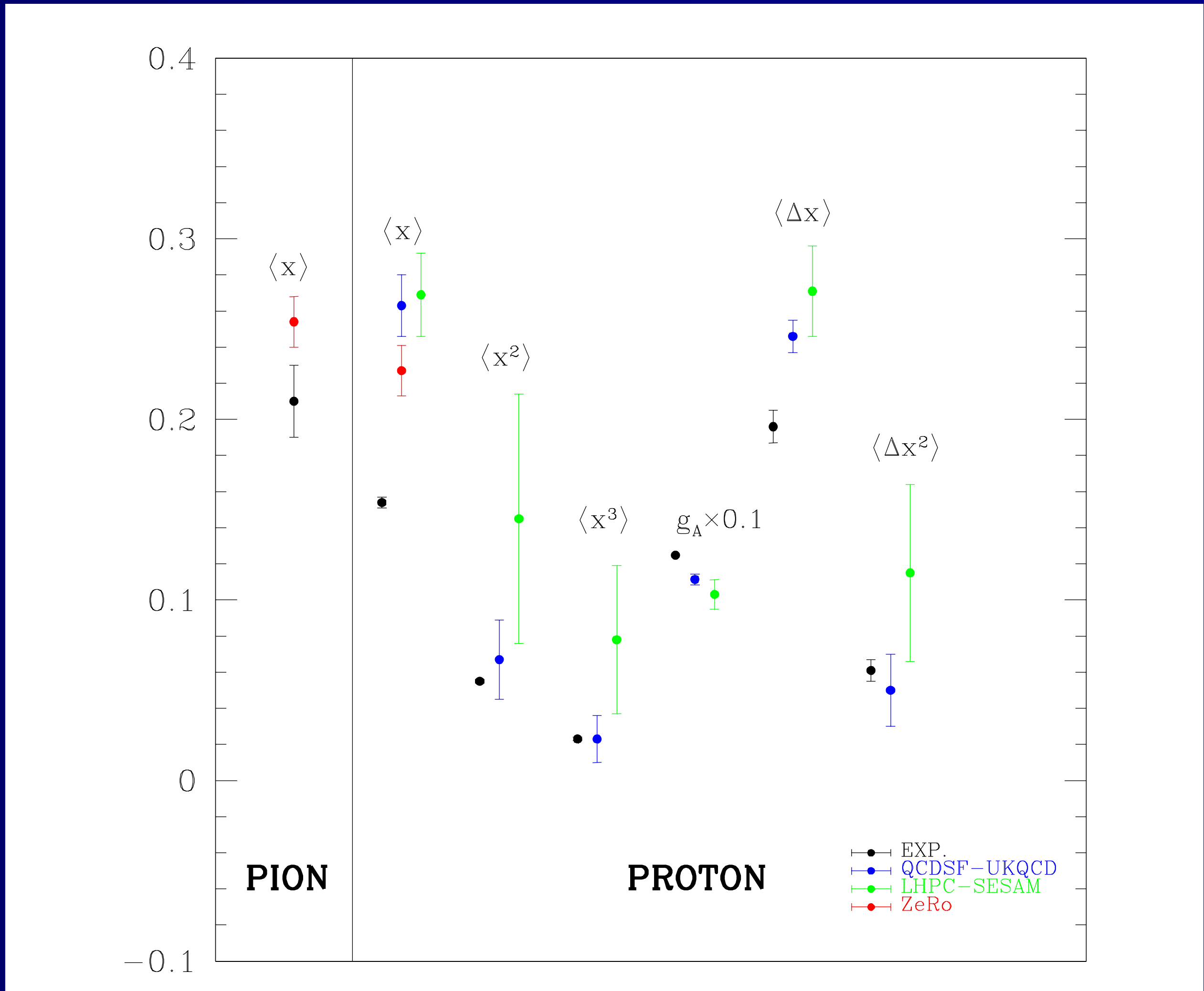
- Singlet

$$\langle x^n \rangle_g \sim \langle h | \mathrm{Tr}(F_\mu^\rho D_{\mu_2} \cdots D_{\mu_n} F_{\rho\mu_1}) | h \rangle$$

Collaborations

- ZeRo : M. Guagnelli, K. Jansen, F. Palombi, R. Petronzio, A. Shindler, I. Wetzorke.
- QCDSF-UKQCD : T. Bakeyev, D. Galletly, M. Göckeler, M. Gürtler, R. Horsley, B. Joo, A.D. Kennedy, B.J. Pendleton, H. Perlt, D. Pleiter, P.E.L. Rakow, G. Schierholz, A. Schiller, T. Streuer, H. Stüben.
- LHPC-SESAM :D. Dolgov, R. Brower, S. Capitani, P. Dreher, J.W. Negele, A. Pochinsky, D. B. Renner, N. Eicker, Th. Lippert, K. Schilling, R.G. Edwards, U.M. Heller.

Structure functions results



Systematics

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- Renormalization
  - Perturbative and non-perturbative
- Continuum limit
  - O(a) Improvement

Systematics

- Renormalization
  - Perturbative and non-perturbative
- Continuum limit
  - O(a) Improvement
- Chiral extrapolation
  - Lattice formulation of the Dirac operator
  - Chiral perturbation theory (χPT)
- Finite volume effects (FVE)
- Quenching

Renormalization

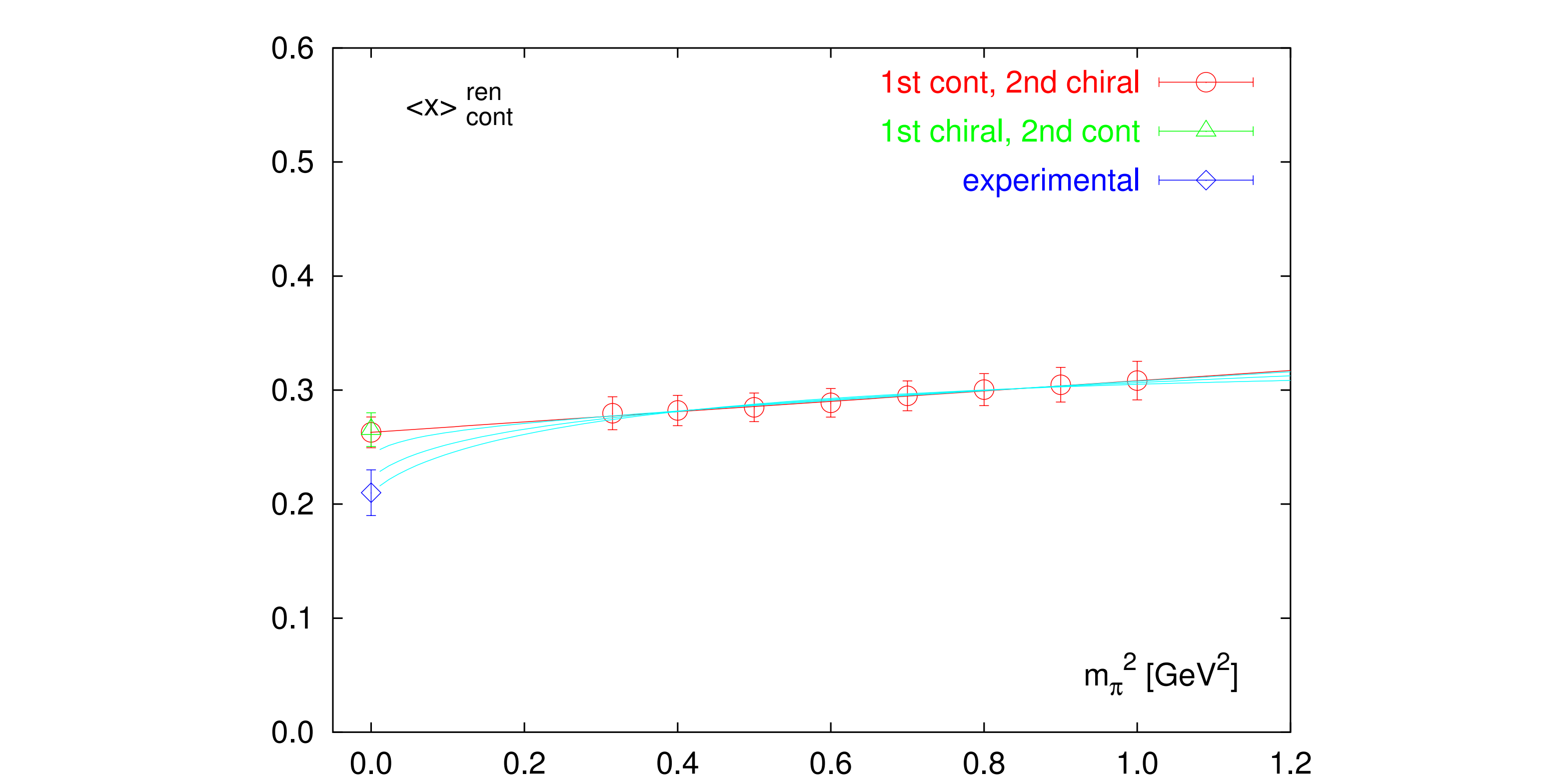
- Perturbative RC computed for all the operators simulated untill now
  - Wilson, clover, overlap (RI-MOM scheme) S.Capitani, QCDSF, LHPC
  - Wilson, clover (SF scheme) ZeRo
- Tadpole improved RC computed for operators related to the unpolarized structure functions (RI-MOM) QCDSF
- Non-perturbative renormalization computed for the first moment of the unpolarized structure function (SF scheme) ZeRo

SF scheme

$\langle x \rangle_{RGI}$	$\longleftarrow$	$\langle x \rangle_{RGI}$
$\downarrow$		$\uparrow$
perturbative evolution		perturbative evolution
$\downarrow$		$\uparrow$
$\langle x \rangle_{\overline{\mathrm{MS}}}$ at a certain scale μ		$\langle x \rangle_{SF}$ at $\mu \simeq 40$ GeV
		$\uparrow$
		non-perturbative evolution
		$\uparrow$
hadronic scheme	$\longrightarrow$	finite-volume scheme $\langle x \rangle_{SF}$ at $\mu \simeq 280$ MeV

ALPHA-ZeRo

NP renormalization (pion)

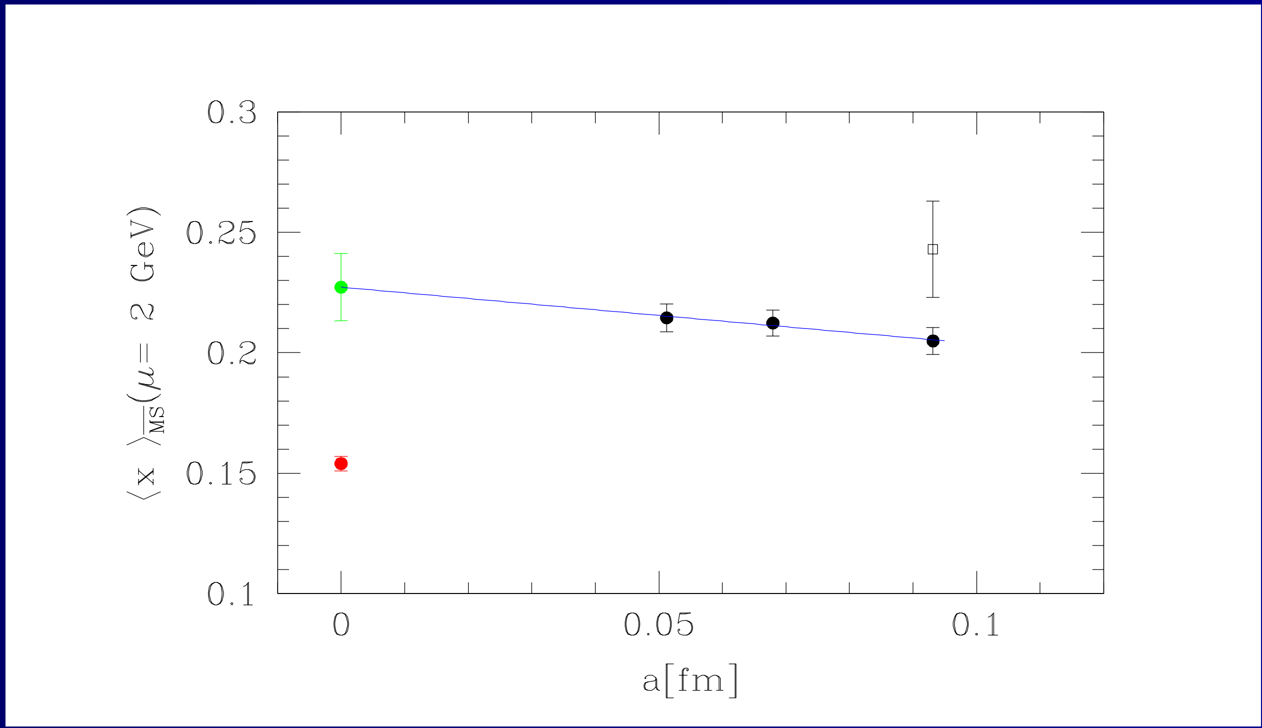


$\langle x \rangle_{\overline{MS}}(2GeV) = 0.254(14)$ ZeRo

$\langle x \rangle_{\overline{MS}}(2GeV) = 0.21(2)$ Exp.

NP renormalization (proton)

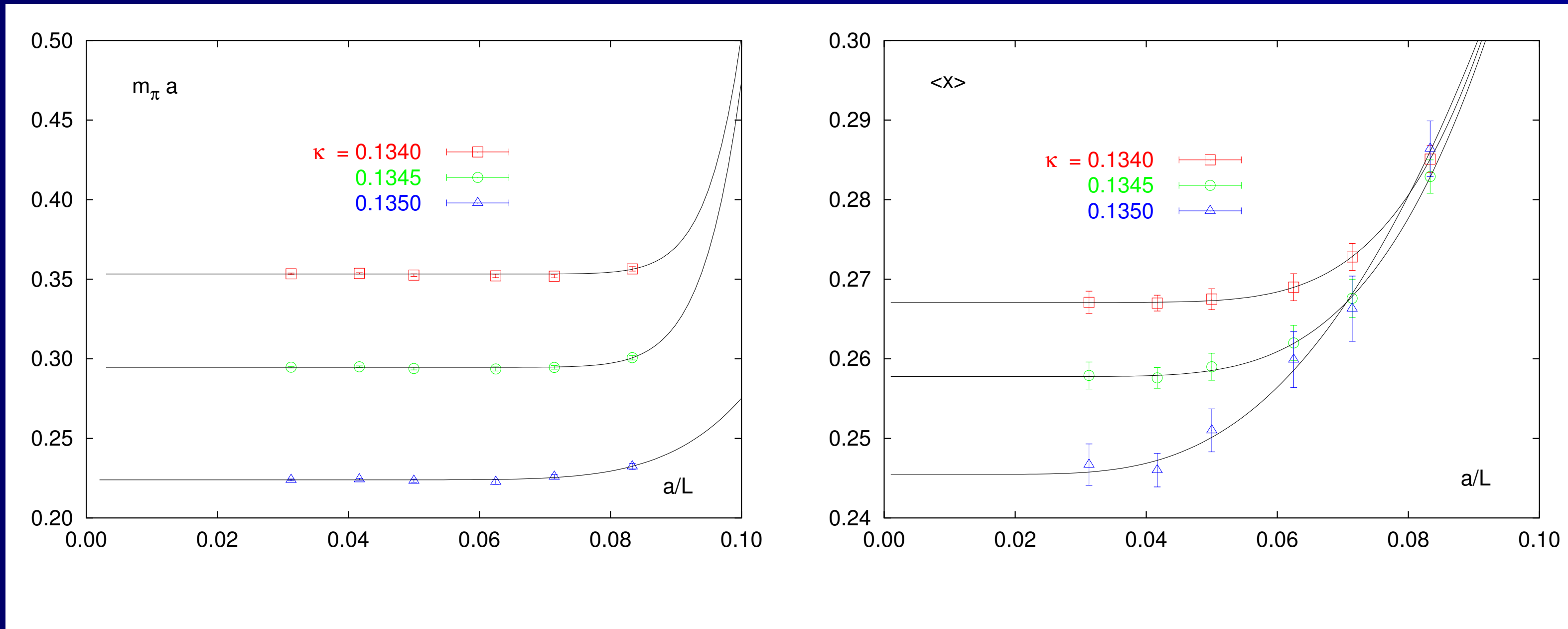
Bare matrix elements from QCDSF and renormalization factor from ZeRo



Finite volume effects

- No studies of FVE for matrix elements of twist ≥ 2
- For the nucleon masses there are big finite volume effects
- From the study of the FVE in the quenched theory for matrix elements between pion states, we have learned that FVE can be an important issue

Finite volume effects



ZeRo

- β = 6.1 L/a = 12,14,16,20,24,32

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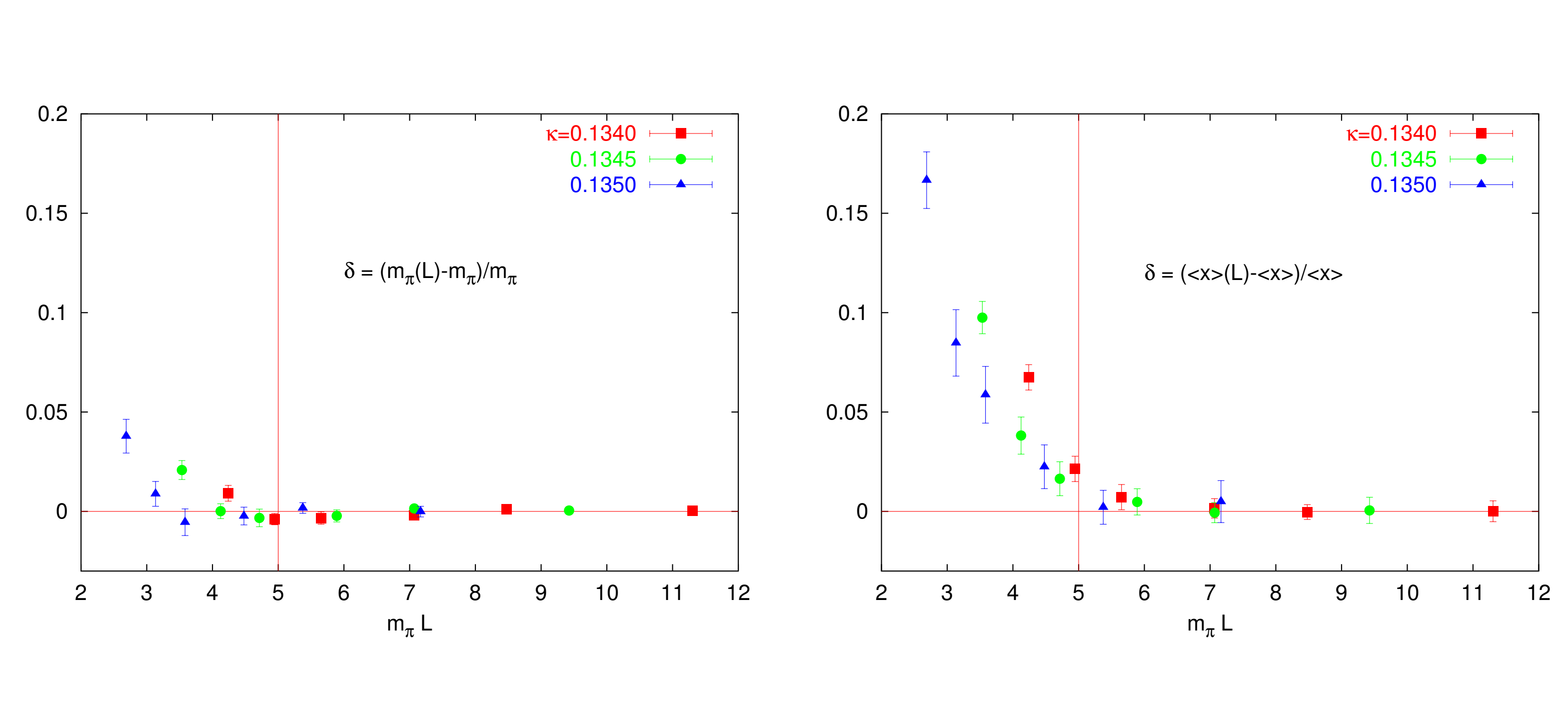
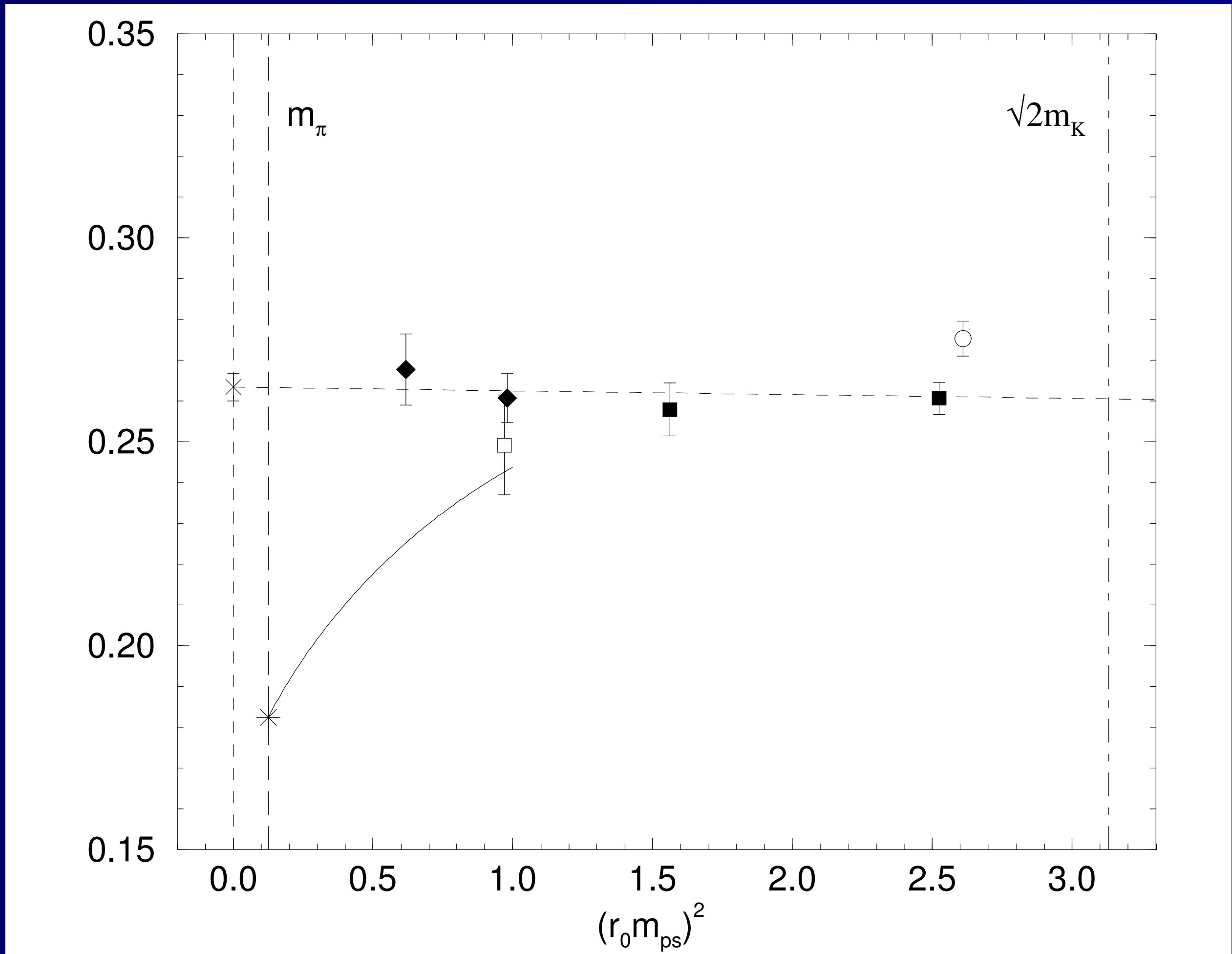
Finite volume effects



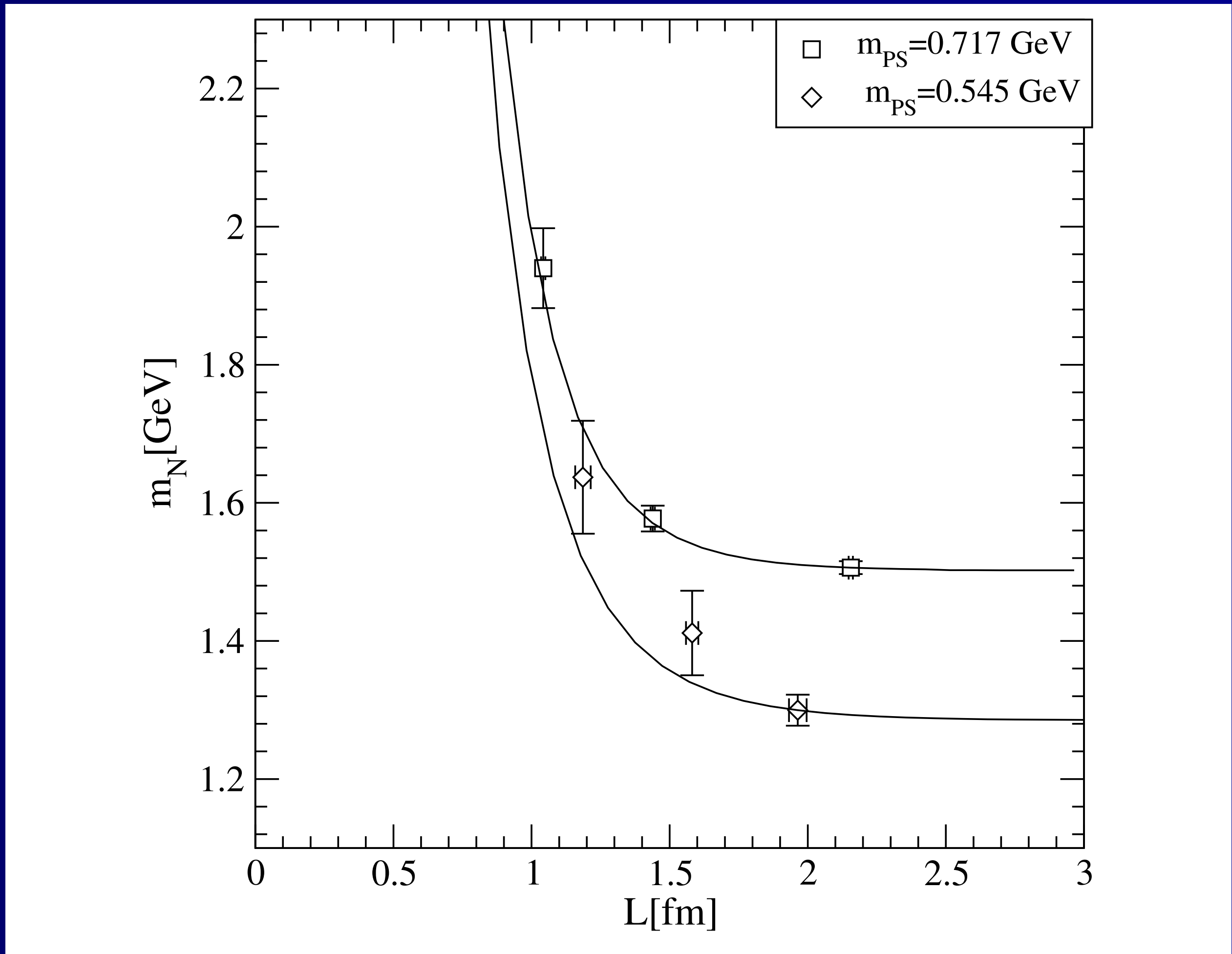
- NO analytic computation of FV corrections
- Quenched matrix element between pion states: GOOD CASE

Finite volume effects



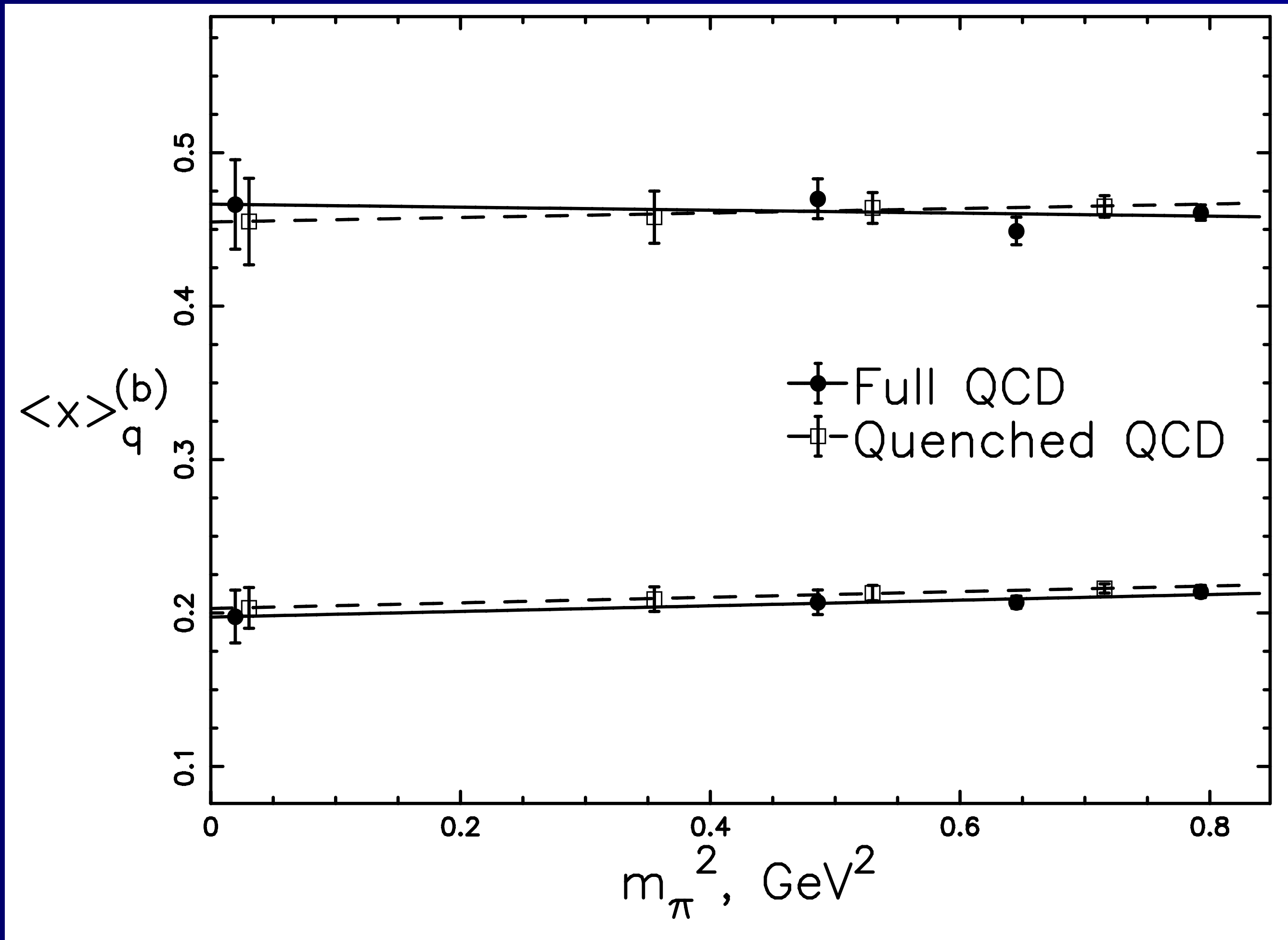
QCDSF

Finite volume effects



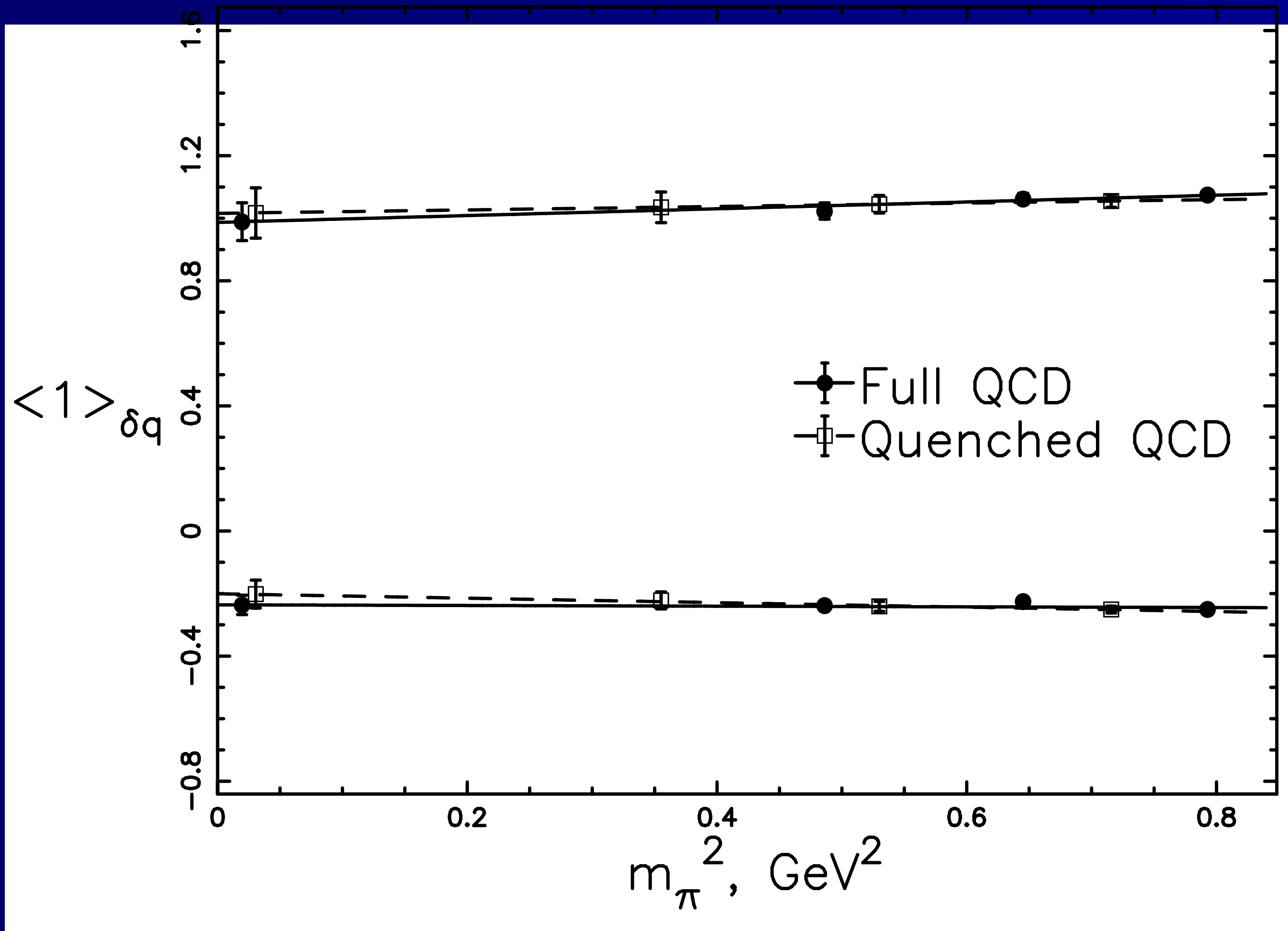
QCDSF

Unquenched results



LHPC-SESAM

Unquenched results



LHPC-SESAM

Chiral extrapolation

- The leading order chiral correction is given by

$$\langle x^n \rangle_{u-d} \sim a_n[1 - \frac{(3g_A^2+1)m_\pi^2}{(4\pi f_\pi)^2} \ln(\frac{m_\pi^2}{(m_\pi^2+\mu^2)})] + b_n m_\pi^2$$

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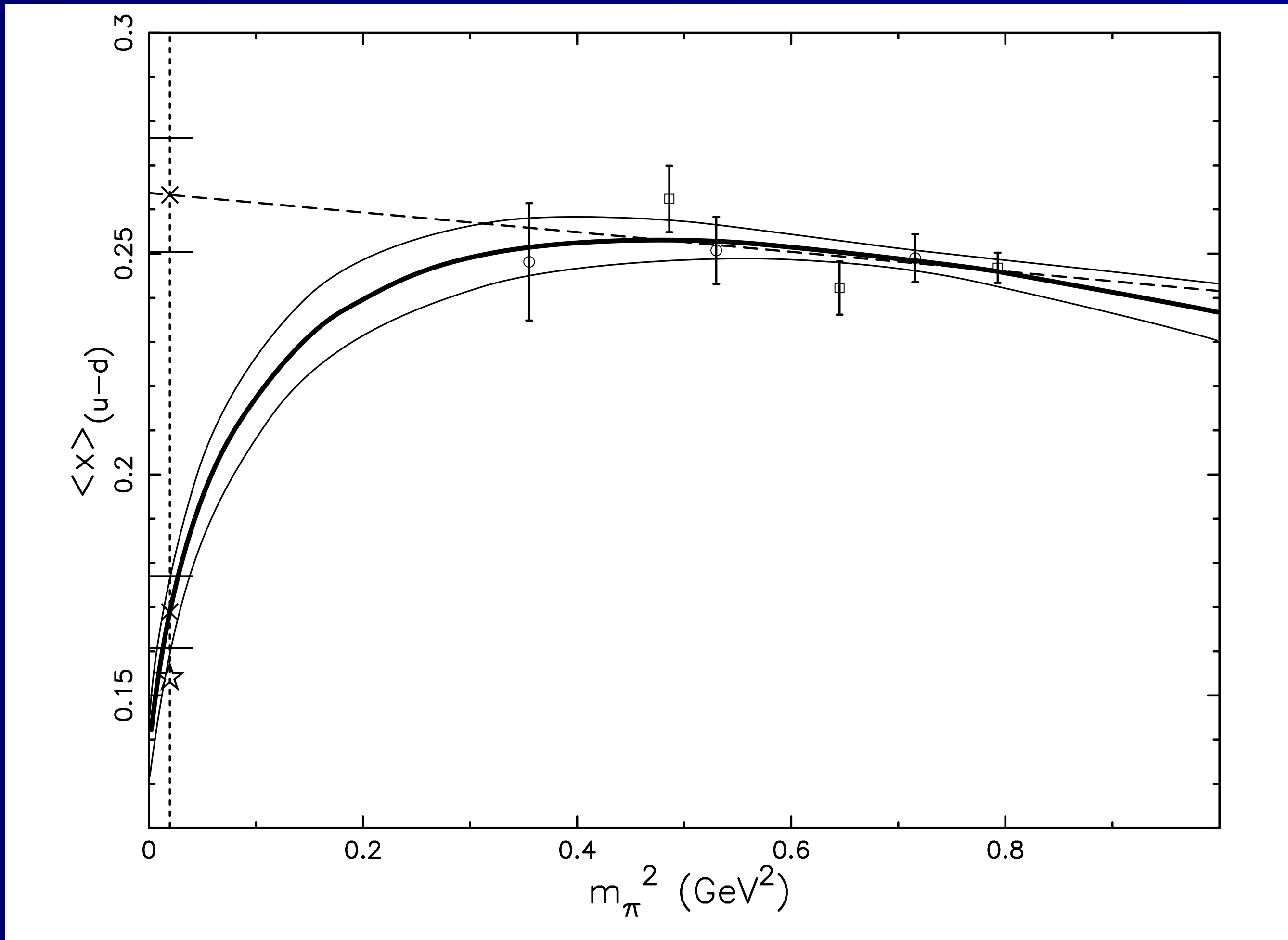
- In quenched χPT the non analytic term vanishes

$$\langle x^n \rangle_{u-d} \sim a_n + b_n m_\pi^2$$

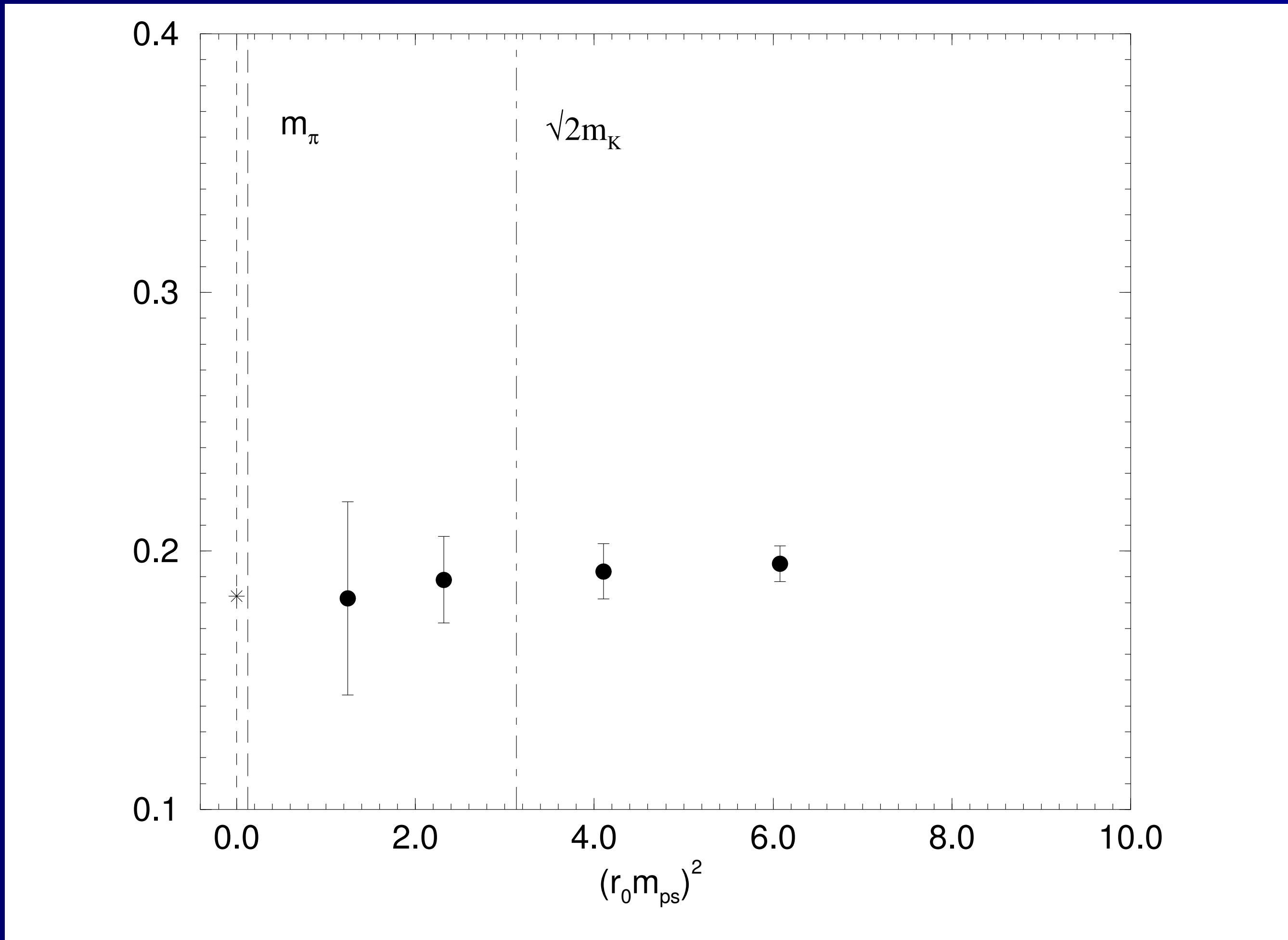
Chiral extrapolation



LHPC-SESAM

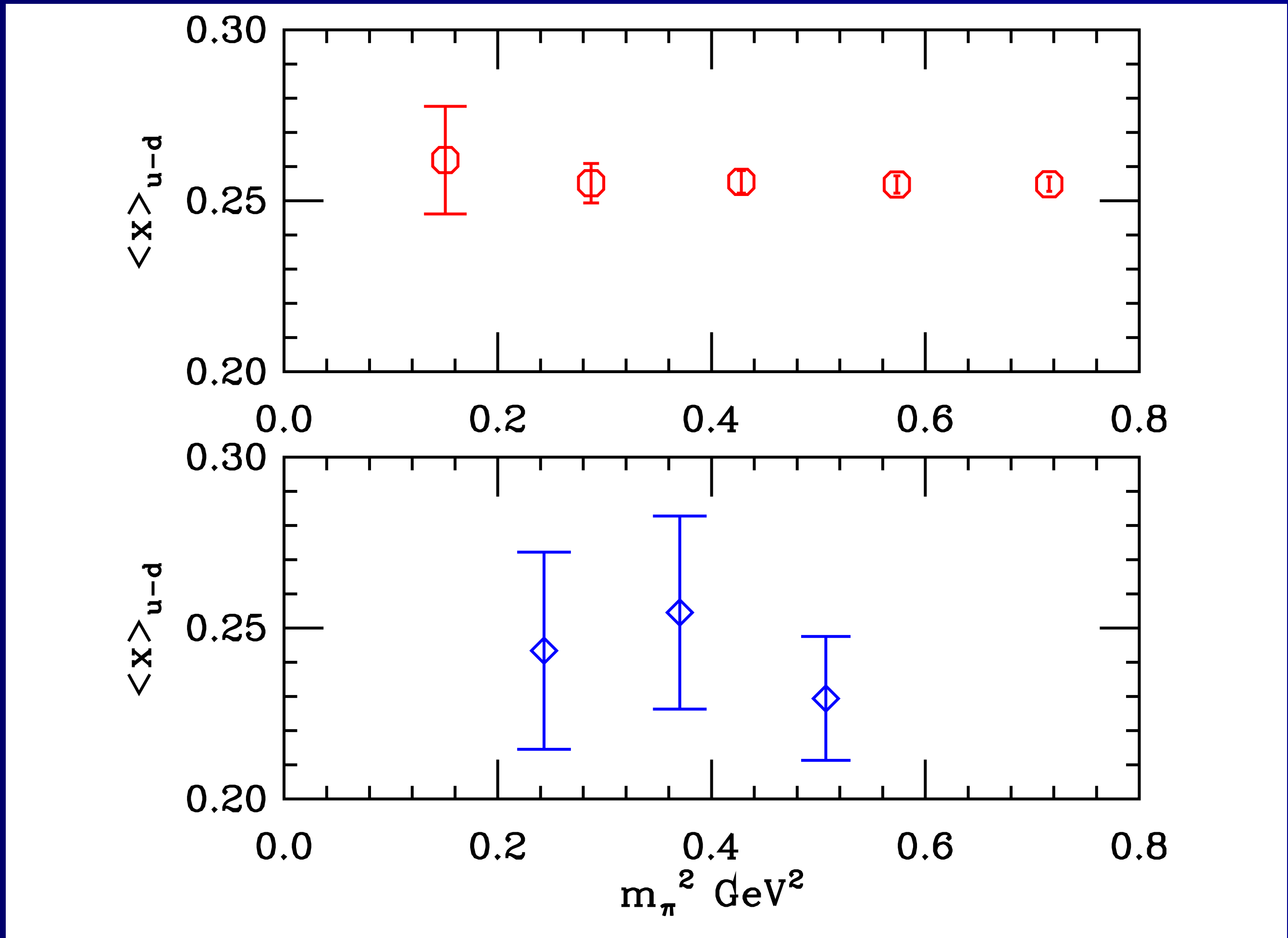
Results with chiral fermions

Overlap fermions



QCDSF

Domain wall fermions



RBC

Summary

- Pion structure functions
  - Disagreement with experiment ($\sim 10\%$) with competitive error
  - NP renormalization, continuum limit, FVE, χ extrapolation YES
  - Unquenched NO

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  - Disagreement with experiment ($\sim 10\%$) with competitive error
  - NP renormalization, continuum limit, FVE, χ extrapolation YES
  - Unquenched NO
- Nucleon (unpolarized) structure function
  - Disagreement with experiment ($\sim 50\%$)
  - NP renormalization, continuum limit, unquenched (high masses) YES
  - χ extrapolation (high masses) YES
  - FVE NO

What do we need in the future?

- NP renormalization of all the other operators (singlet)
- Continuum limit
- Quenched with high precision and unquenched

What do we need in the future?

- NP renormalization of all the other operators (singlet)
- Continuum limit
- Quenched with high precision and unquenched
- χPT with finite volume corrections
- Numerical studies of FVE, expecially in the unquenched case for the nucleon
- Simulation at lower values of quark masses (overlap, domain wall,...)

Aknowledgments

- Thank you for the invitation
- In particular: A. M. Stasto and D. Bruncko for their patience ;-)