Fall and rise of the gluon splitting function (at small x)

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DIS 2004 — Štrbské Pleso

16 April 2004

- Small-x gluon splitting function has logarithmic enhancements:

$$xP_{gg}(x) = \sum_{n=1} \alpha_s^n \ln^{n-1} \frac{1}{x}$$

$$+ \sum_{n=2} \alpha_s^n \ln^{n-2} \frac{1}{x} + \dots$$

Perturbative structure

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Leading Logs (LLx):

$$\bar{\alpha}_s + \frac{\zeta(3)}{3} \bar{\alpha}_s^4 \ln^3 \frac{1}{x} + \frac{\zeta(5)}{60} \bar{\alpha}_s^6 \ln^5 \frac{1}{x} + \cdots$$

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Next-to-Leading Logs (NLLx):

$$A_{20}\bar{\alpha}_s^2 + A_{31}\bar{\alpha}_s^3 \ln\frac{1}{x} + A_{42}\bar{\alpha}_s^4 \ln^3\frac{1}{x} + \dots$$

Fadin & Lipatov '98

Camici & Ciafaloni '98

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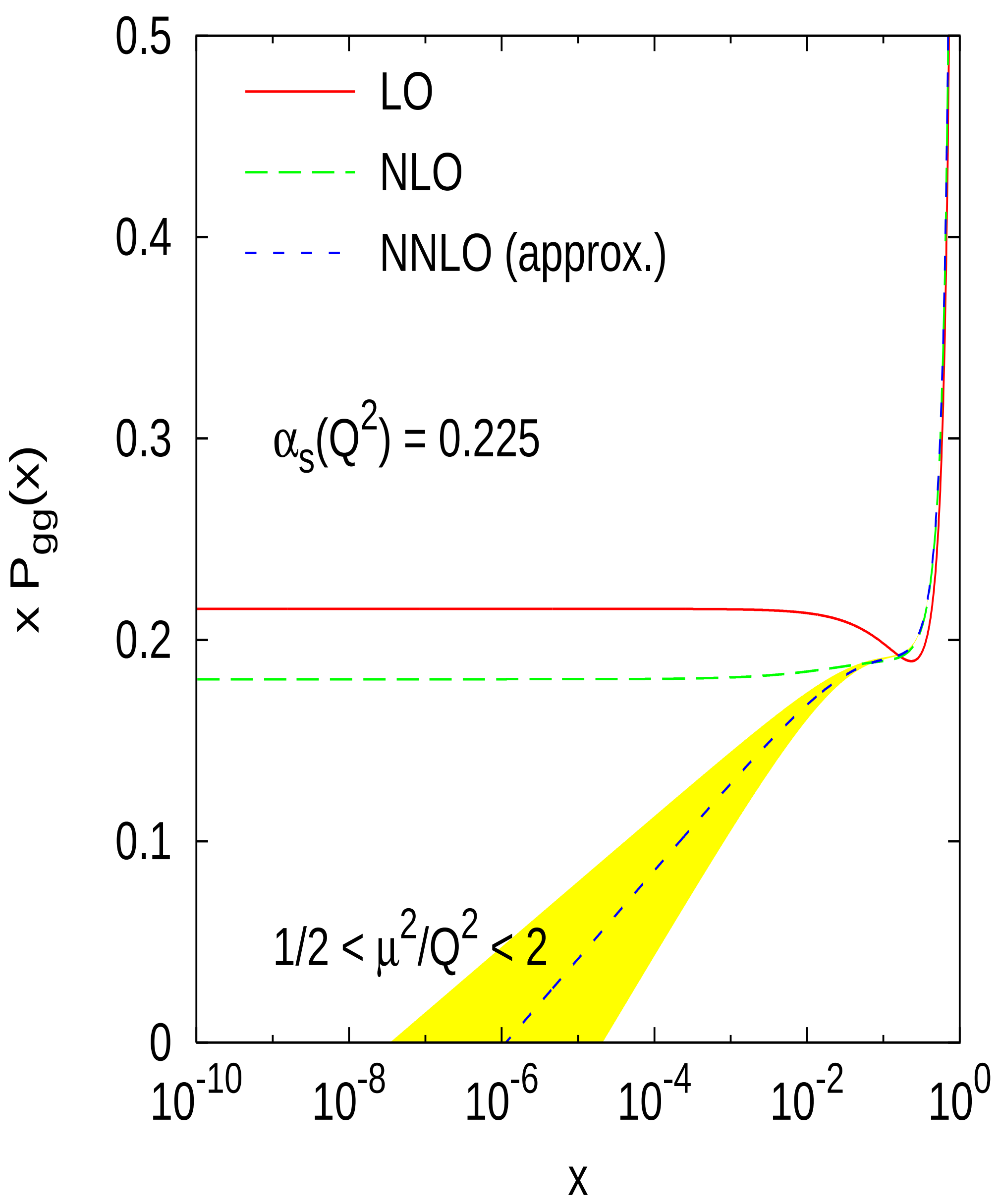


Reminder

- LLx terms rise very fast, $xP_{gg}(x) \sim x^{-0.5}$.

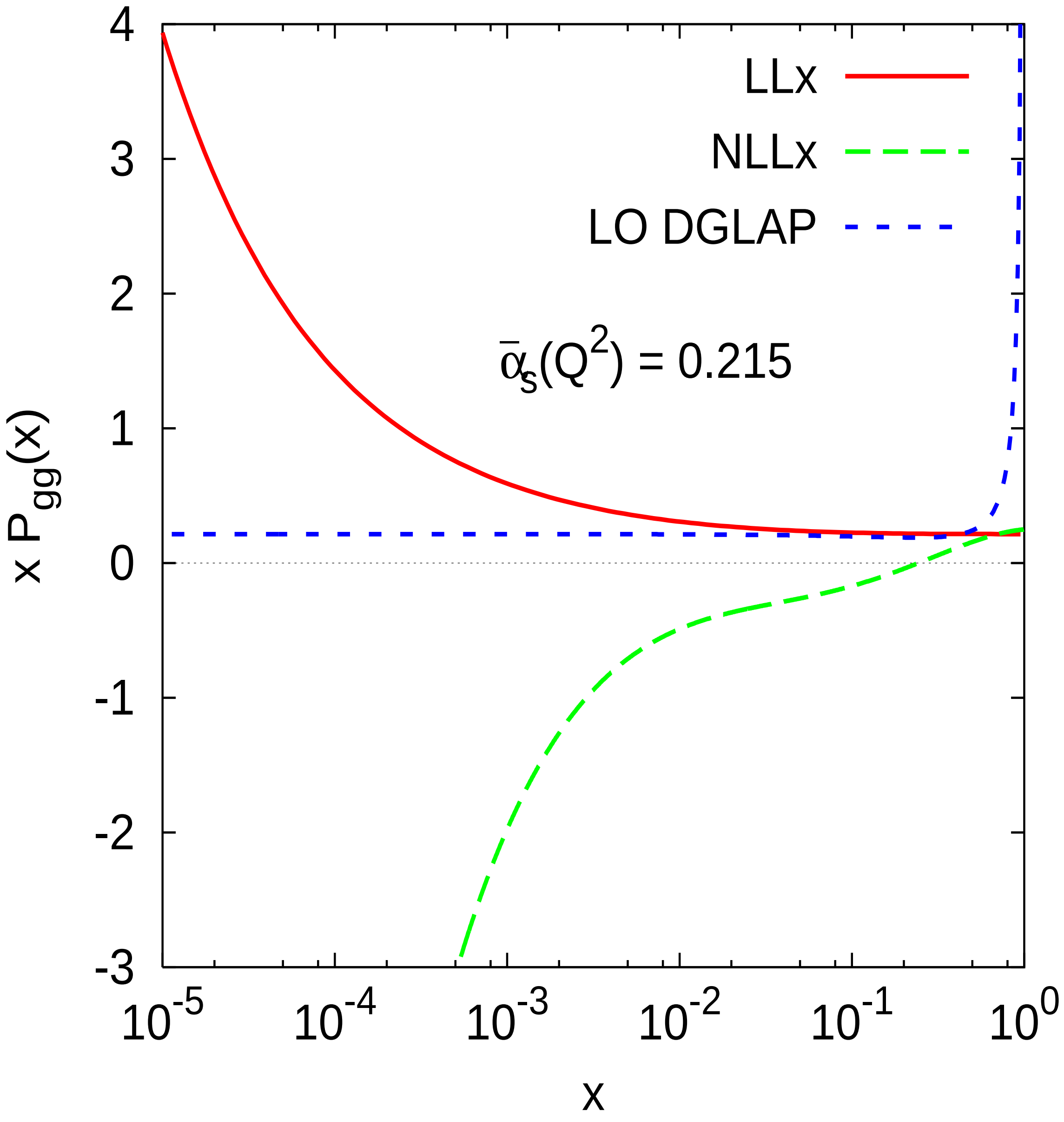
  Incompatible with data.

  Ball & Forte '95

- NLLx terms go negative very fast.

  No one's even tried fitting the data!

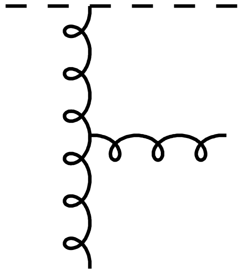
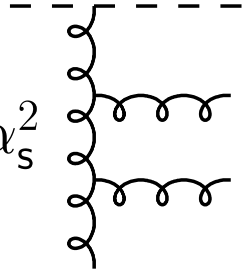
[NB: Taking NLLx terms of P_{gg} is almost the worst possible expansion]



‘Improving’ on NLLx? Start with kernel. . .

BFKL

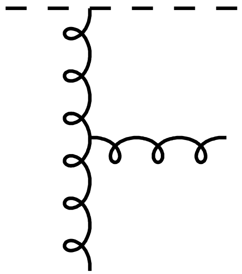
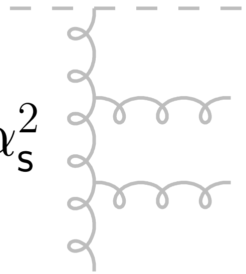
$x \ll x_0$

α_s  $+\ \alpha_s^2$  $\times \ln \frac{x_0}{x}$

x_0

DGLAP

$Q^2 \gg Q_0^2$

α_s  $+\ \alpha_s^2$  $\times \ln \frac{Q^2}{Q_0^2}$

Q_0^2

$+\ Q^2 \Leftrightarrow Q_0^2$

anti-DGLAP

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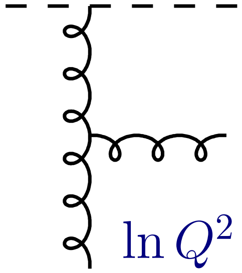
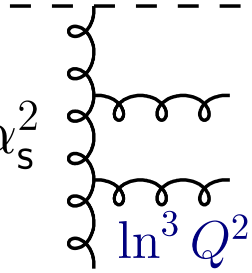
α_s



x_0

+ α_s^2

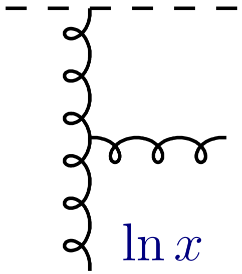


$$\left(\alpha_s \,[\ln Q^2] + \alpha_s^2 \,[\ln^3 Q^2] \right) \mathsf{x} \ln \frac{x_0}{x}$$

DGLAP

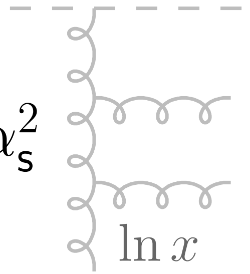
$Q^2 \gg Q_0^2$

α_s



Q_0^2

+ α_s^2



$$\left(\alpha_s \,[\ln x] + \alpha_s^2 \,[\ln x] \right) \mathsf{x} \ln \frac{Q^2}{Q_0^2}$$

+ $Q^2 \Leftrightarrow Q_0^2$

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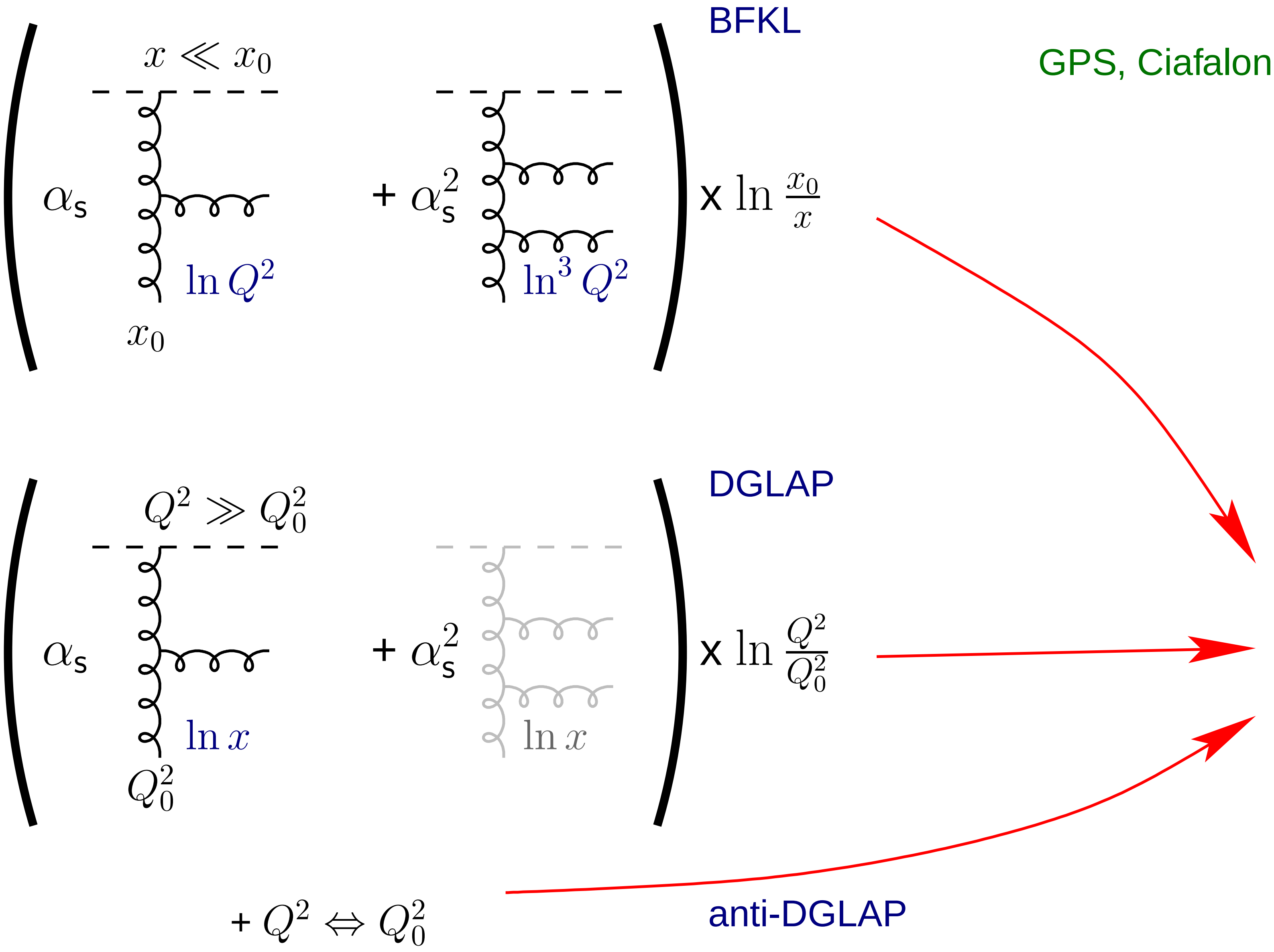


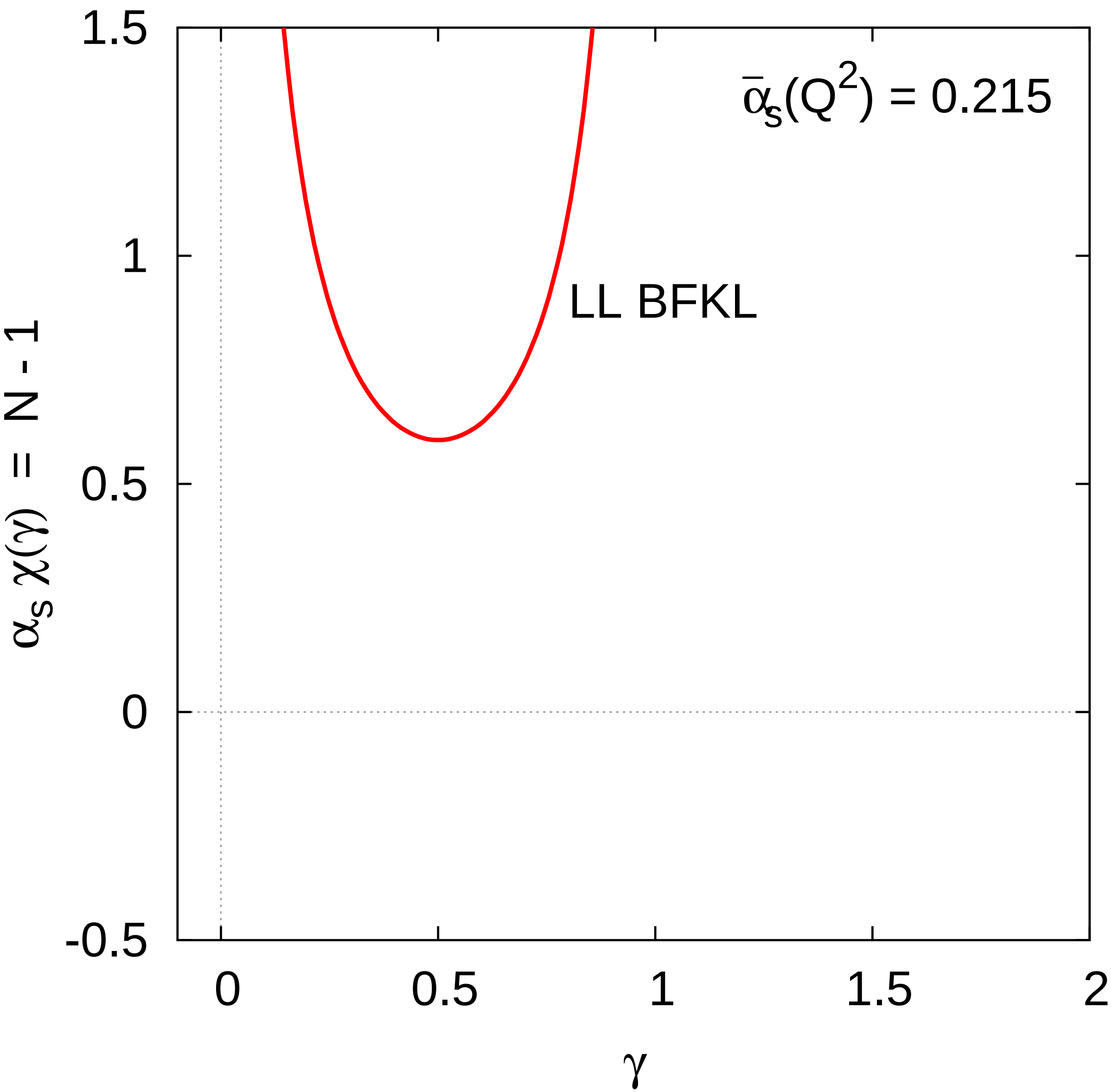
GPS, Ciafaloni, Colferai ’98–99

ibid. + Staśto ’03

combined
BFKL+DGLAP
kernel $\mathcal{K}$

(beware
double counting)

Building up the kernel. . .



Build up *characteristic function*, i.e. the Mellin transform of kernel (fixed coupling)

$$\bar{\alpha}_s \chi(\gamma) =$$
$$= \int \frac{dk^2}{k^2} \left(\frac{k^2}{k_0^2}\right)^{\gamma} \mathcal{K}(k, k_0)$$

Height of minimum is ‘BFKL power’

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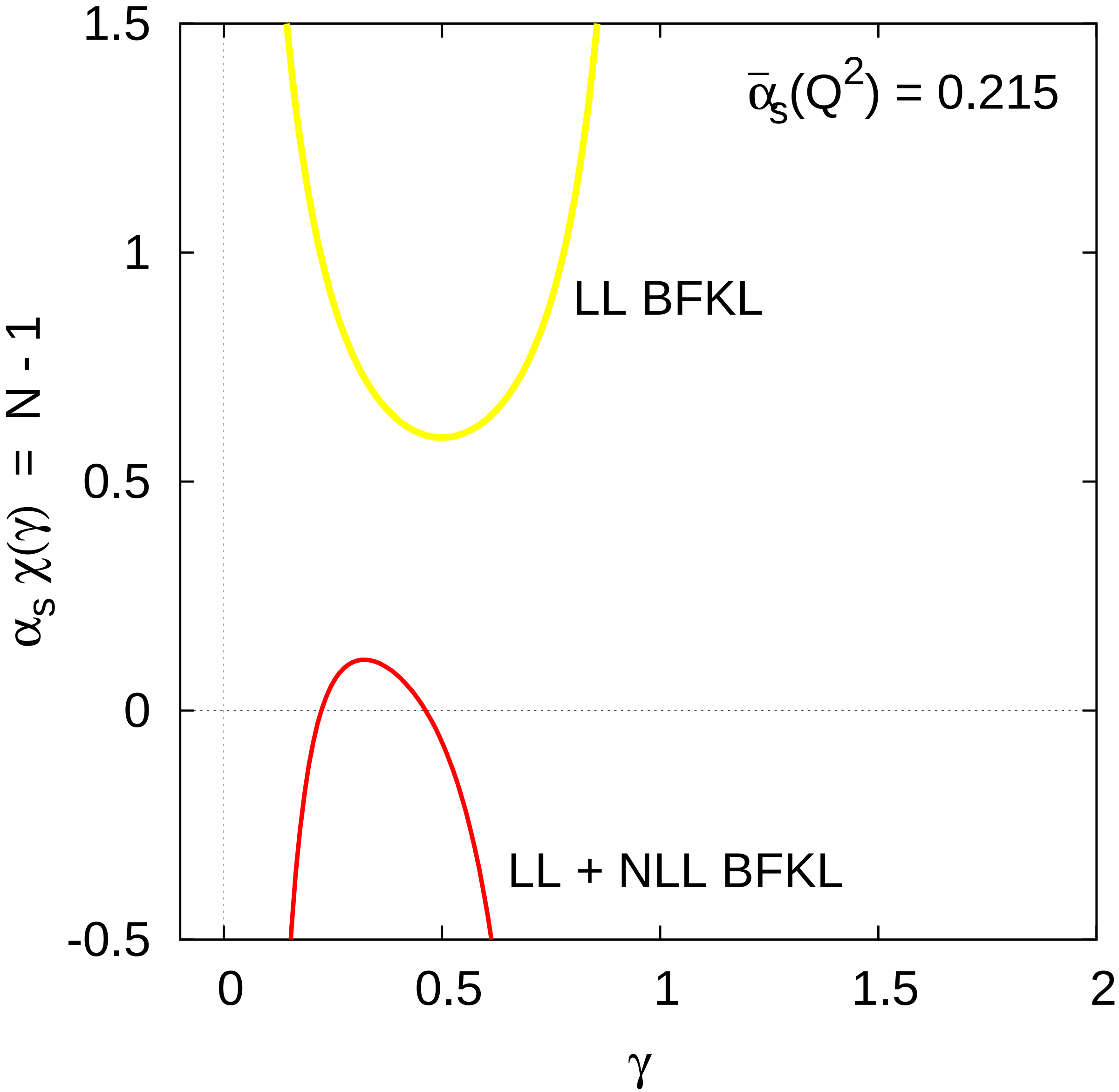

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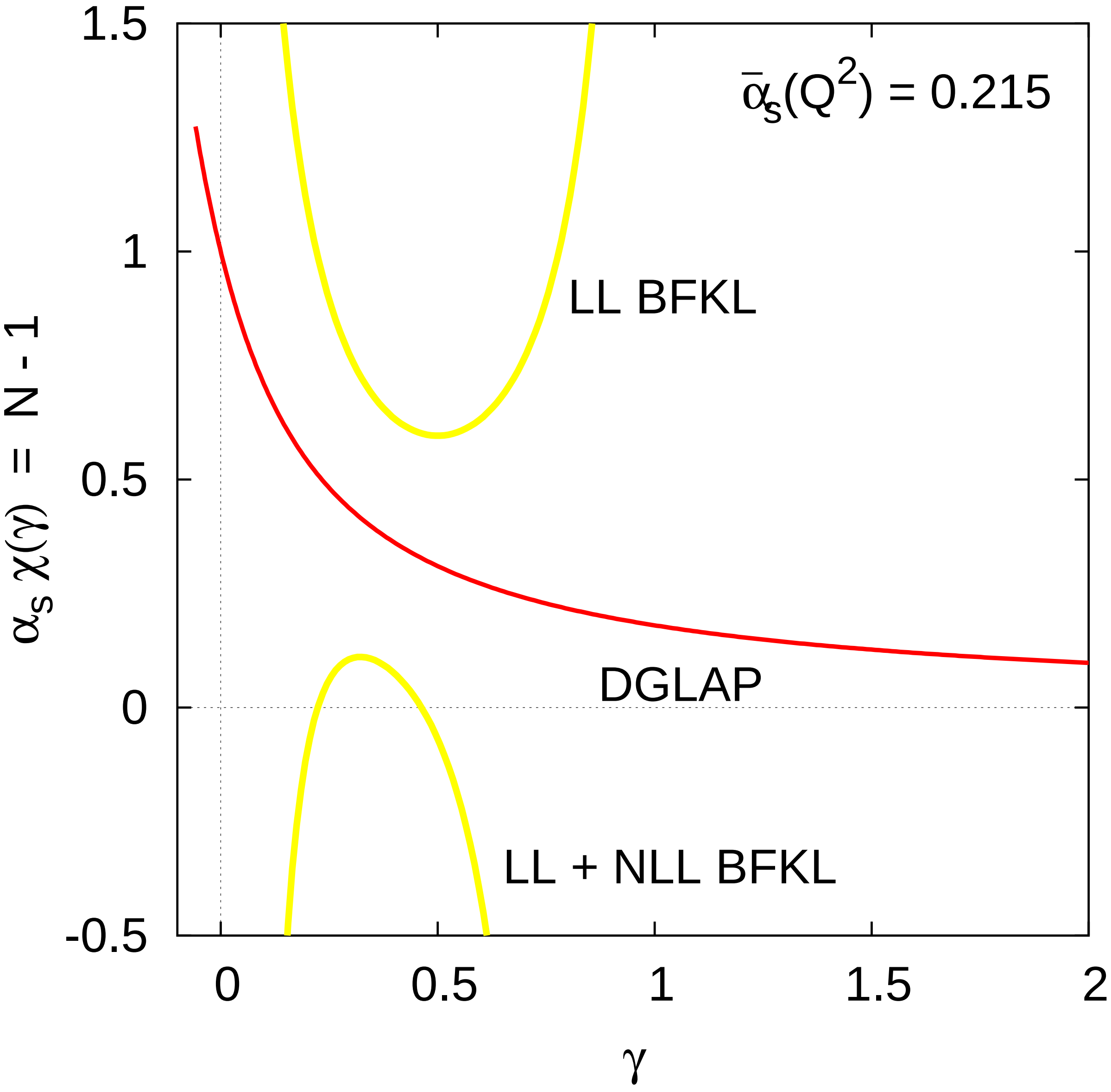

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NB: DGLAP = 'rotated' plot of

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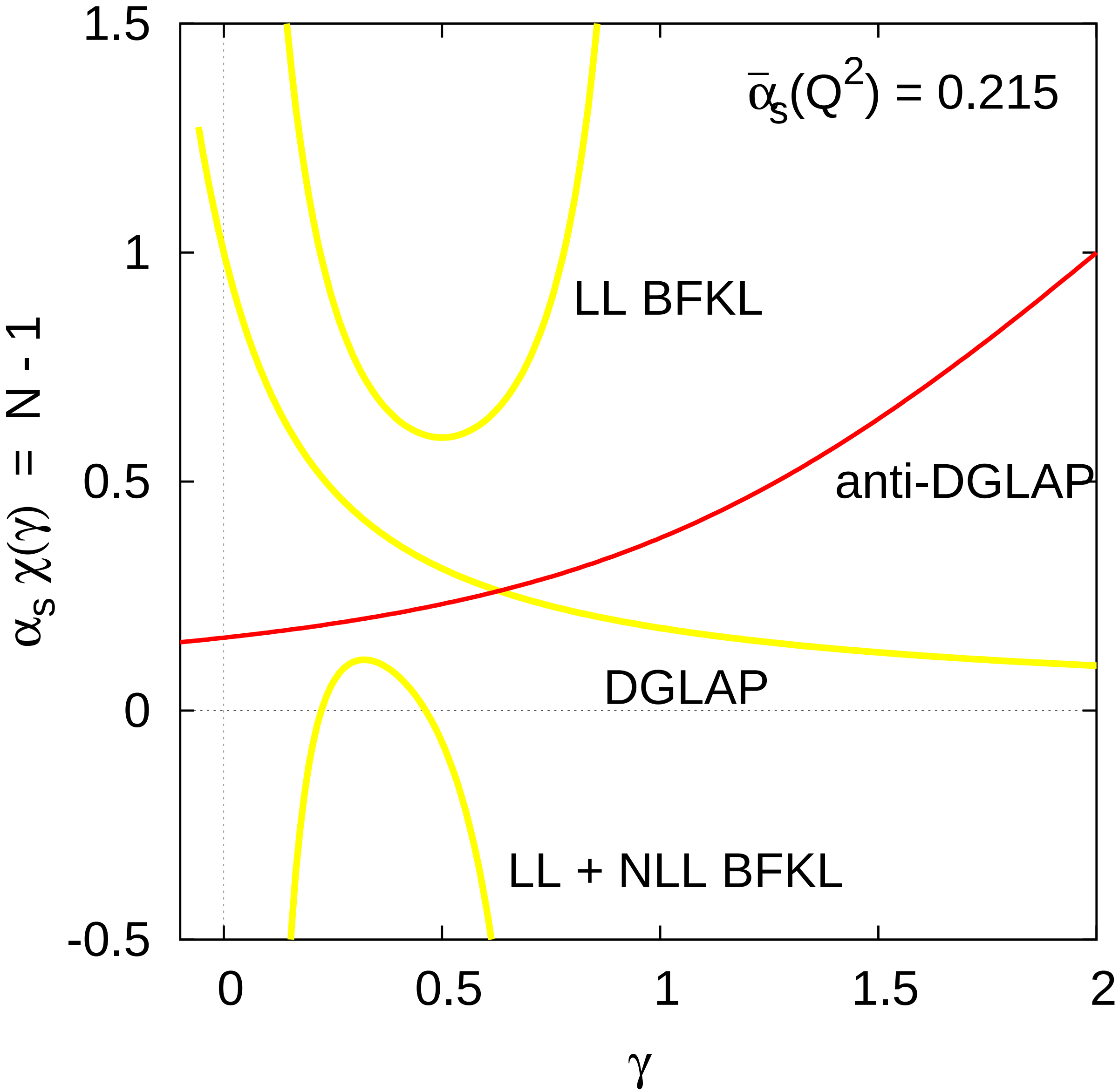

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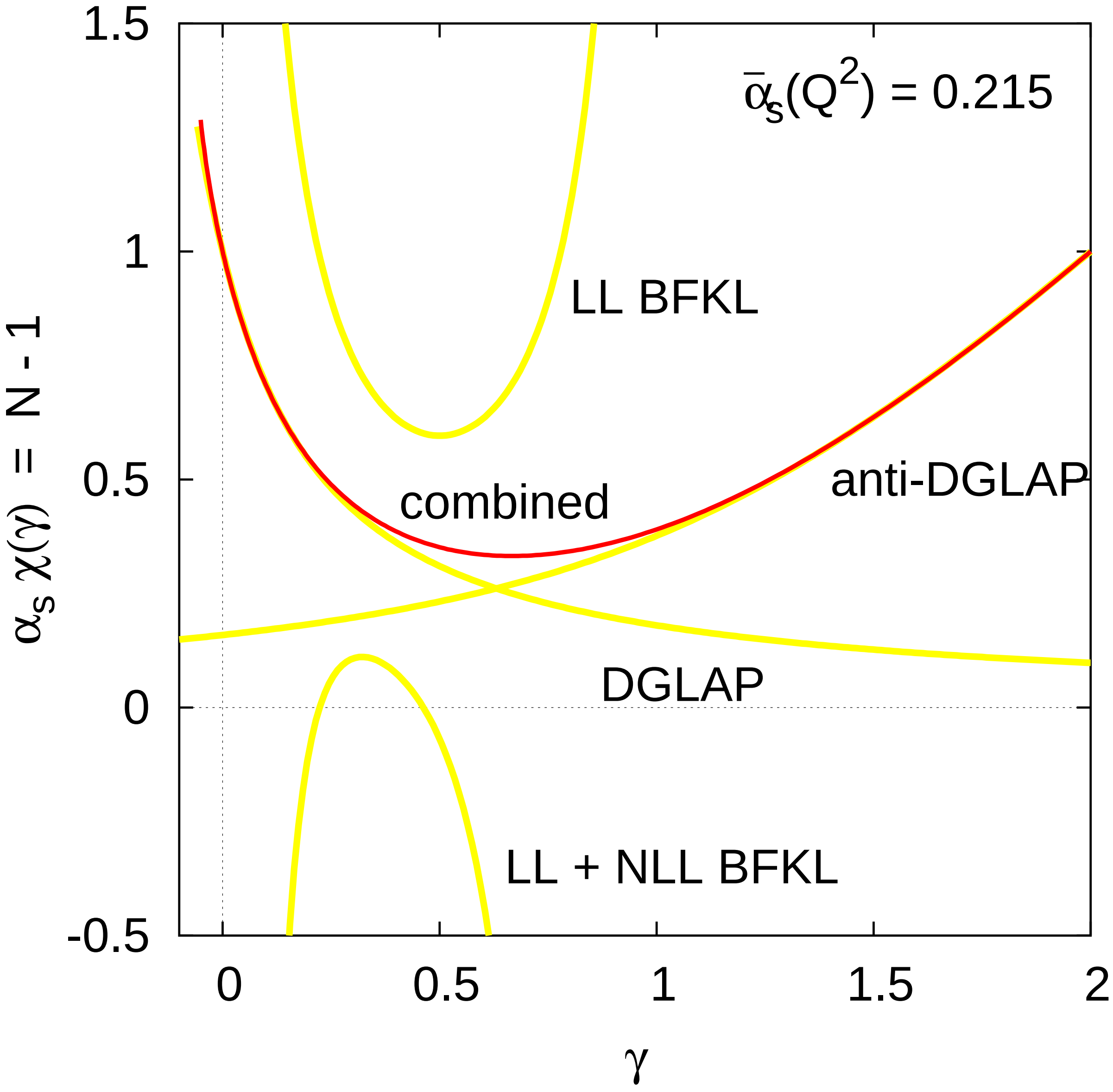

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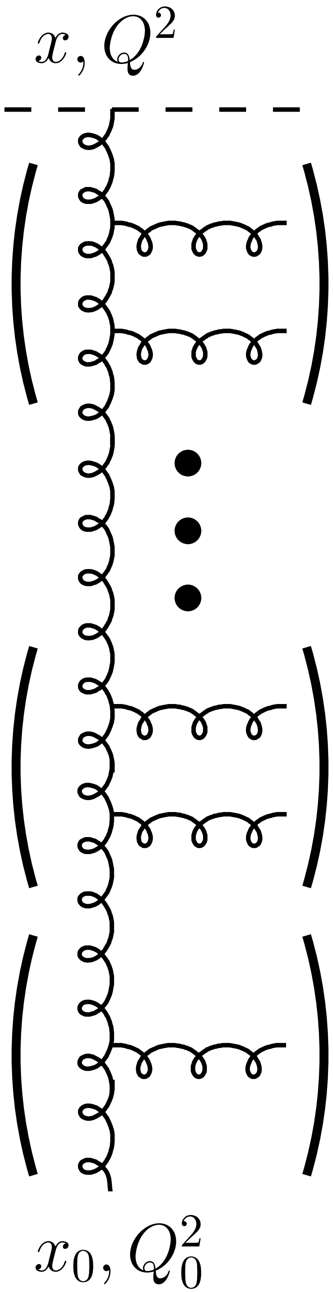
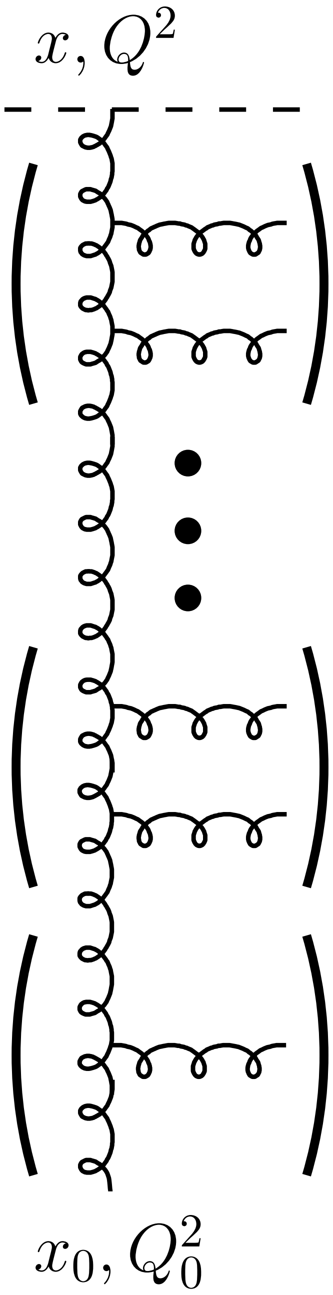
Iteration of kernel $\Rightarrow$ Green function



Green function: $G\left(\ln\frac{x}{x_0}; Q_0, Q\right)$

Iteration of kernel ⇒ Green function



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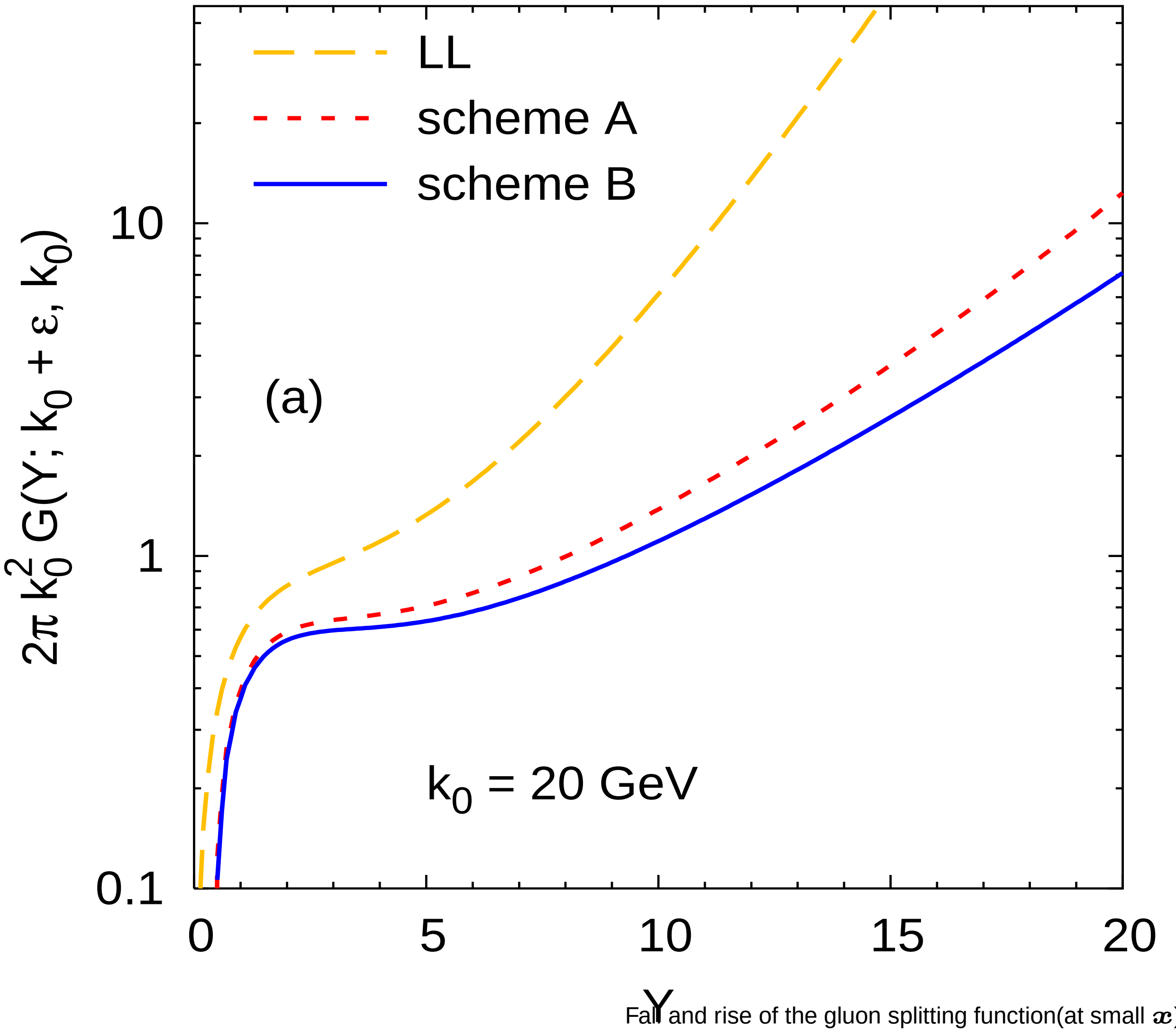

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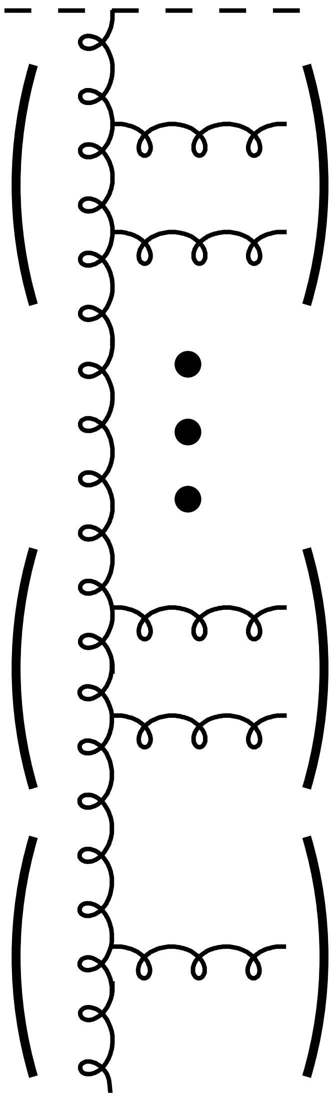

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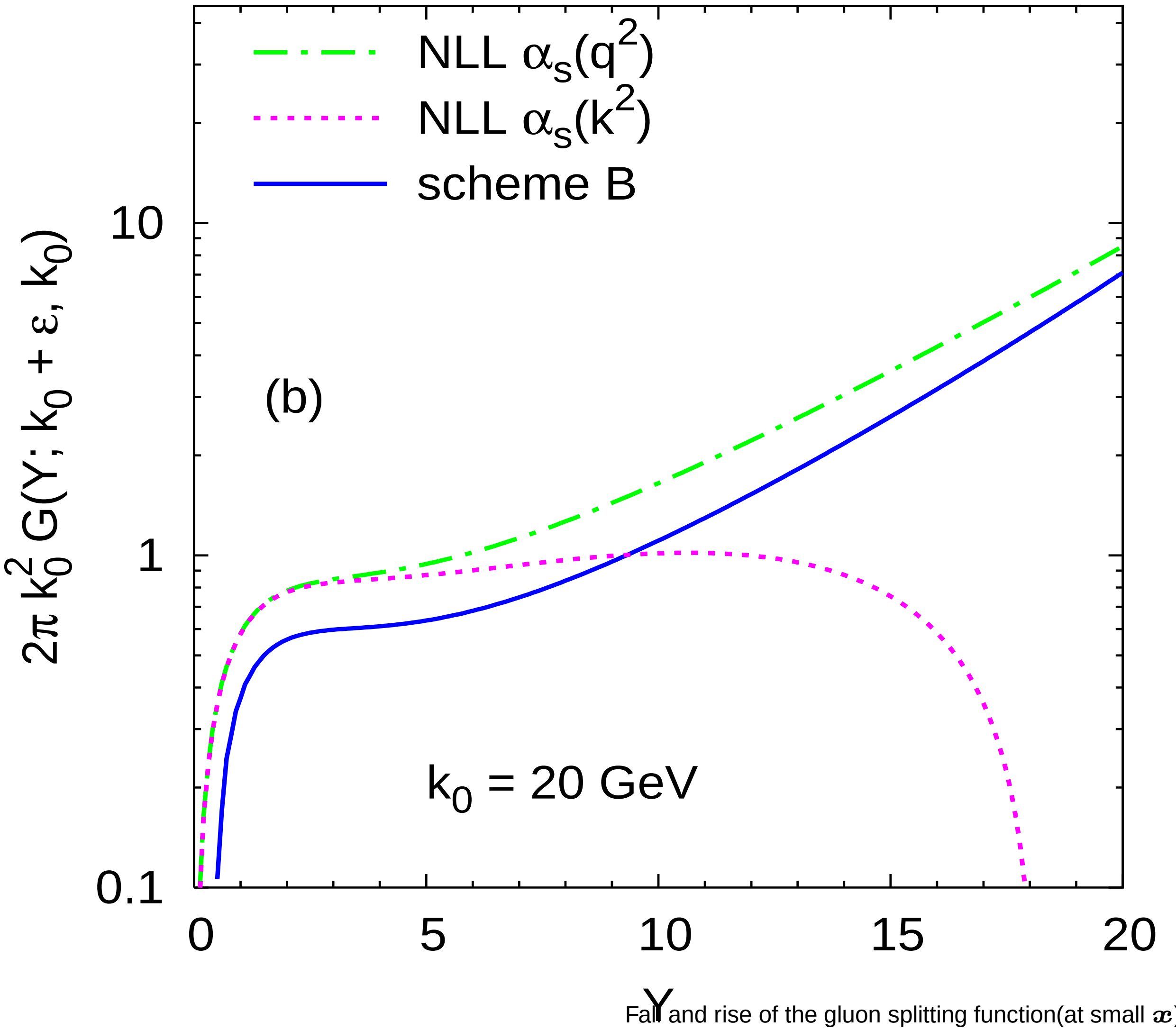

Green function $\Rightarrow$ effective DGLAP splitting function

Construct a gluon density from Green function (take $k \gg k_0$):

$$xg(x, Q^2) \equiv \int^Q d^2k \; G^{(\nu_0 = k^2)}(\ln 1/x, k, k_0)$$

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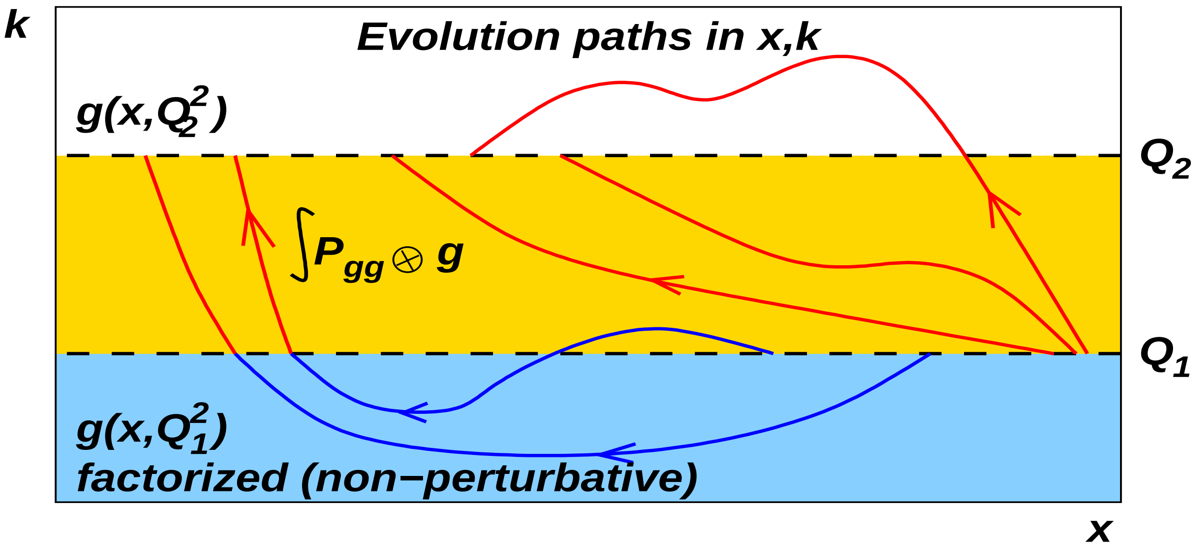
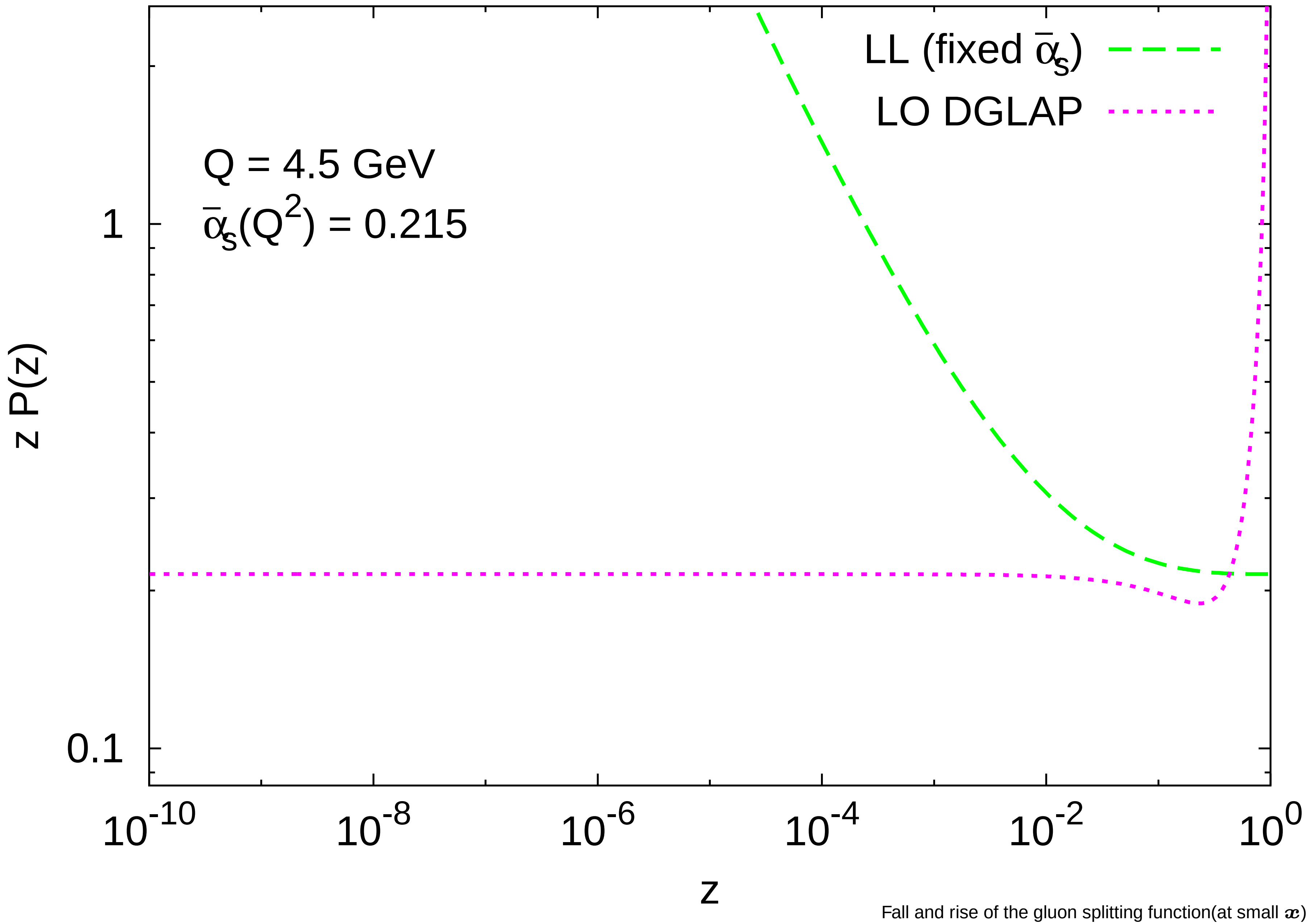
Factorisation

- Splitting function:
  red paths
- Green function:
  all paths



$P_{gg}(z)$ splitting function results



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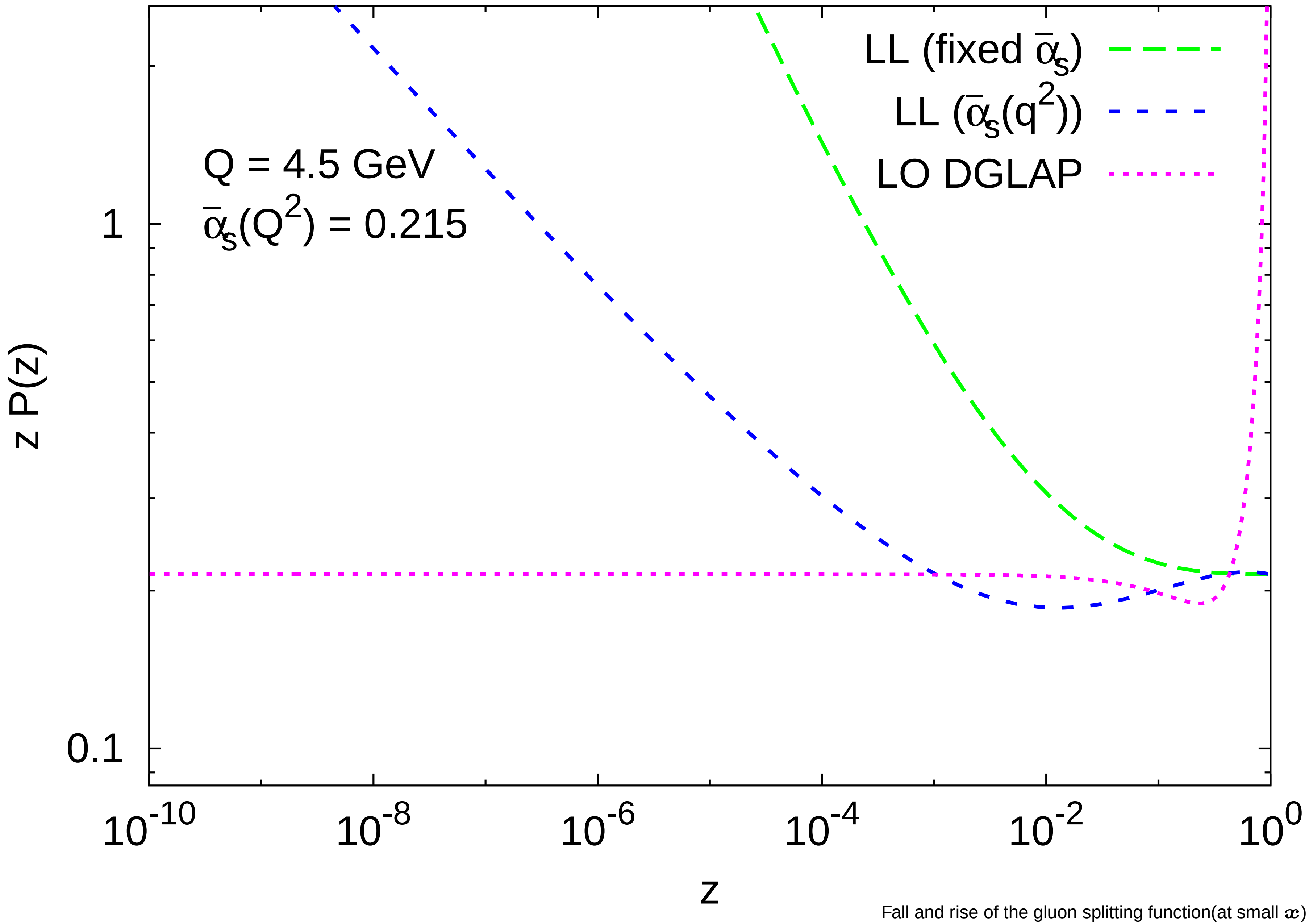

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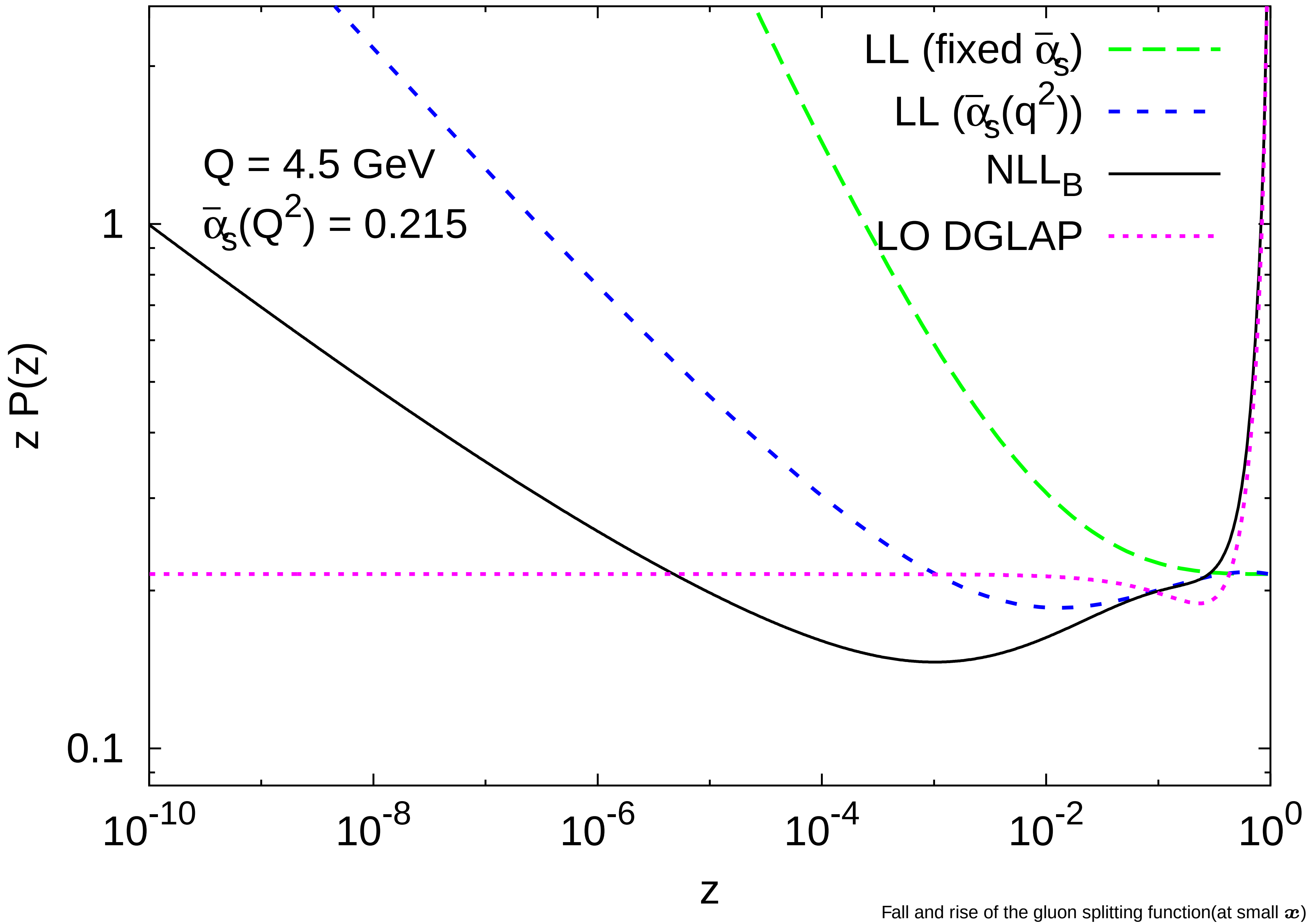

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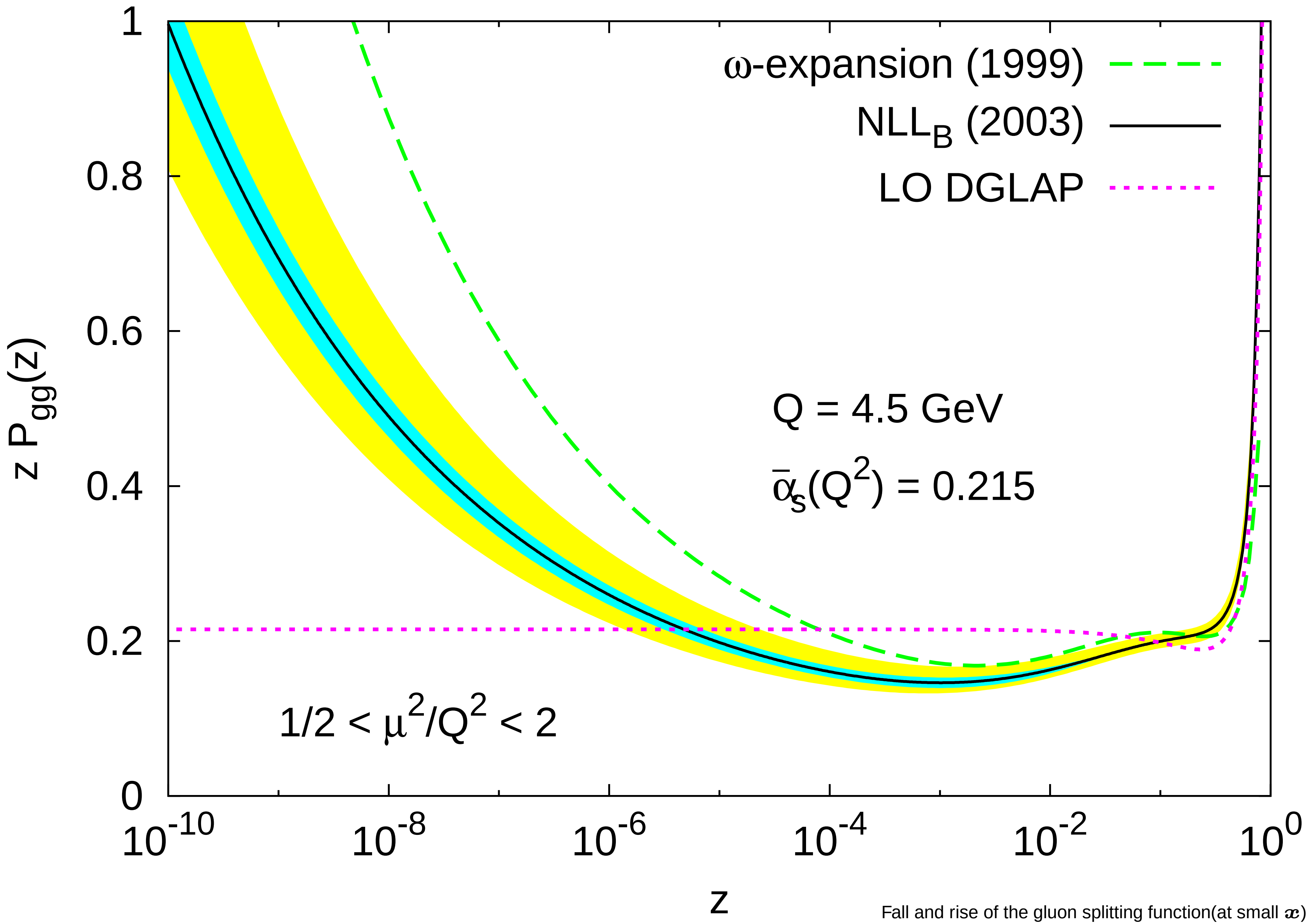

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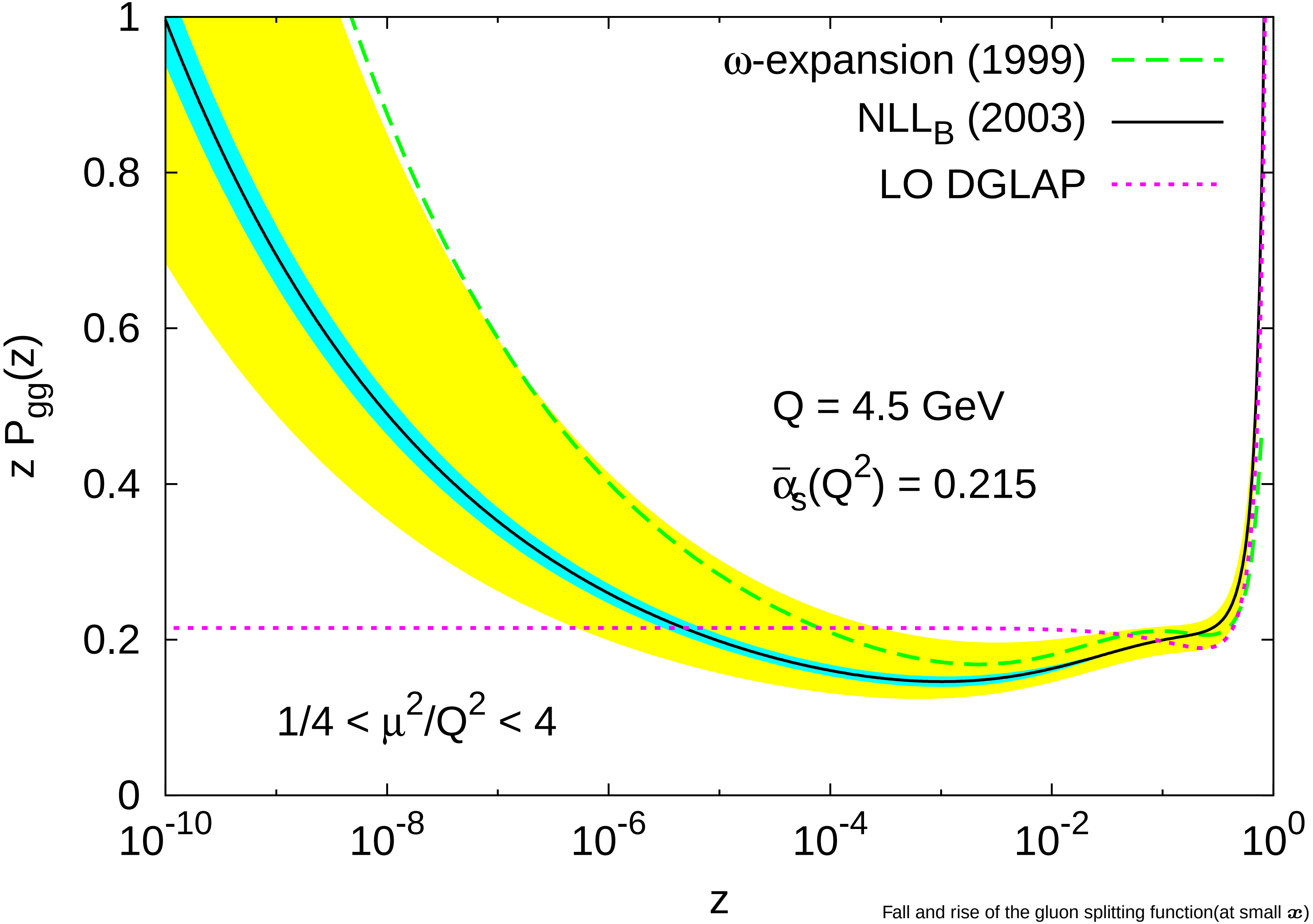
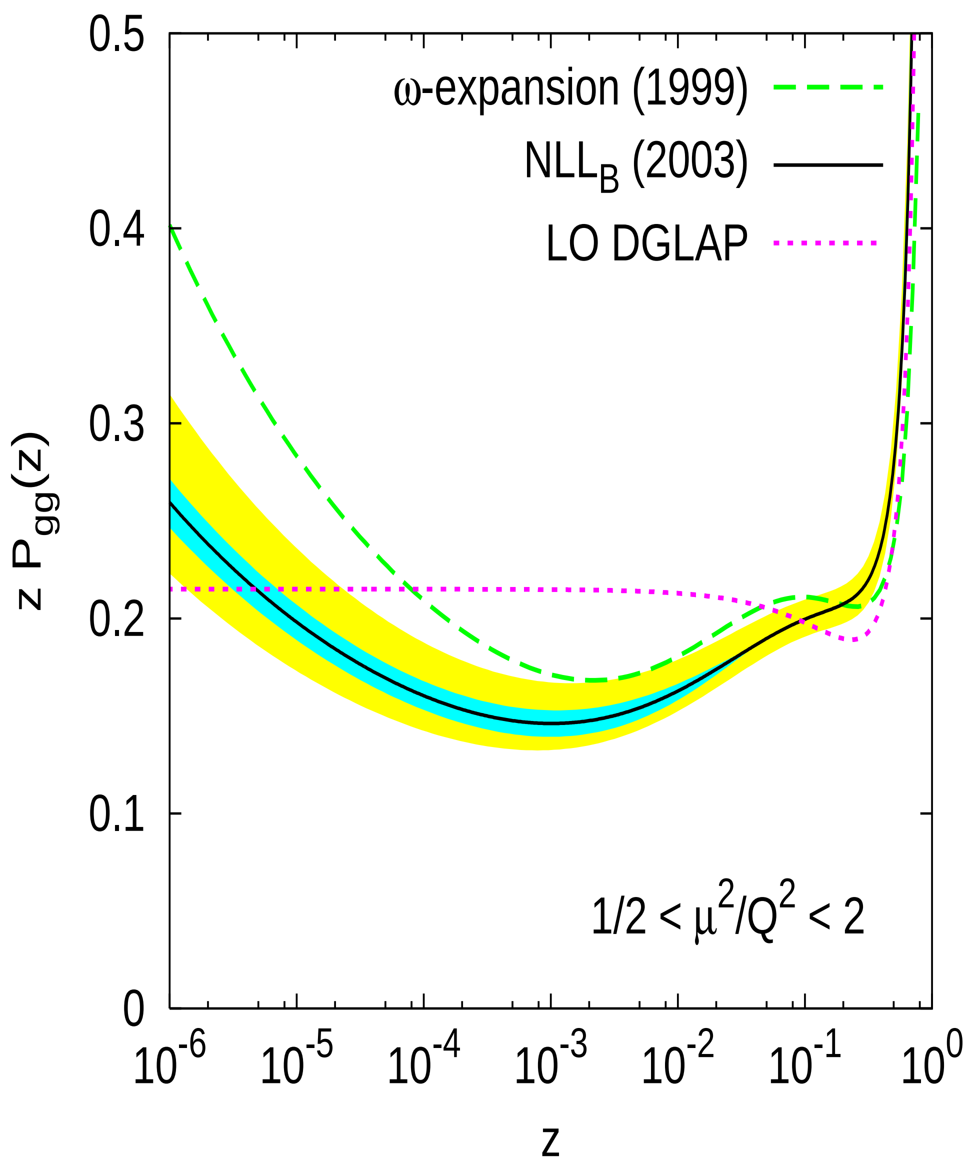


Dominant phenomenological structure is dip

- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at* $x \sim 10^{-3}$



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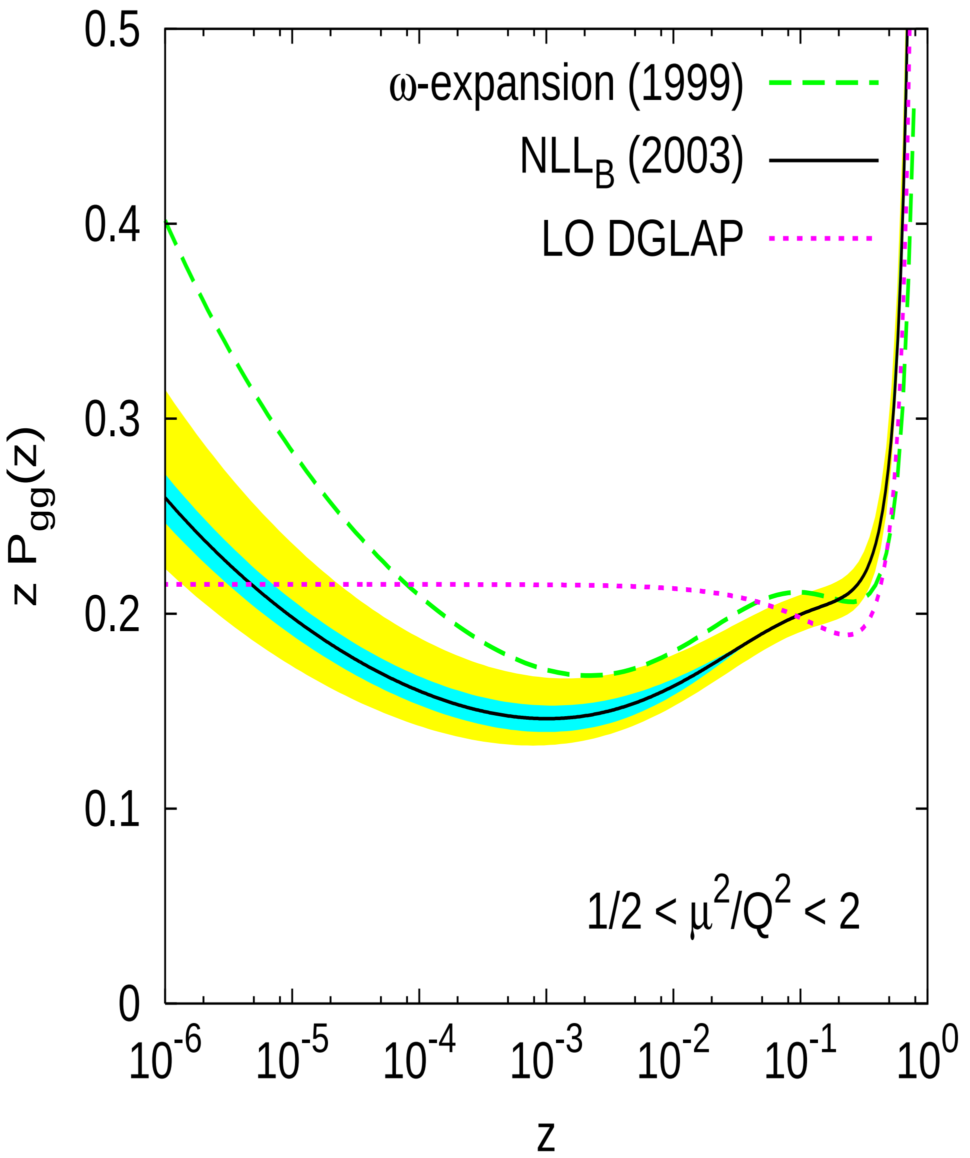

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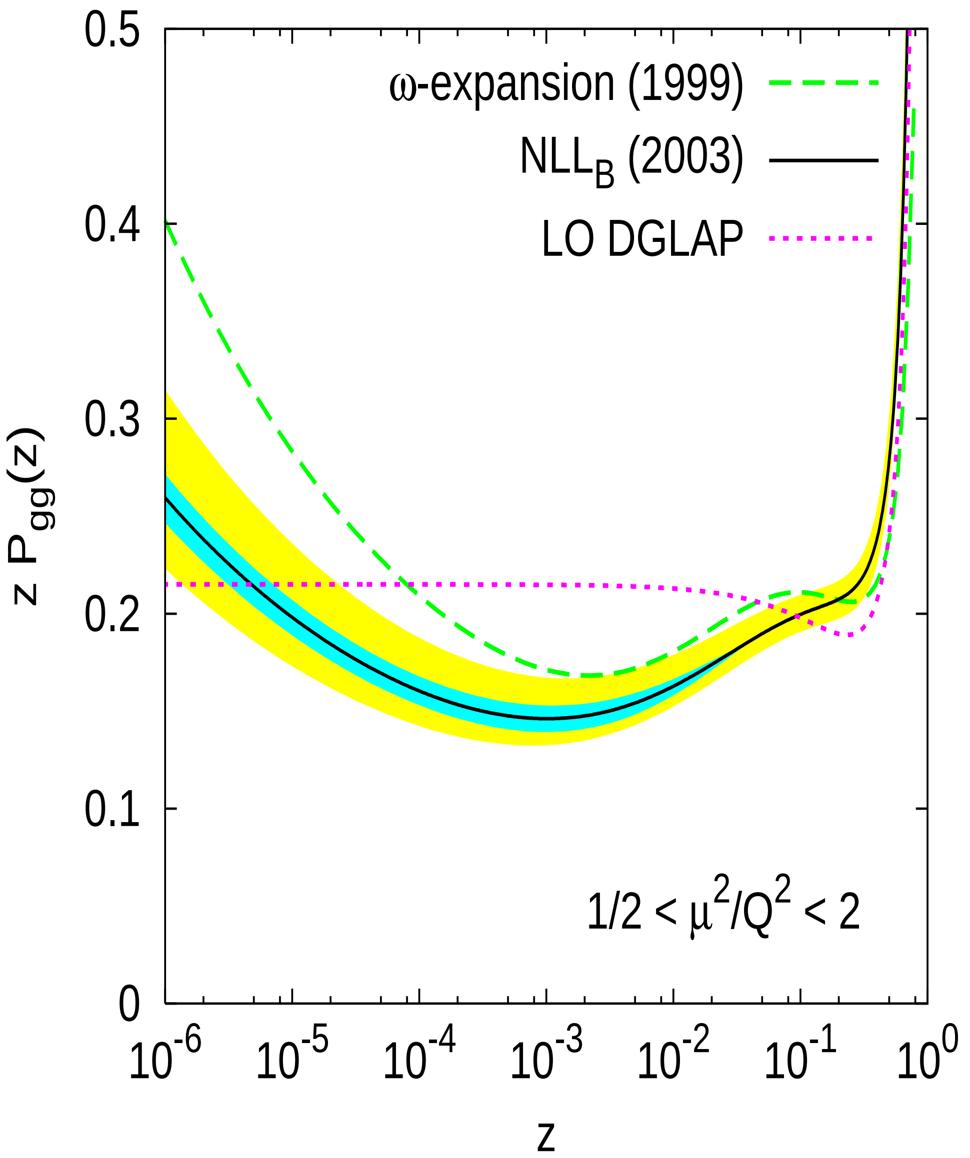

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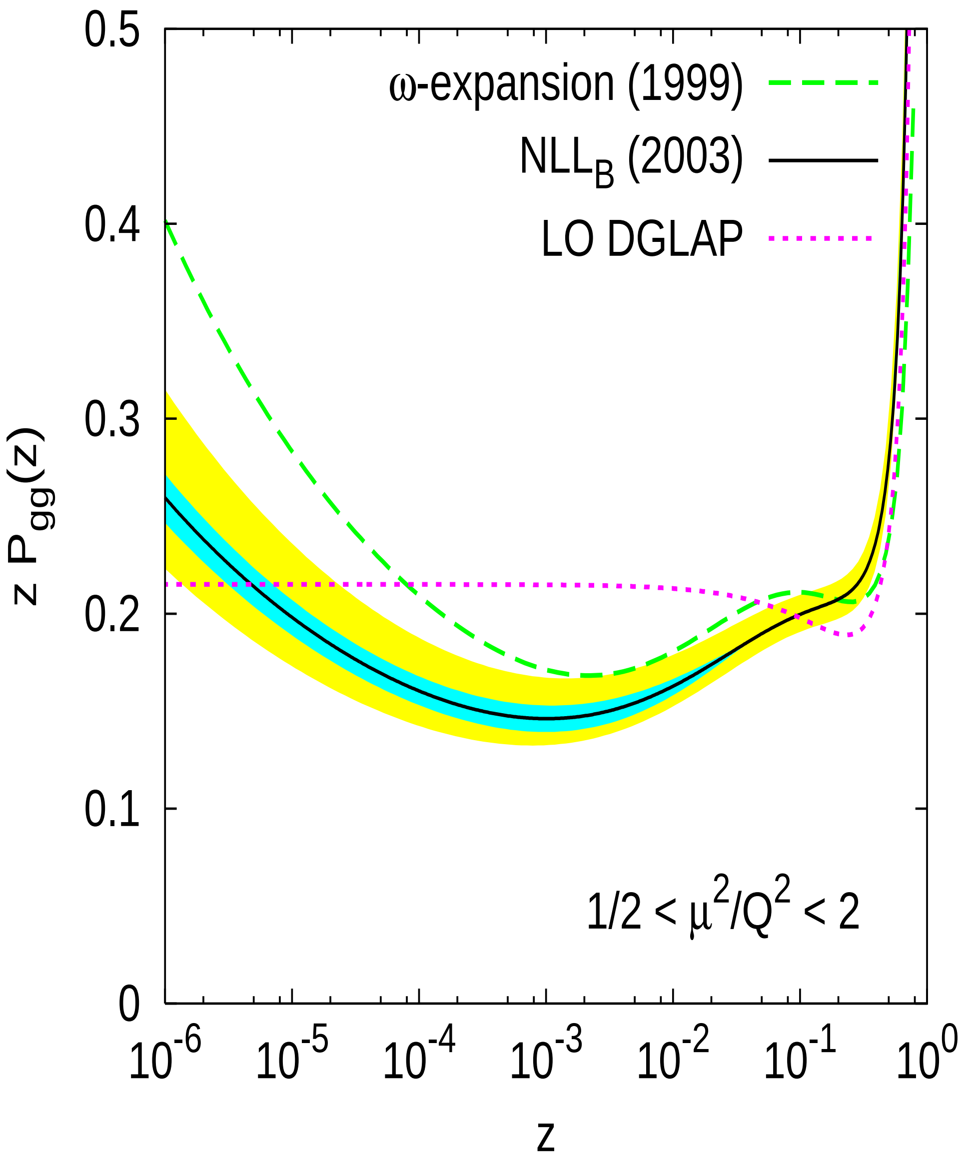

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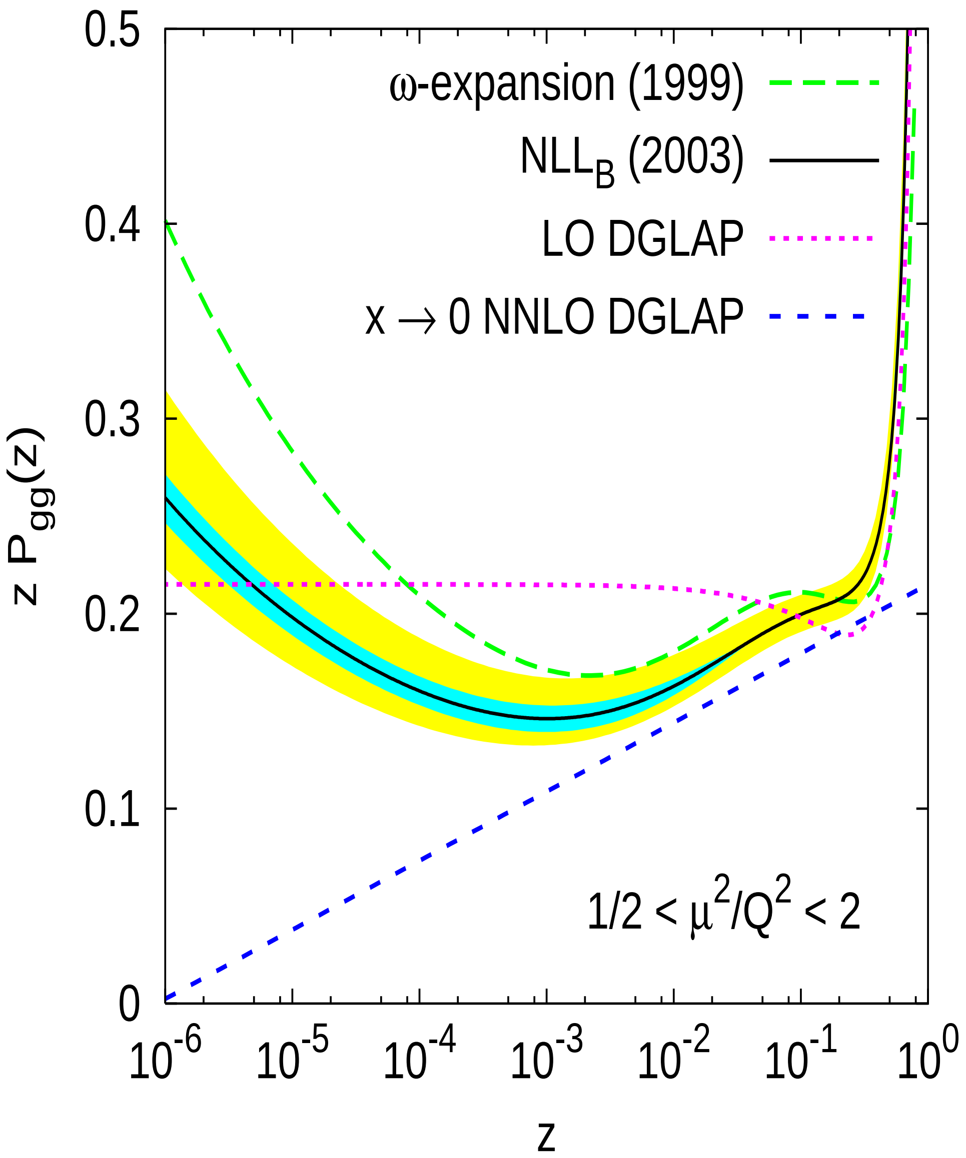

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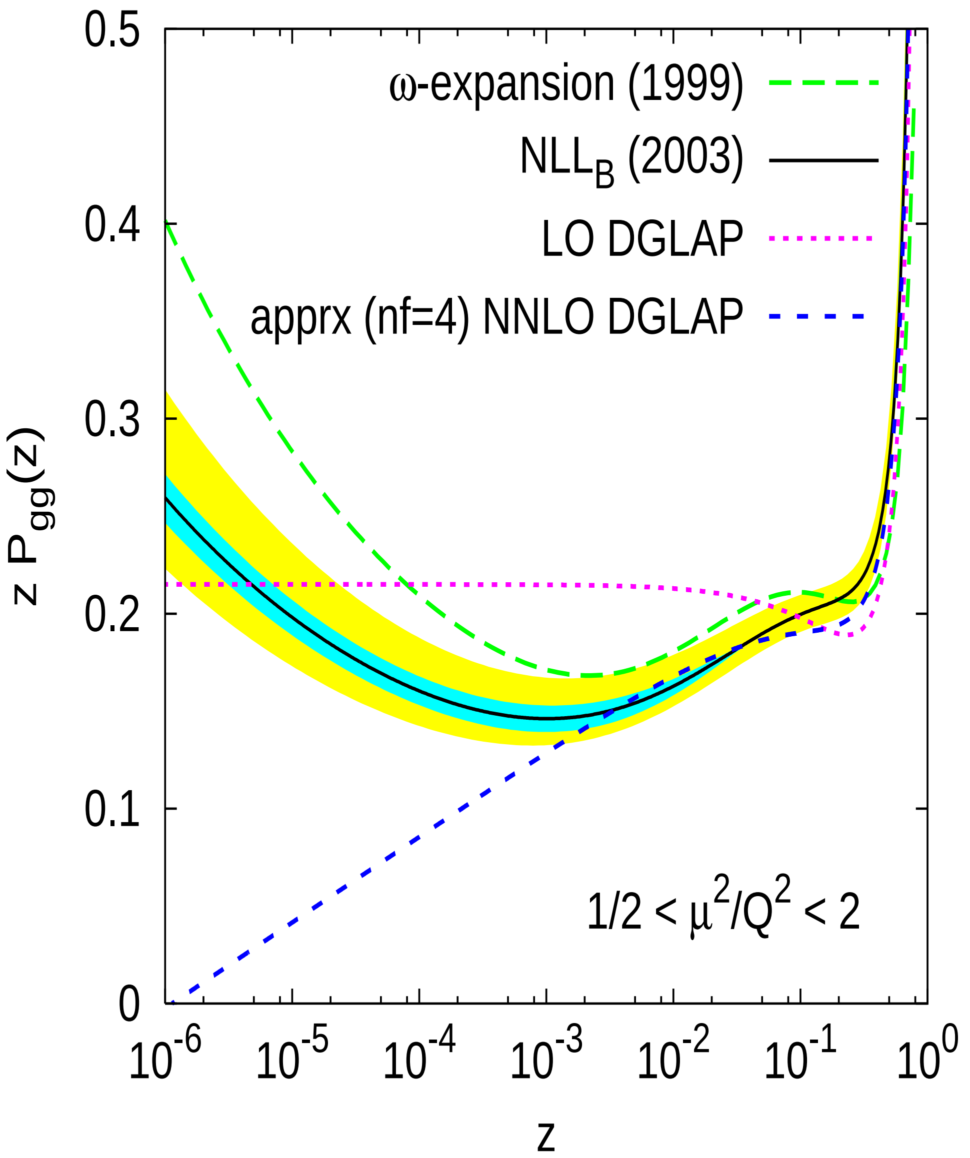

Reorganise perturbative series

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	const.
α_s^5	0	x	x	ln 1/x
$\vdots$			$\ln^3 1/x$	$\ln^2 1/x$

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$\ln 1/x$

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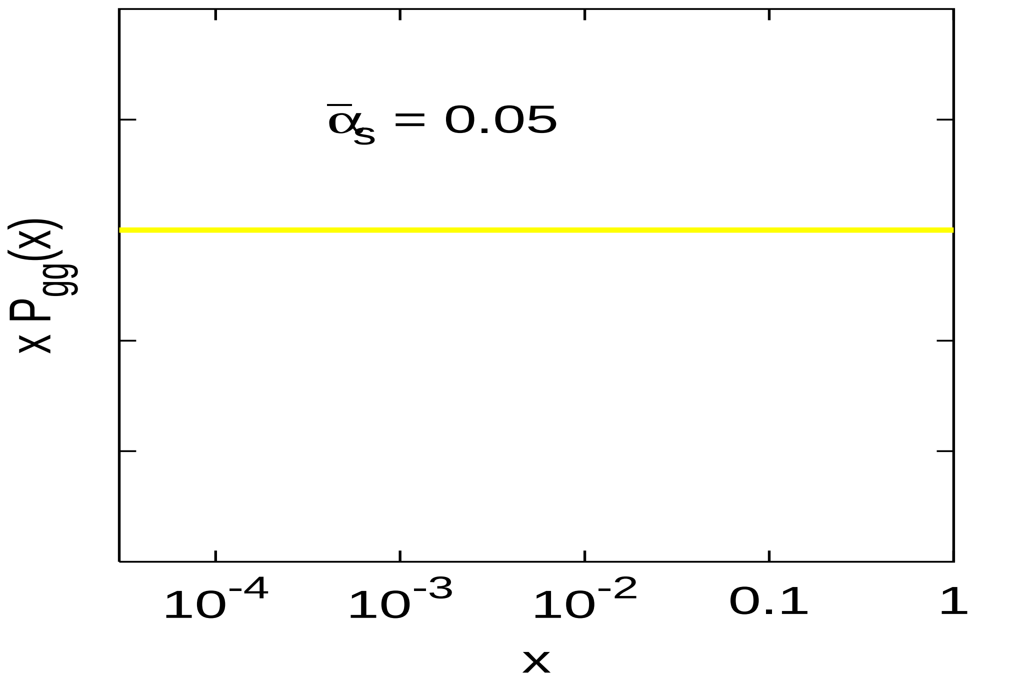
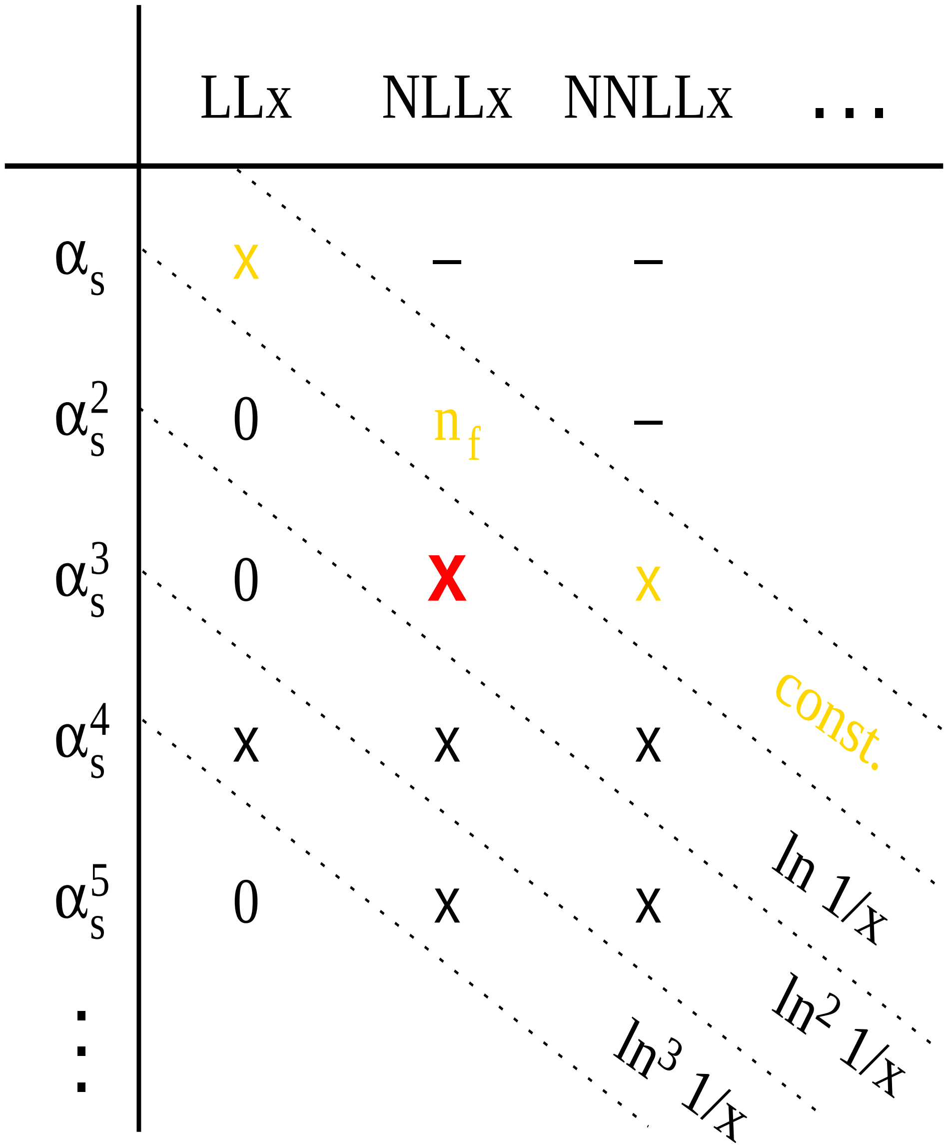
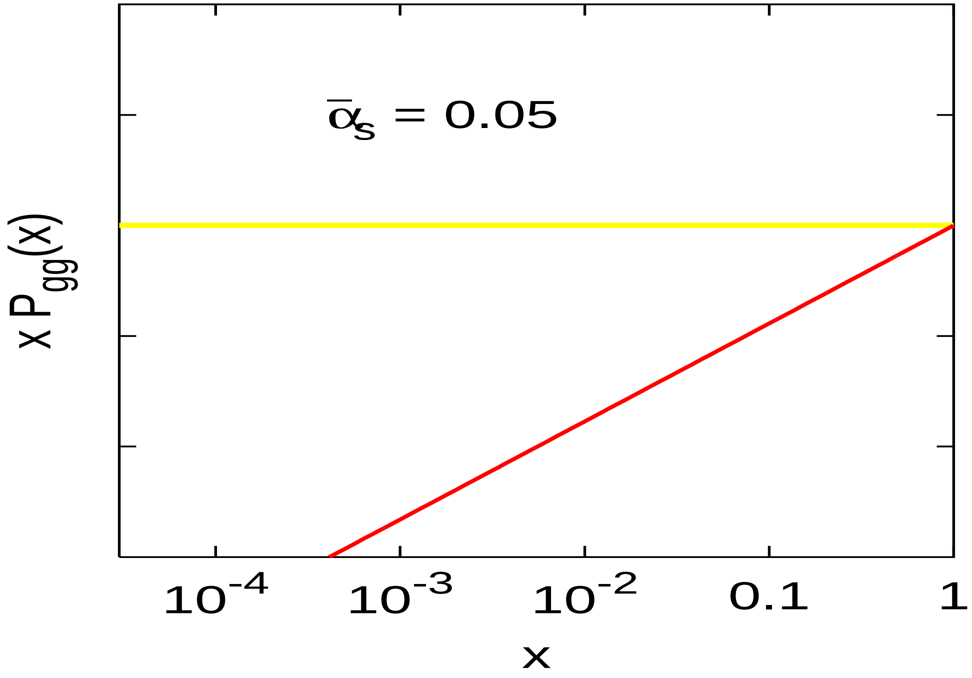




At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x}$$



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⋮				

const.

ln 1/x

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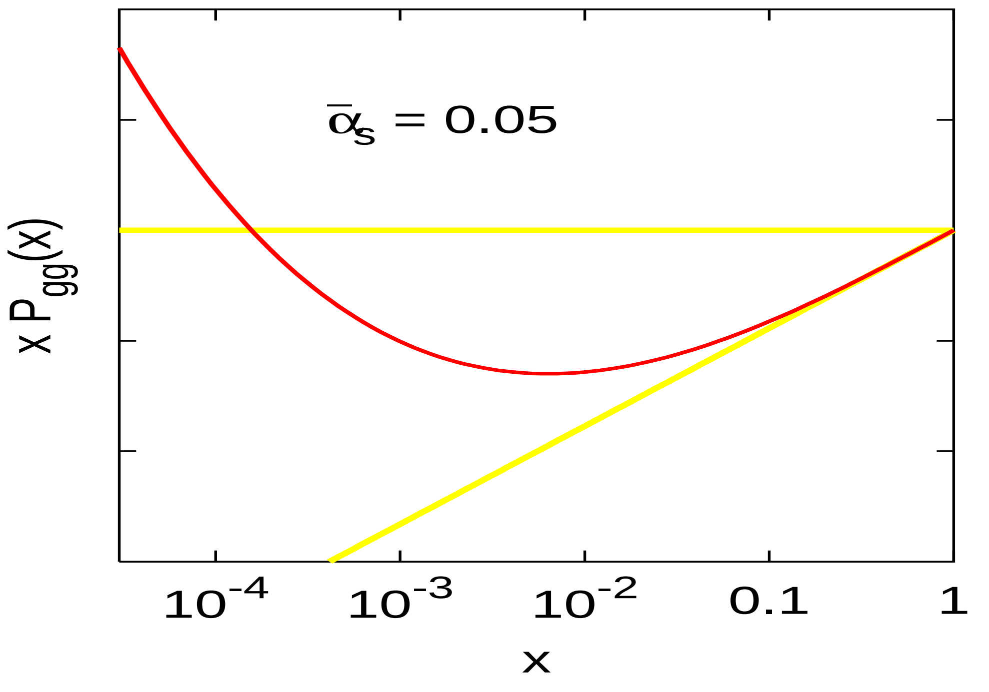
$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x} + \mathbf{0.401}\,\boldsymbol{\bar{\alpha}_s^4 \ln^3 \frac{1}{x}}$$



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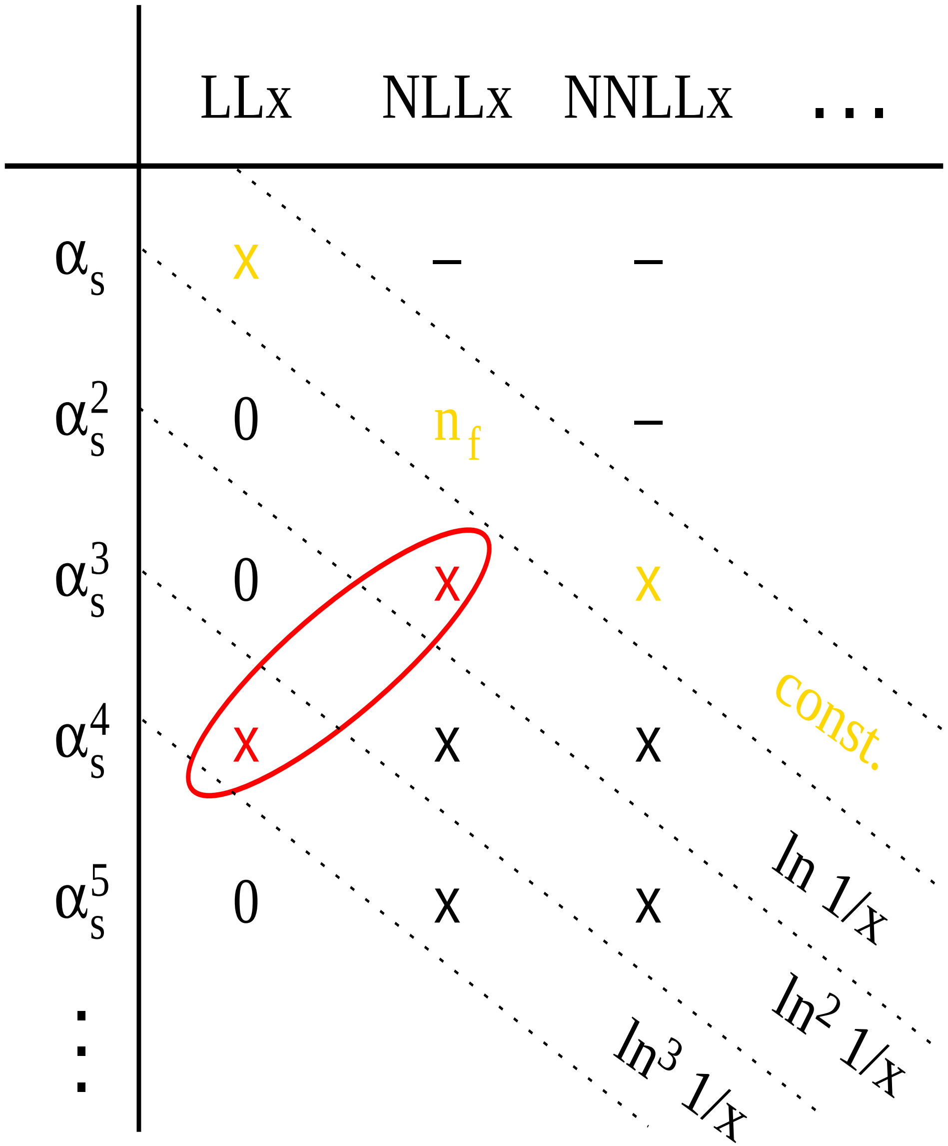

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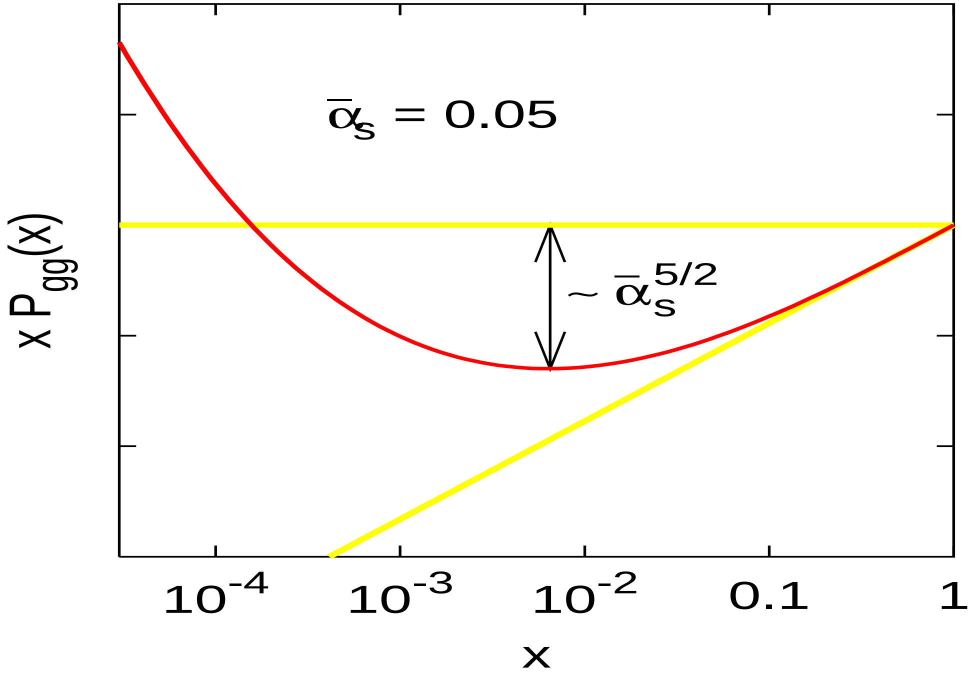
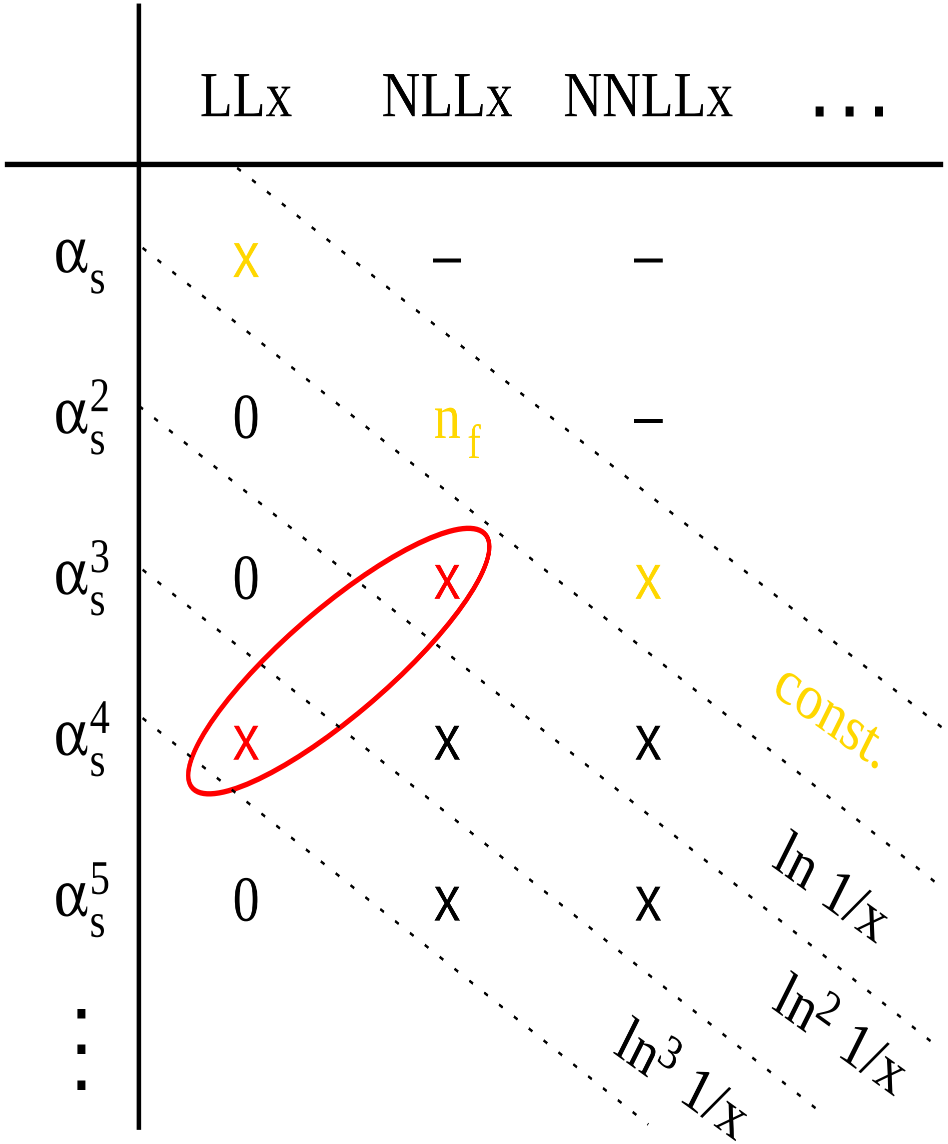
Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$



Systematic expansion in $\sqrt{\alpha_s}$



Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}}$$

Depth of dip

$$-d \simeq -1.237\bar{\alpha}_s^{5/2}$$

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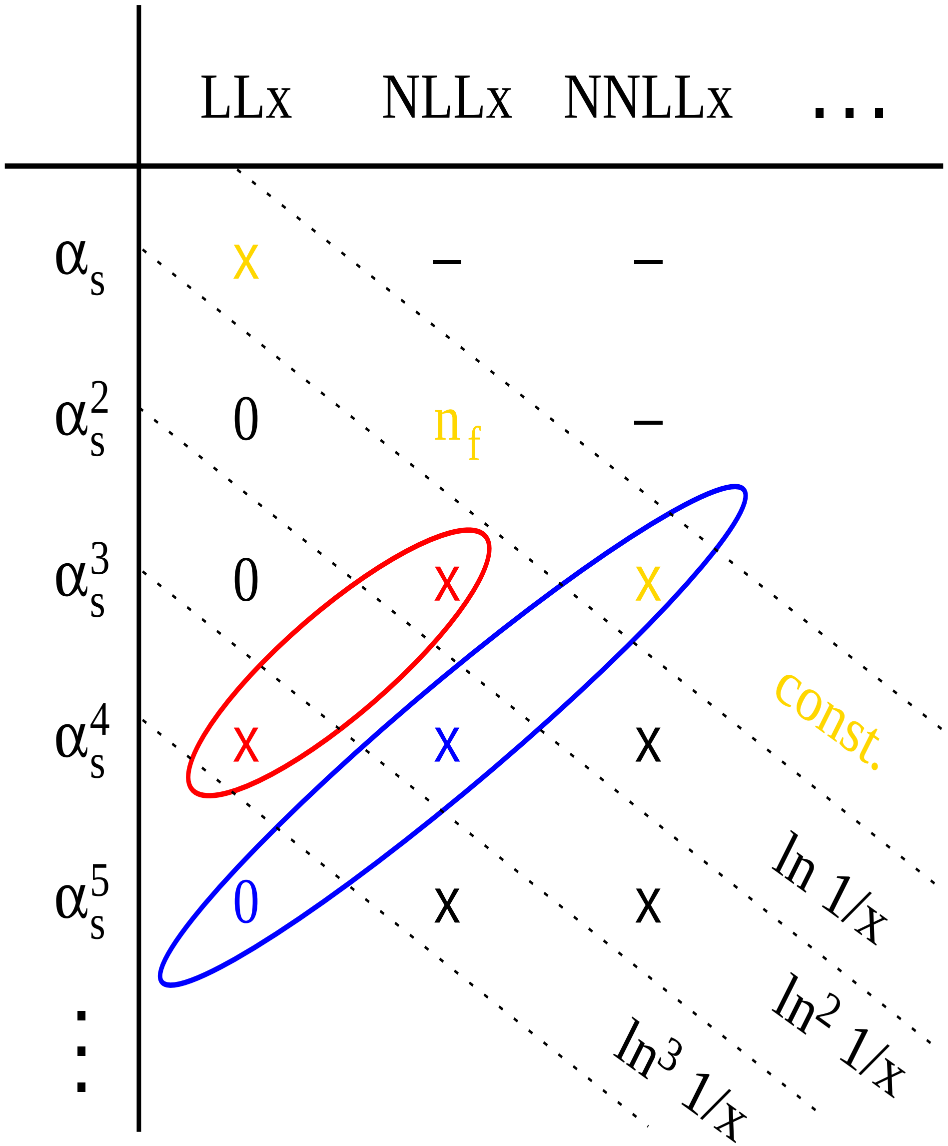

Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947$$

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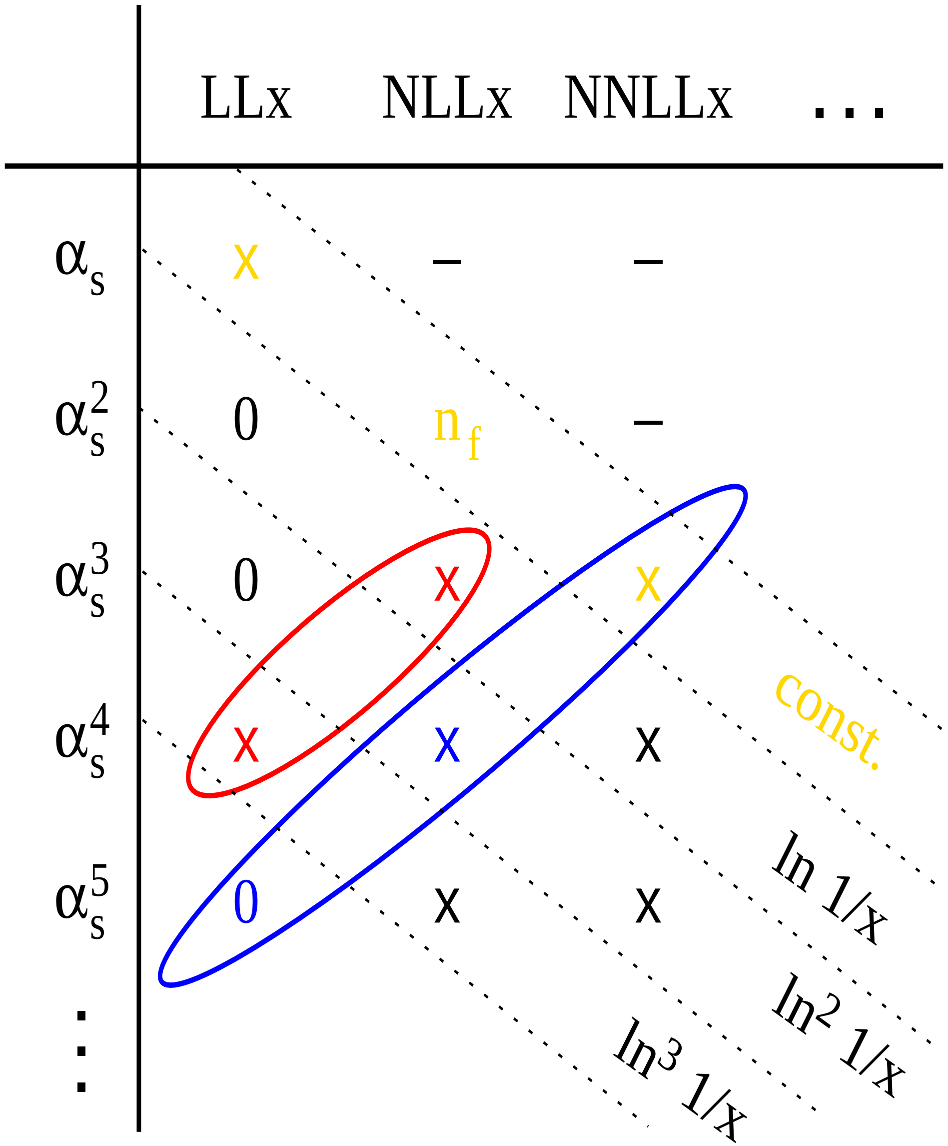

Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}} + 6.947 + \cdots$$

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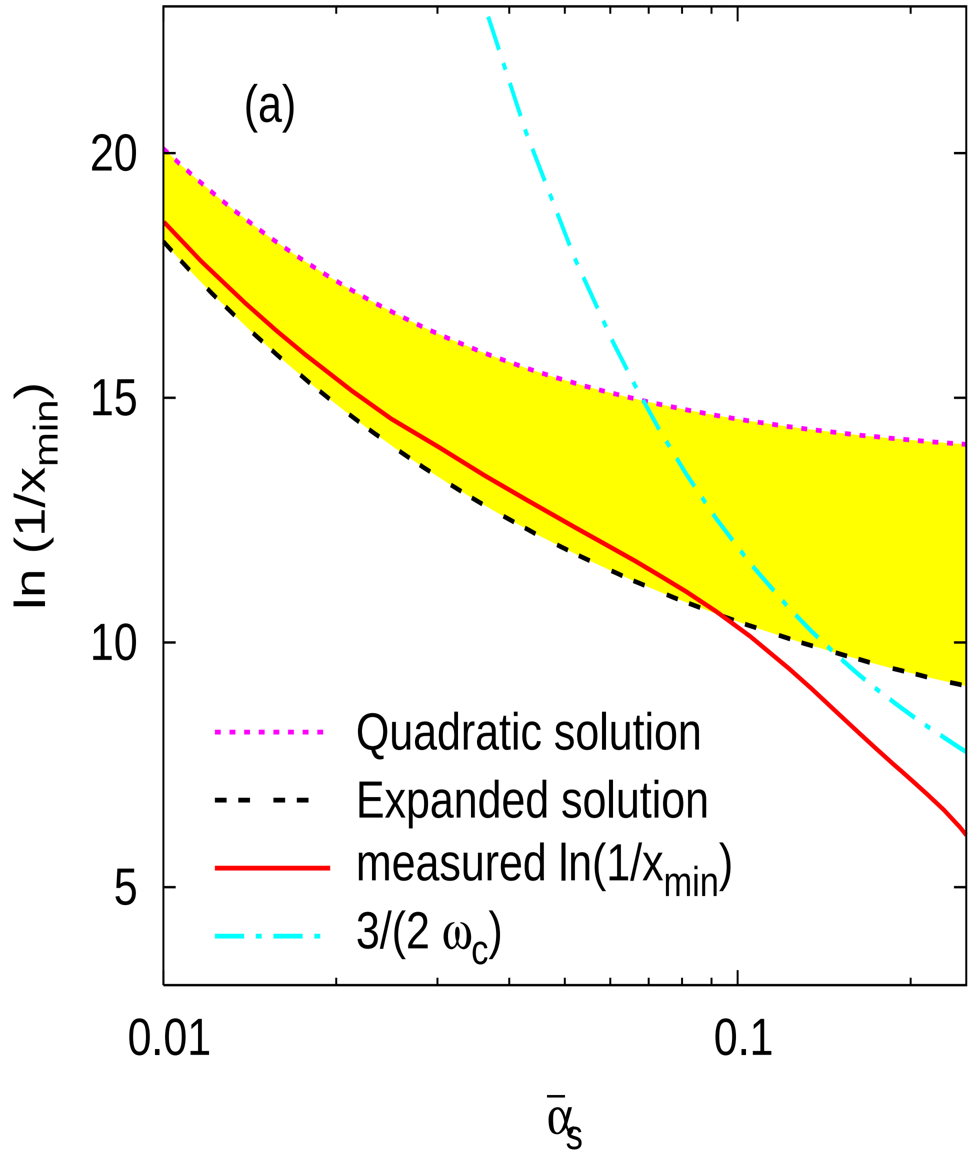
NB:

- convergence is very poor

  As ever at small x!

- higher-order terms in expansion need NNLLx info

Test dip properties v. BFKL+DGLAP resummation



Test position of dip v. α_s

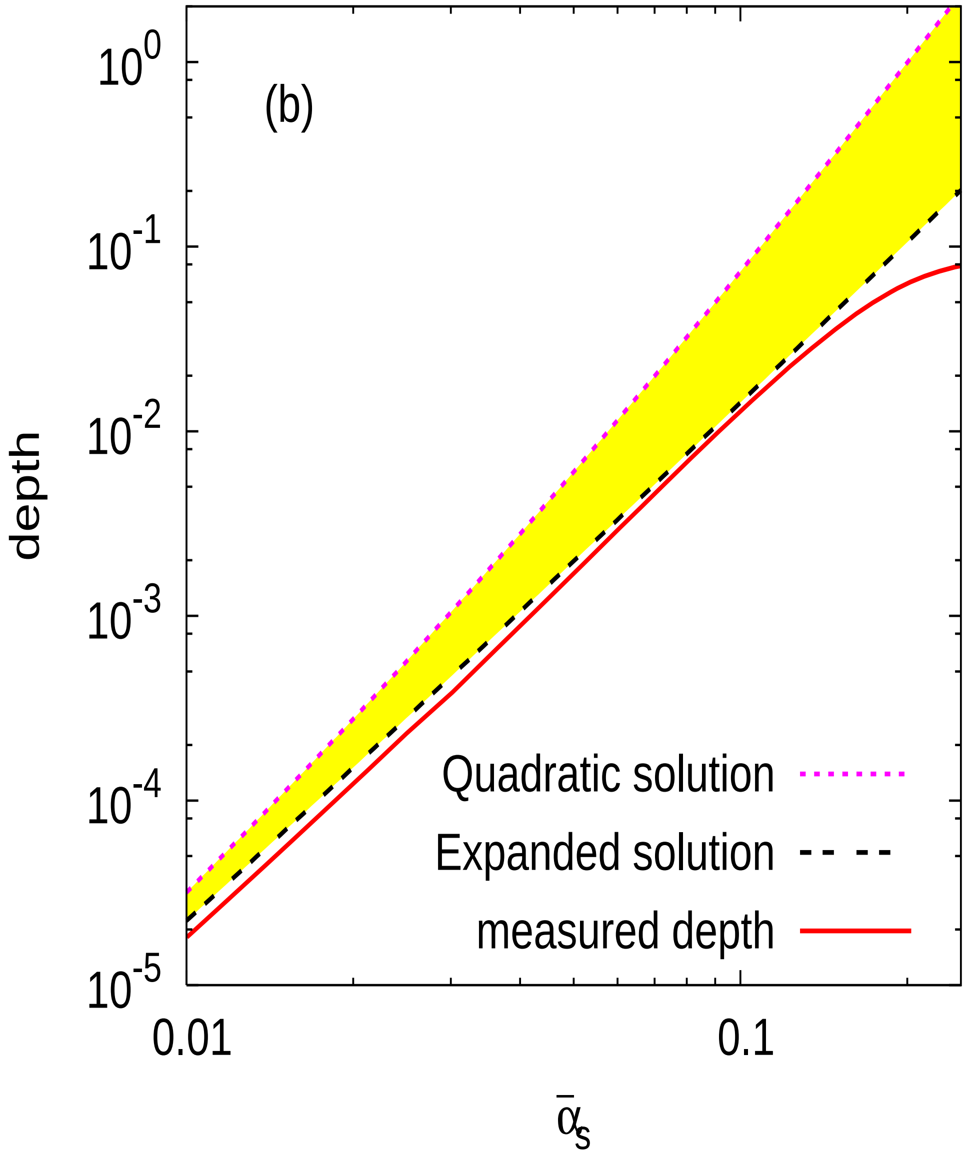
- Band is uncertainty due to higher orders in $\sqrt{\alpha_s}$
- At small α_s, good agreement $\longrightarrow$ confirmation of 'dip mechanism'
- At moderate α_s, normal small-x resummation effects 'collide' with dip

$$\ln \frac{1}{x_{\min}} \lesssim \frac{3}{2\omega_c}$$

Dip then comes from interplay between $\alpha_s^3 \ln x$ (NNLO) term and full resummation.

[Actually, story more complex]

Test dip properties v. BFKL+DGLAP resummation



Test depth of dip v. α_s

- similar conclusions!

Conclusions

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  - dip signals start of significant resummation effects —limit of applicability of NNLO DGLAP
- Further work needed on various phenomenological fronts. . .
  - Inclusion of quarks $\longrightarrow$ matrix of splitting functions
  - Coefficient functions (depending on scheme)
  - Comparison to data