Saturation and Traveling Waves

Robi Peschanski[a]
(SPhT, Saclay)
DIS 2004, Štrbské Pleso, April 14-18

- Saturation and Non-Linear Equations:

  Non-linear Density Effects in QCD.

- Mathematics:

  "Universal" Traveling Wave Solutions

- Physics:

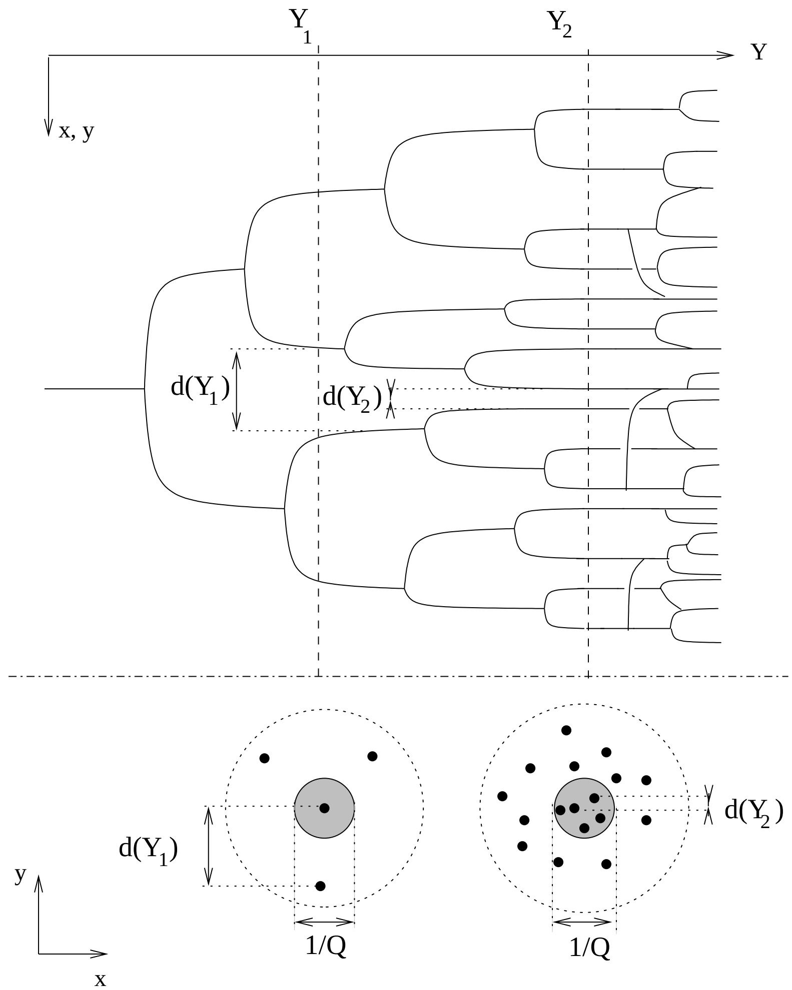
  Geometrical Scaling and Anomalous Diffusion

---

[a]With Stéphane Munier (CPhT, Palaiseau): hep-ph/0309177 /0310357 /0401215

Saturation and Non-Linear Equations

The Tree of Partons/Dipoles



$d(Y) \to 0$ = Non-Linear Density effects

$Y \sim Y_1$: Exponential regime: BFKL

$Y \sim Y_2$: Transition to Saturation: BK

$Y > Y_2$: Deeper in Saturation: JIMWLK, CGC

The Balitskii-Kovchegov Equation

- **The Dipole Tree Observed in DIS:**

$$\sigma^{\gamma^*}(Y,Q) = \int_0^\infty x_{01}^3\, dx_{01}\, |\psi(x_{01}Q)|^2 \int k dk J_0(kx_{01})\, \mathcal{N}(Y,k)$$

x_{01} : Dipôle Size

$\psi(x_{01}Q)$: $q\bar{q}$ Dipole Wave Function

$\mathcal{N}(Y,k)$: $\sim$ Unintegrated Gluon k-Distribution

- **The Non-Linear BK Equation for $\mathcal{N}$:**

$$\partial_Y \mathcal{N} = \bar{\alpha}\chi\,(-\partial_L)\,\mathcal{N} - \bar{\alpha}\,\mathcal{N}^2$$

  – BFKL kernel

$$\chi\,(-\partial_L) = 2\psi(1) - \psi\,(-\partial_L) - \psi\,(1+\partial_L)\ \ ;\ \ L \equiv \log\frac{k^2}{\Lambda^2}$$

  – QCD Coupling (fixed or running)

$$\bar{\alpha}\ = cste.\ \mathit{or}\ \bar{\alpha}\ = \frac{1}{bL}$$

  – Equation valid for uncorrelated probes

(cf. "∞-size nucleus")

Mathematical Problem

1^{rst} step: $\rightarrow$ **Non-Linear Diffusion**

- **Diffusive Approximation of BK ($\bar{\alpha} = cst.$)**

$$\bar{\chi}\,(-\partial_L) \sim \chi\,(\tfrac{1}{2}) + \frac{D}{2} \times \left(\partial_L + \frac{1}{2}\right)^2$$

- **Equation BK $\Rightarrow$ F-KPP (1938)**

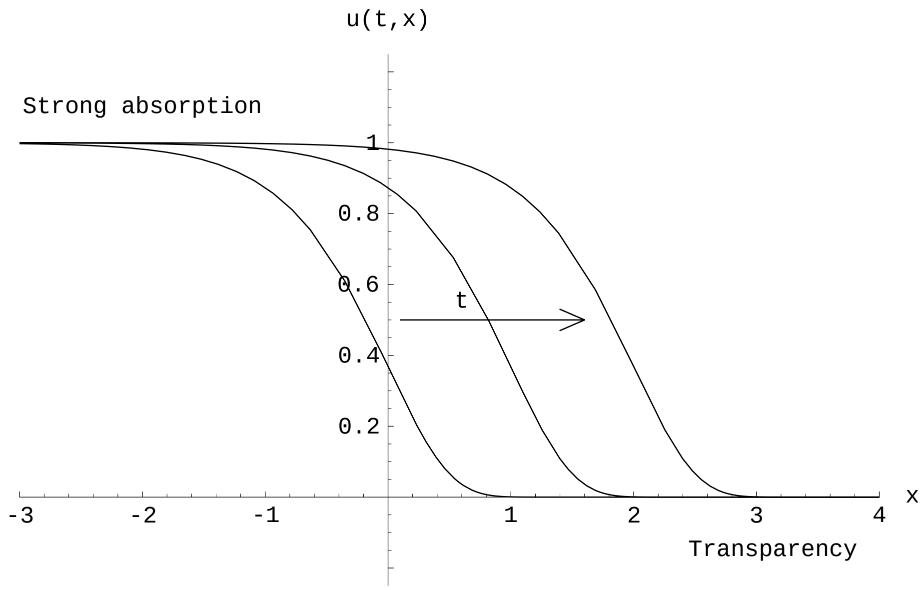
$$\partial_t u(t,x) = \partial_x^2 u(t,x) + u(t,x)(1 - u(t,x))$$

- **"Dictionnary"**

$$\begin{aligned} Time &= t \propto Y \\ Space &= x \propto \left(L + \frac{\bar{\alpha} D}{2}\,Y\right) \\ Wave\ Front &= u(t,x) \ \propto\ \mathcal{N}(Y,k) \end{aligned}$$

Traveling wave Solutions

Uniform in x, $t \to \infty$ limit, Bramson (1983)



- $\to$ Geometrical Scaling

$$u(t,x) \xrightarrow[t \to \infty]{} w(x - m_{\bar\gamma}(t)) \Rightarrow \mathcal{N}(Y,k) = \mathcal{N}\left(\log k - \log Q_s(Y)\right)$$

- $\to$ Saturation Scale: "Universal terms"

$$\log Q_s^2(Y) = \bar\alpha \frac{\chi(\bar\gamma)}{\bar\gamma} Y - \frac{3}{2\bar\gamma} \log Y - \frac{3}{(\bar\gamma)^2} \sqrt{\frac{2\pi}{\bar\alpha \chi''(\bar\gamma)}} \frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y)$$

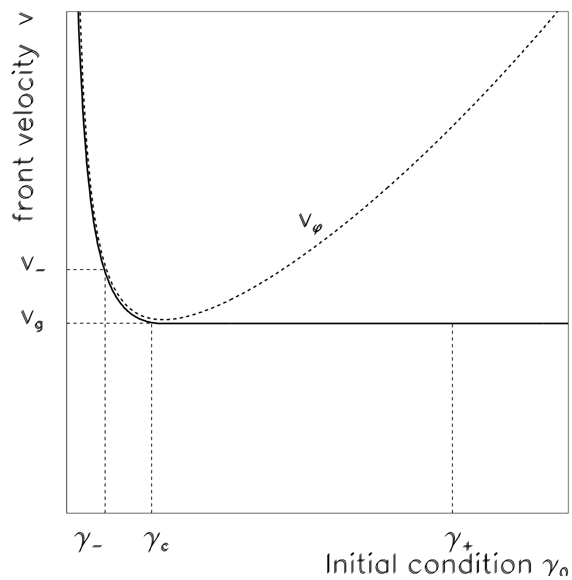
- $\to$ Critical Initial Condition vs. Transparency

$$\mathcal{N}(k, Y_0) \sim 1/k^2 \ll 1/k^{2\bar\gamma} \Rightarrow \bar\gamma = .6275\ldots \quad \left(\bar\gamma \equiv \frac{\chi(\bar\gamma)}{\chi'(\bar\gamma)}\right)$$

- *2 first terms: Mueller Triantafyllopoulos (2002)*

Generic Wave Solutions for BK

2^{nd} step: **Beyond the Diffusive Approximation**



- Sub-critical regime: phase velocity

$$\gamma_0 = \gamma_- \Rightarrow v \equiv v_\varphi(\gamma) = \bar{\alpha}\chi(\gamma)/\gamma$$

- Critical regime: phase $\equiv$ group

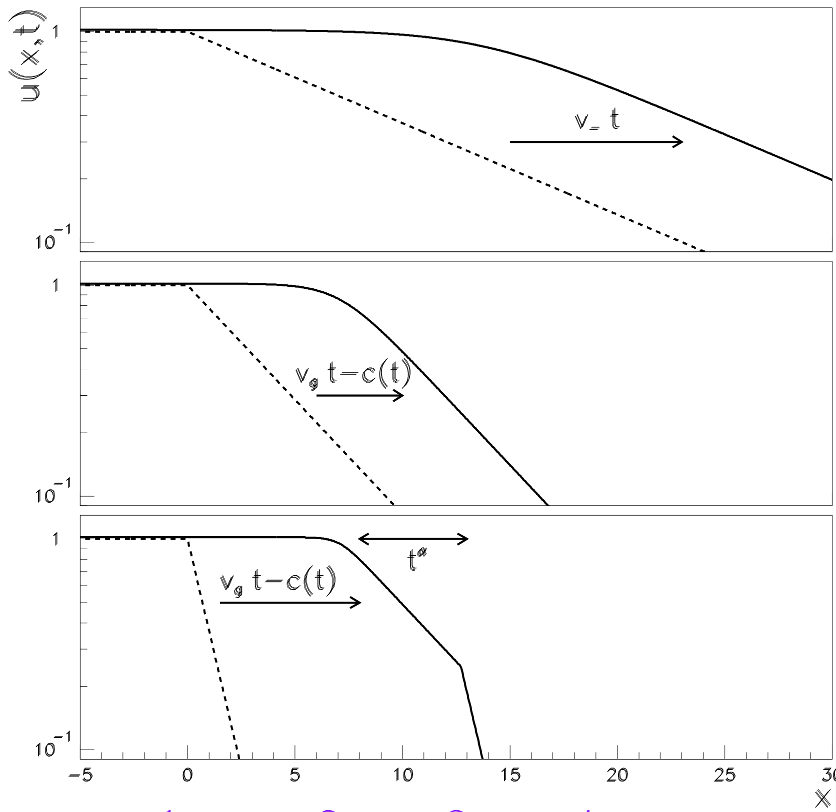
$$\gamma_0 = \bar{\gamma} = .6275... \Rightarrow v_\varphi(\bar{\gamma}) \equiv v_g(\bar{\gamma}) = \bar{\alpha}\chi'(\bar{\gamma})$$

- Super-critical regime (cf. QCD at $\gamma_+ = 1$)

$$\gamma_0 = \gamma_+ \Rightarrow \bar{v} \equiv v_g(\bar{\gamma}) < v_\varphi(\gamma)$$

The Wave Front Structure

Derrida, Van Saarlos: “Pulled vs. Pushed fronts”



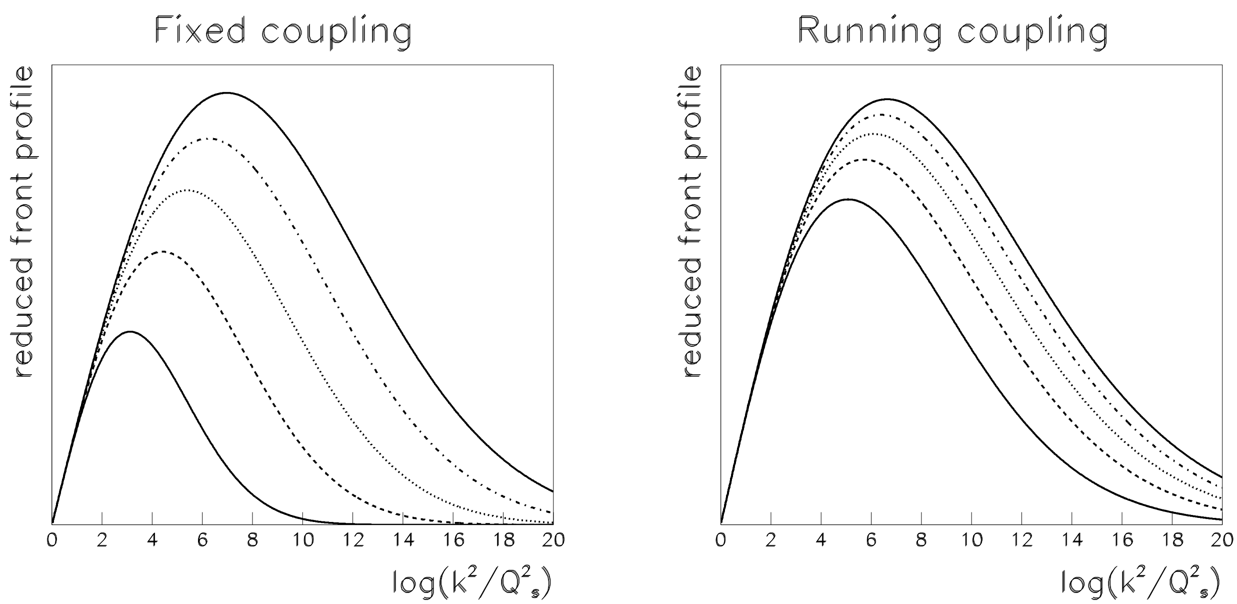
−1 − −2 − −3 − −4−

Supercritical “Pulled” Fronts: 4 Regions

- -1- “Absorptive”: Deep Saturation
- -2- “Interior”: Geometrical Scaling
- -3- “Leading Edge”: Transition to Saturation
- -4- “Conserved Velocity”: Transparency limit

BK Solutions: Results

Diffusive Transition to Saturation

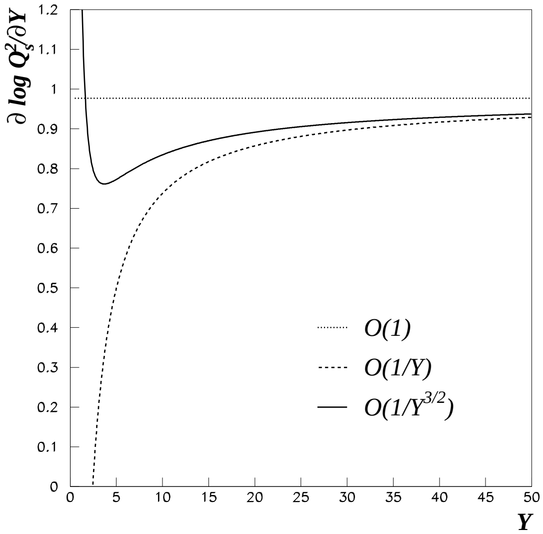


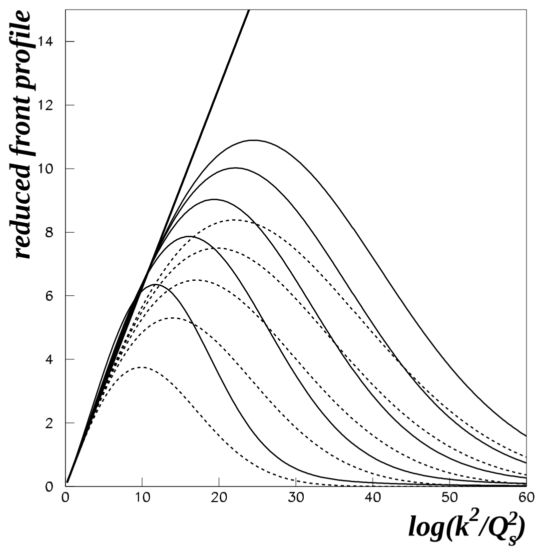
The "Reduced" Front profile

$$(k^2/Q_s^2)^{\gamma_c}\,\mathcal{N}(k/Q_s(Y),Y)$$

- **Fixed α : Normal diffusion**
  variance $\Delta \sim t^{1/2} \equiv Y^{1/2}$
- **Running α : Anomalous "Airy" Diffusion**
  variance $\Delta \sim t^{1/3} \equiv Y^{1/6}$

Physics Results:





INTERCEPT

$$\partial \log(Q_s^2/k^2)/\partial Y$$

PROFILE

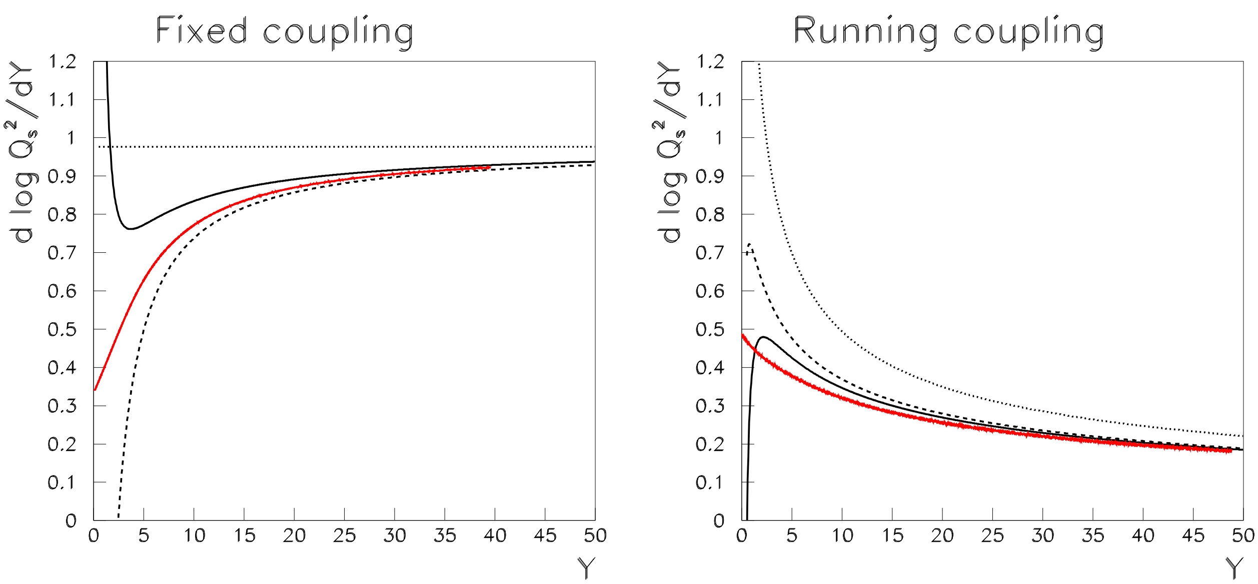
$$(k^2/Q_s^2)^{\bar{\gamma}}\, \mathcal{N}(k/Q_s(Y), Y)$$

Universal Approach to Asymptotia (α fixed)

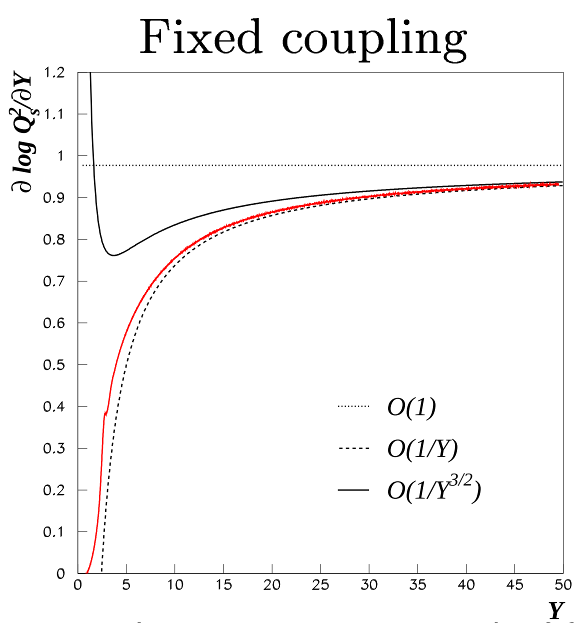
$\rightarrow$ **3 Universal Terms:** $\mathcal{O}(1), \mathcal{O}(1/Y), \mathcal{O}(1/Y^{3/2})$

Universality vs. Initial Conditions

Comparison with Numerical Simulations



$\sim Mc\ Lerran{-}Venugopalan$



$\sim Golec{-}Biernat\ Wusthoff$

Some Comments:

Where are the non-linear modifications?

$$\log Q_s^2(Y) = \bar{\alpha}\frac{\chi(\bar{\gamma})}{\bar{\gamma}}Y - \frac{3}{2\bar{\gamma}}\log Y - \frac{3}{(\bar{\gamma})^2}\sqrt{\frac{2\pi}{\bar{\alpha}\chi''(\bar{\gamma})}}\frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y)$$

- 1^{rst} *term* $\mathcal{O}(Y)$:

  Linear BFKL amplitude set to a constant

  Iancu, Itakura, Mc Lerran (2002)

- 2^{nd} *term* $\mathcal{O}(\log Y)$:

  Linear BFKL + Absorptive B.C. : $\frac{1}{2\bar{\gamma}} \to \frac{3}{2\bar{\gamma}}$

  Proof of Mueller, Triantafyllopoulos (2002)

- 3^{rd} *(and last universal) term* $\mathcal{O}(1/\sqrt{Y})$:

  New Universal term

- *Beyond* : $\mathcal{O}(1/Y)$

  $Y \to Y + Y_0$: Non-Universal

New Universal term for running α?

Questions and Prospects :

- **Correlations and Multiple Interactions**

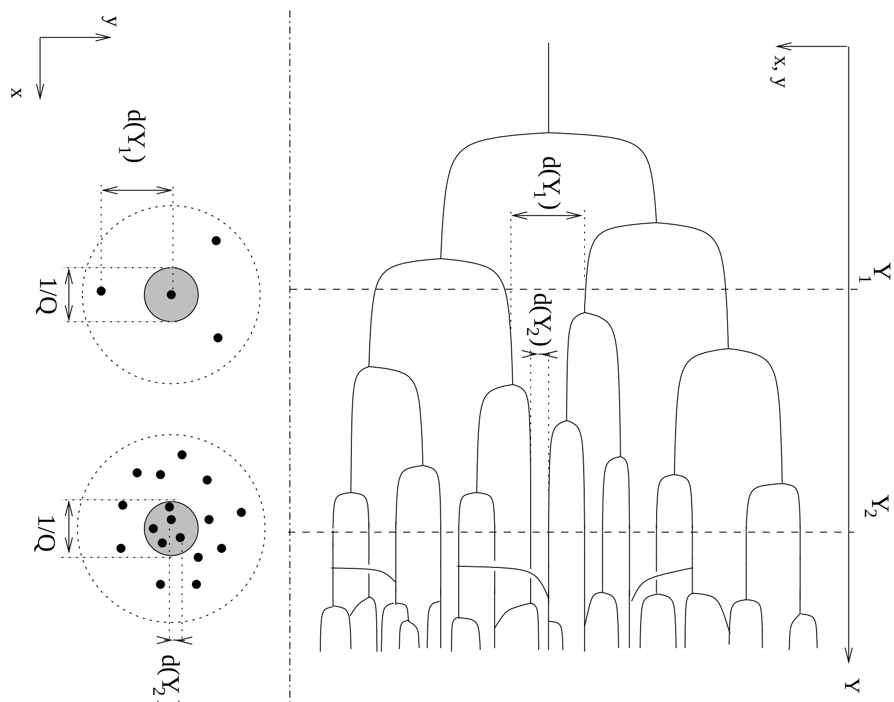
  Extension to Eqn. JIMWLK?

  cf. problem by Mueller, Shoshi (2004)

- **CGC and Spin Glasses**

  "Diffusion Trees and F-KPP Equation "

  Derrida, Spohn (1988)



Conclusions

- **Balitskii-Kovchegov Saturation Equation**
  Traveling wave solutions, Independence from B.C. and specific Non-Linearities → “Pulled Fronts”
- Physics Properties:
  Geometric Scaling and its Diffusive Transition at large rapidity
- Results:
  Derivation of universal expansion terms for the saturation scale and the gluon distribution (cf. wave velocity and front profile)
- Saturation Physics:
  A hint for new relations with Mathematics (non-linear Eqs.) and Physics (Disordered systems, Polymer diffusion and Spin glass phase transitions)