Target Size Effects in Nonlinear Evolution

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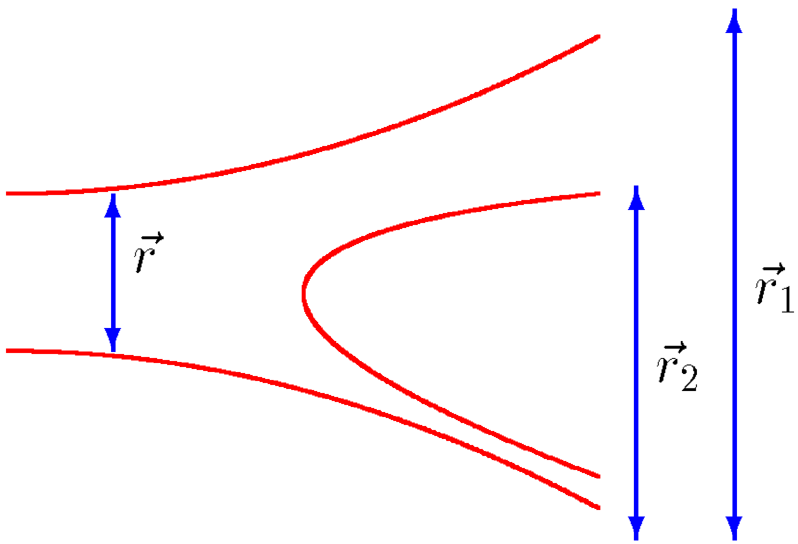
E. Gotsman, M. Kozlov, E. Levin, U. Maor & E.N., `hep-ph/0401021`

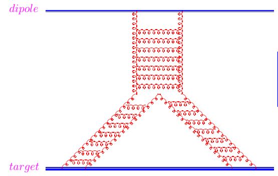
Outline

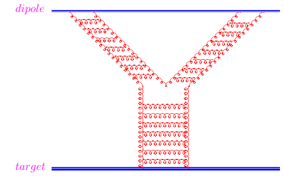
- The Balitsky-Kovchegov (BK) equation
- Validity region of the BK equation
- The shape & characteristics of the scattering amplitude
- Comparison with other solutions
- Summary

The BK equation is for $N(y, \vec{r}; \vec{b})$, the ($\vec{b}$-dependent) imaginary part of the amplitude for dipole-nucleon elastic scattering.



$$\frac{\partial N(\vec{r})}{\partial y} \propto \int d^2 r_2 \frac{r^2}{r_1^2 r_2^2} \left[N(\vec{r}_1) + N(\vec{r}_2) - N(\vec{r}_1)\, N(\vec{r}_2)\right].$$

The BK equation includes contributions from fan diagrams like 

but not 

$\Rightarrow$ Applicable only for $r_1, r_2 \ll R$.

$$\frac{\partial N(y,\vec{r};\vec{b})}{\partial y} = \frac{C_F \alpha_s}{2\pi^2} \int d^2 r_2 \frac{r^2}{r_1^2 r_2^2} \theta(R - |\vec{r}_1|)\, \theta(R - |\vec{r}_2|) \times$$
$$\Big[2N(y, \vec{r}_1; \vec{b} - \tfrac{1}{2}\vec{r}_2) - N(y, \vec{r}_1; \vec{b} - \tfrac{1}{2}\vec{r}_2)\, N(y, \vec{r}_2; \vec{b} - \tfrac{1}{2}\vec{r}_1)$$
$$- N(y, \vec{r}; \vec{b}) \Big] \,.$$

Initial Conditions

- To obtain a solution at arbitrary b, we need to specify the initial conditions of N at $y = y_0$, as a function of $\vec{r}$ and $\vec{b}$.

- A natural choice of the initial conditions is the Glauber-Mueller from:

$$N(y_0, \vec{r}; \vec{b}) = 1 - e^{-\kappa(y_0,\vec{r})\, S(\vec{b})/S(0)},$$

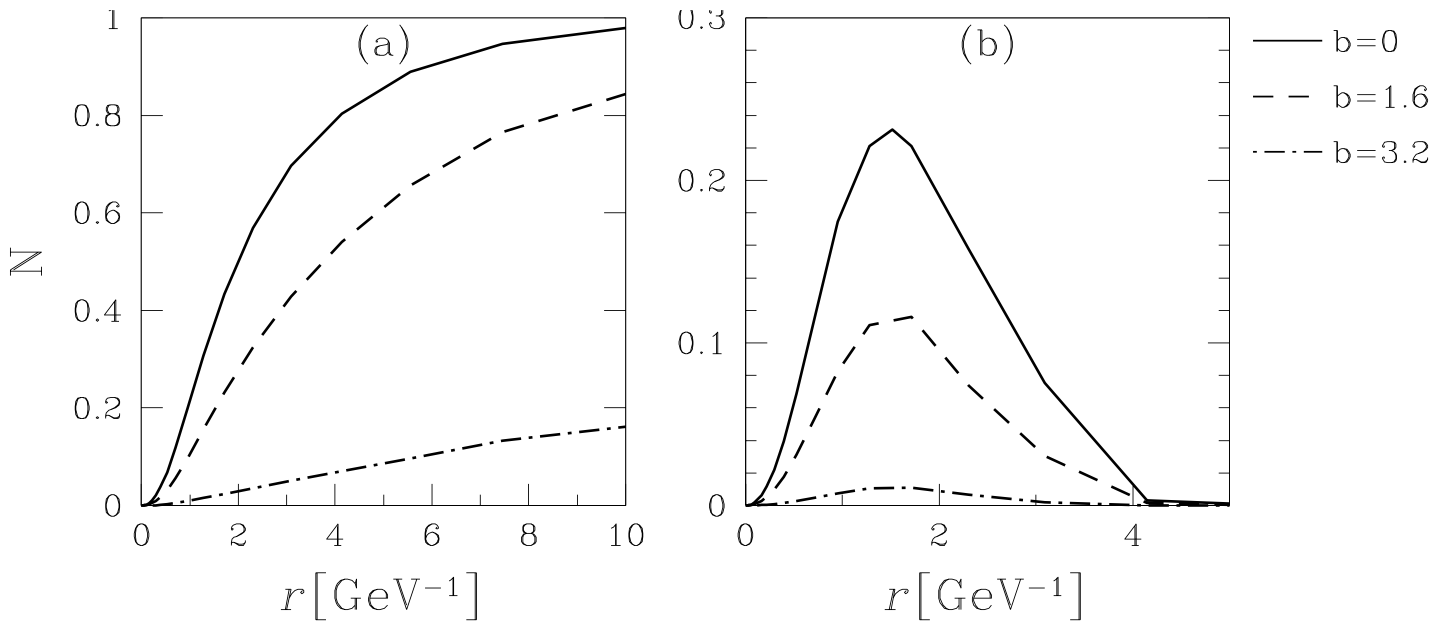
- In the Born approximation, however, the scattering amplitude decreases for large r already at $b = 0$.

- Motivated by the Born approximation result, we make the replacement $b \longrightarrow \sqrt{b^2 + r^2}$:

$$N(y_0, \vec{r}; \vec{b}) = 1 - e^{-\kappa(y_0, \vec{r})\, S(\sqrt{b^2+r^2})/S(0)},$$

where κ is taken from the approximated solution at fix b [Lublinsky, Gotsman, Levin, Maor, *Eur. Phys. J. C* (2003)]

fix b solution　　　　our input



Method of the Solution

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3. Add $\Delta N^{(1)}$ to the R.H.S. of the BK equation and repeat the evolution $\Rightarrow$ $\boxed{N^{(2)}}$

$$\frac{\partial N^{(2)}(y,\vec{r};\vec{b})}{\partial y} = \frac{C_F \alpha_s}{2\pi^2} \int_{r_1,\, r_2 < R} d^2 r_2 \frac{r^2}{r_1^2 r_2^2} \ldots + \Delta N^{(1)}$$

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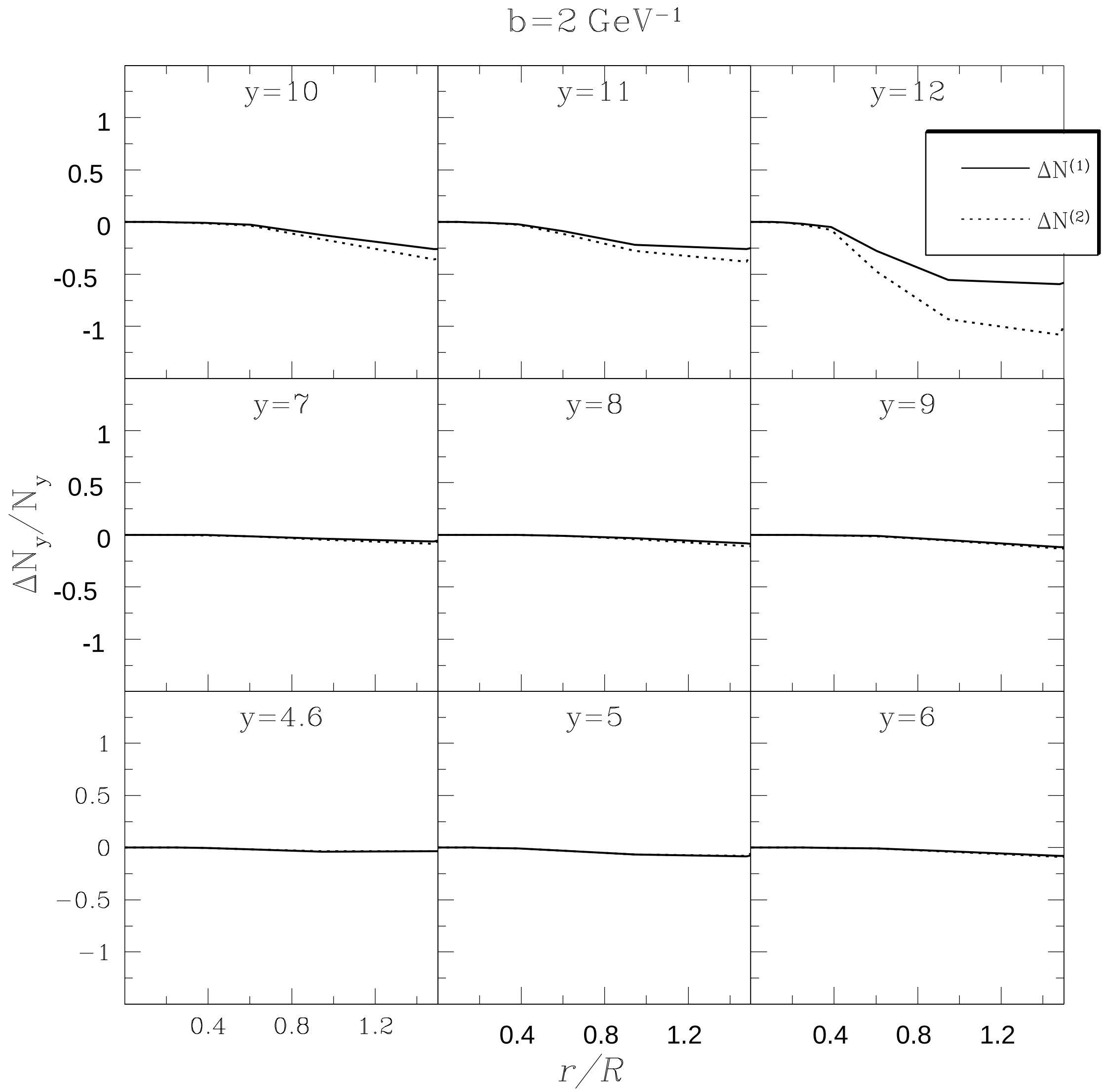
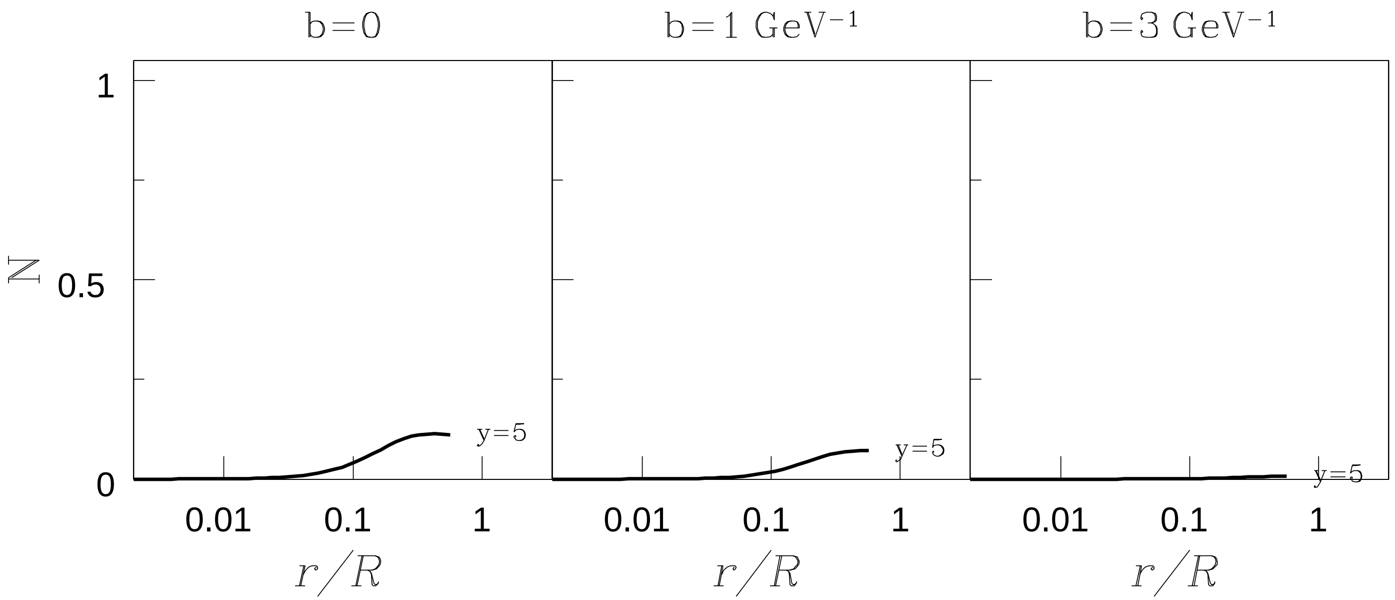
4. Repeat step 2 with $N^{(2)} \;\Rightarrow\; \boxed{\Delta N^{(2)}}$

Comparison between $\Delta N^{(1)}$ and $\Delta N^{(2)}$



The evolution process



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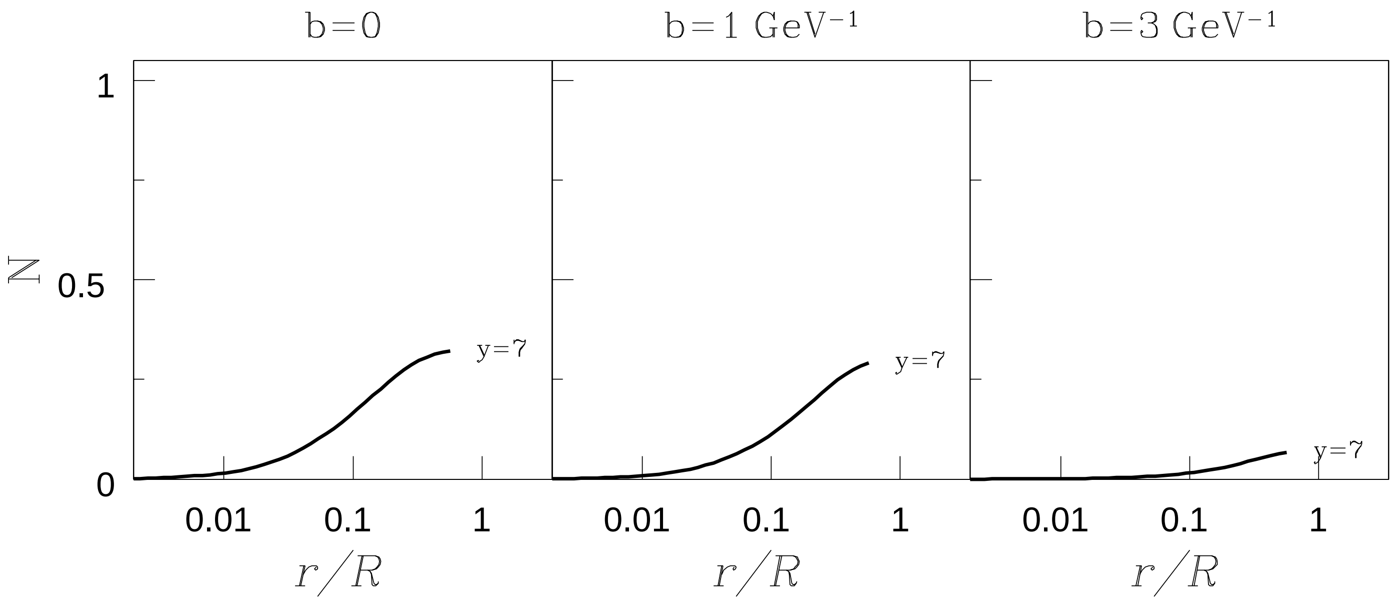

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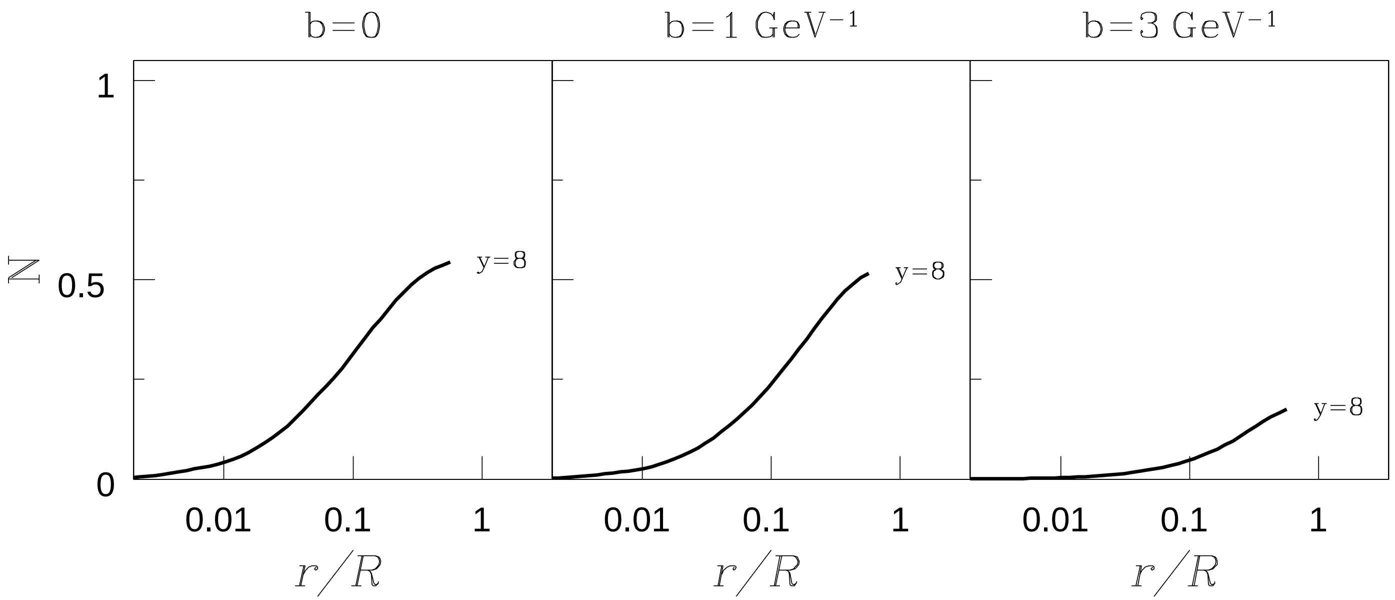

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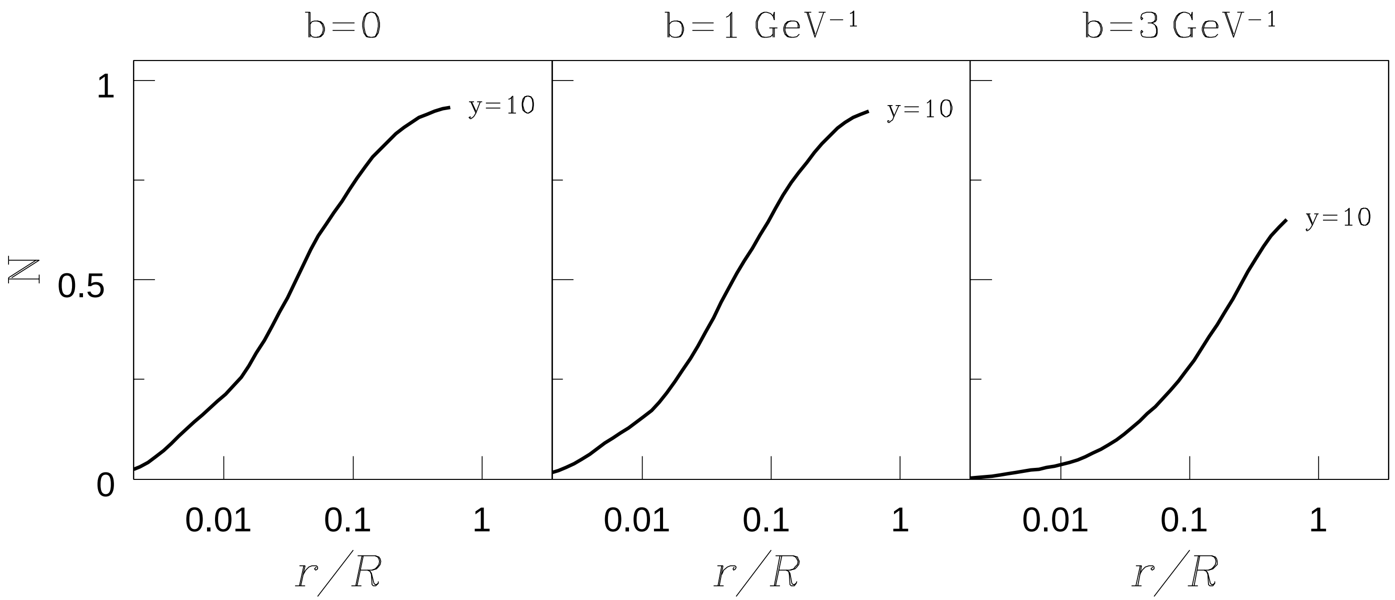
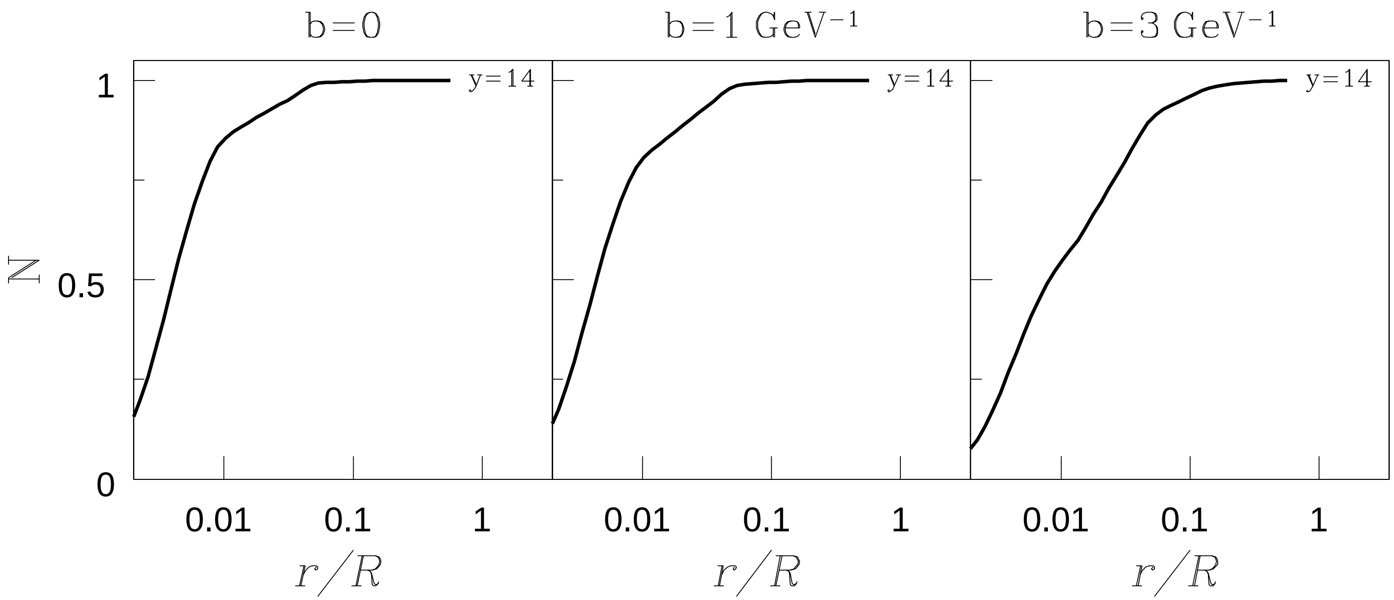
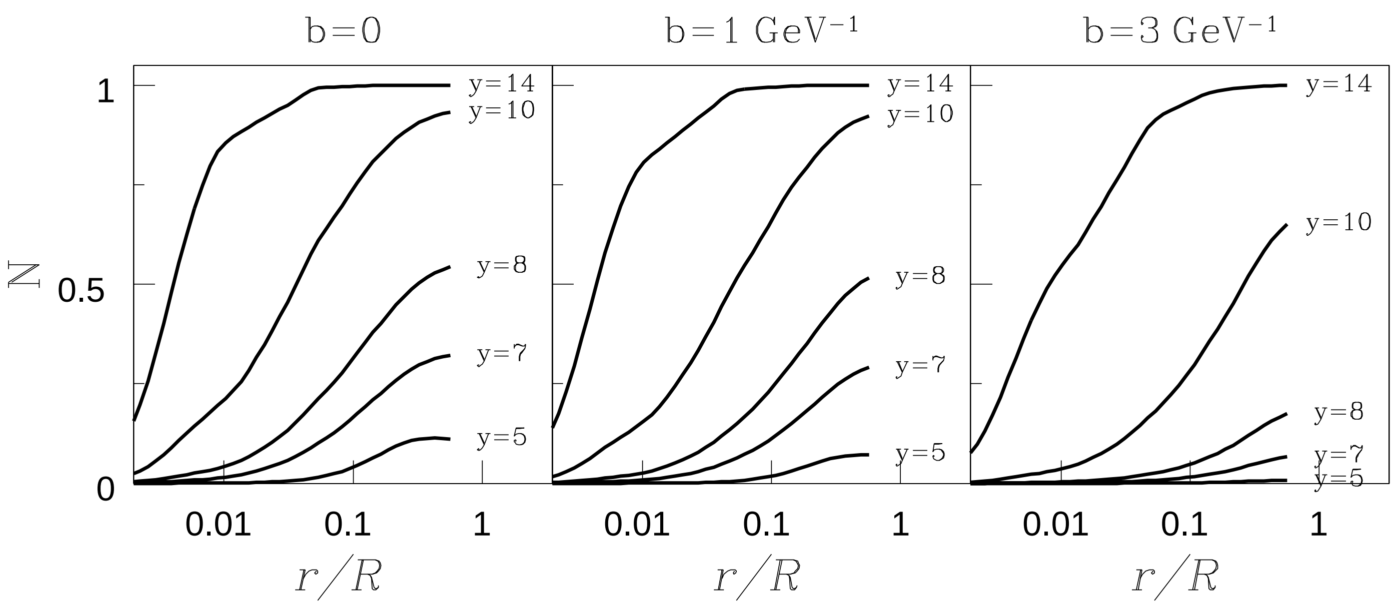


The evolution process

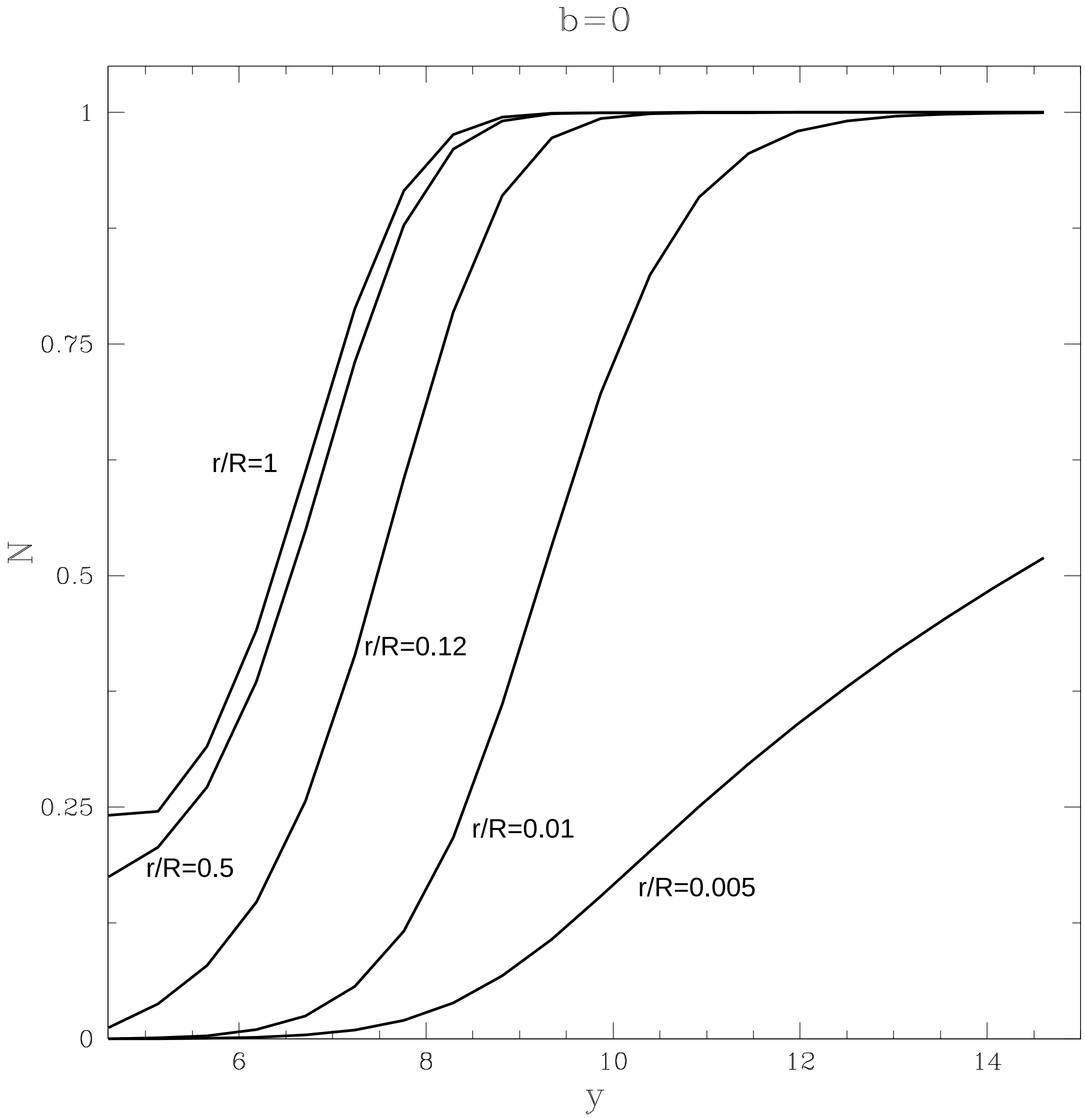


The evolution process

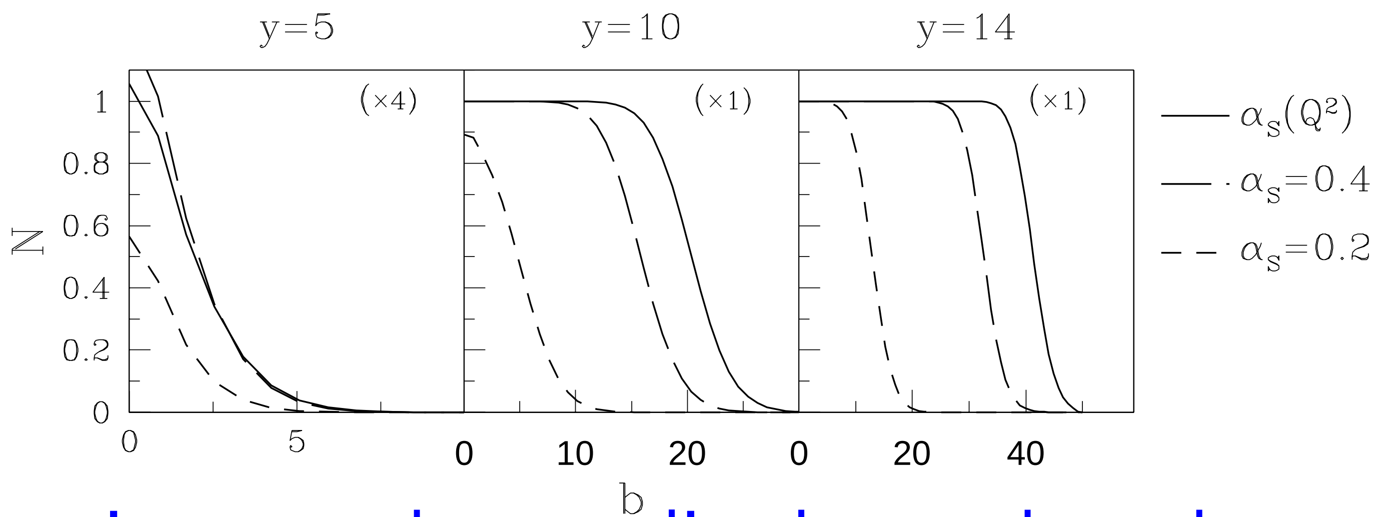


- Steep increment of N with the dipole size for large values of the rapidity.
- The rise with r is tamed when the impact parameter becomes large.

Energy dependence



Impact parameter dependence for fixed and running α_s



- For running α_s, the amplitude reaches the saturation boundary for small values of y.

- For fixed α_s, the growth of N with energy depends on the value of α_s.

- No power-like b-dependence at large b.

Saturation scale

- In the semiclassical solution for fixed α_s the saturation scale, Q_s, grows rapidly for large y and decreases rapidly for large b:

$$Q_s(y,b) \;\propto\; e^{\lambda_F y}\, e^{-\frac{b}{R}},$$

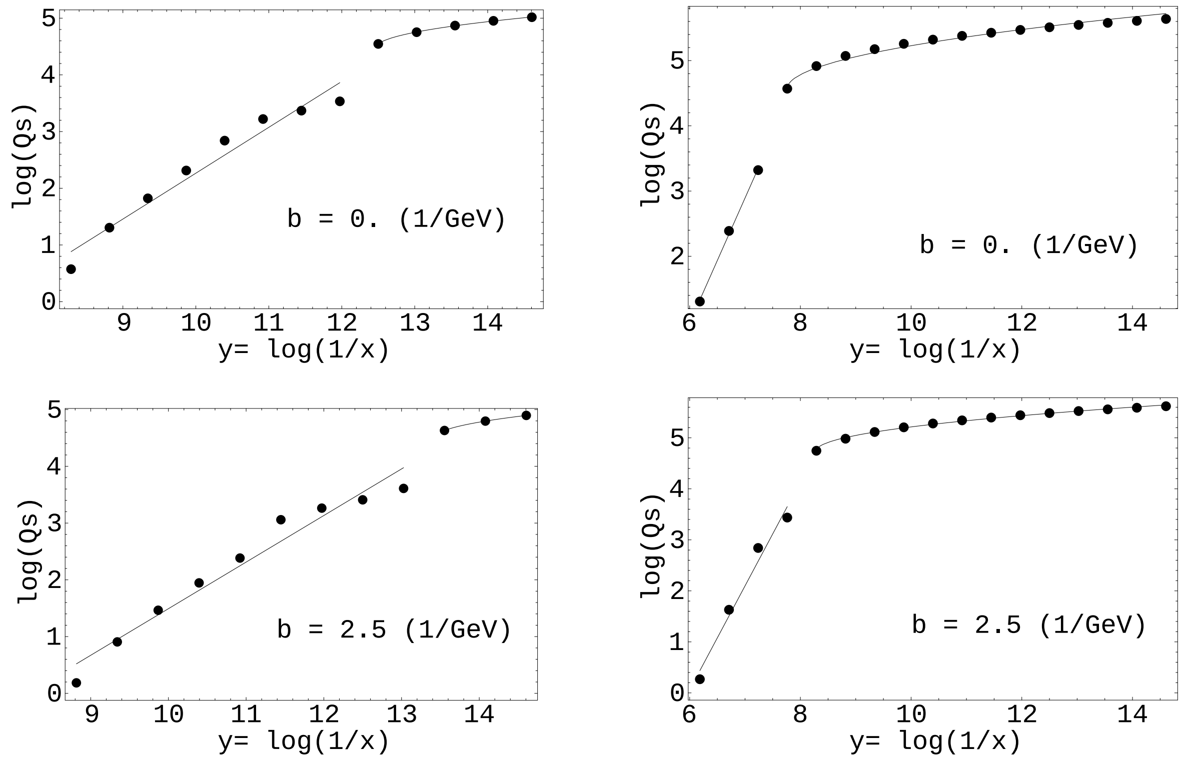
- For running α_s, one expects different behavior for small and large b:

$$b < R\lambda_R\, y \Rightarrow \quad Q_s \propto e^{\sqrt{\lambda_R y}};$$
$$b > R\,\lambda_R\, y \Rightarrow \quad Q_s \propto e^{-\frac{b}{R}}$$

with $\lambda_F \approx 2\alpha_s$ and $\lambda_R \approx 2.5$.

y-dependence of the saturation scale

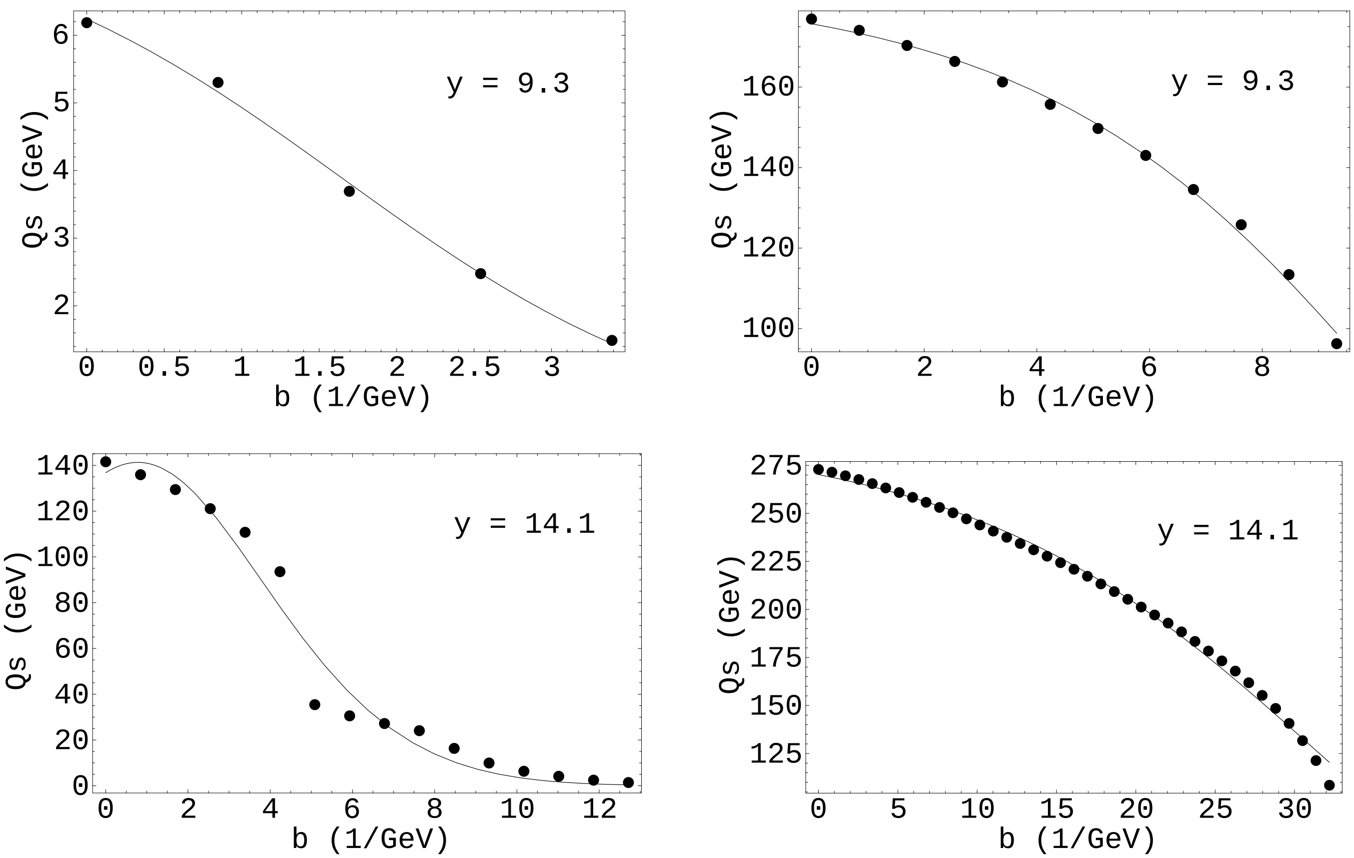
fixed α_s running α_s



- small y: $Q_s \propto e^{\lambda_F\, y}$
- large y: $Q_s \propto e^{\sqrt{\lambda_R\, y}}$

b-dependence of the saturation scale

fixed α_s | running α_s



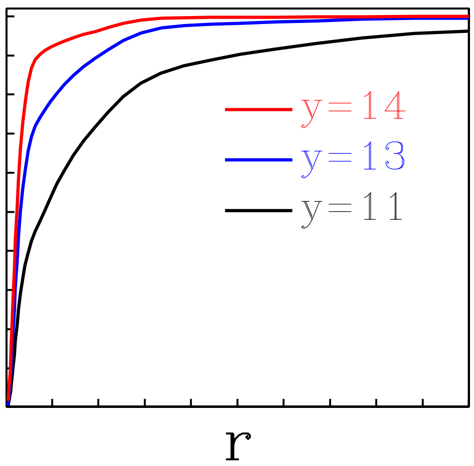
- $Q_s \propto \left(A(y) + e^{(b/R(y))^2}\right)^{-1}$ both for fixed and running α_s.

Geometrical Scaling

In the saturation region, the amplitude is a function of one variable $\tau \equiv rQ_s(y,b)$.

for fixed α_s:

$N(y,r,b)$



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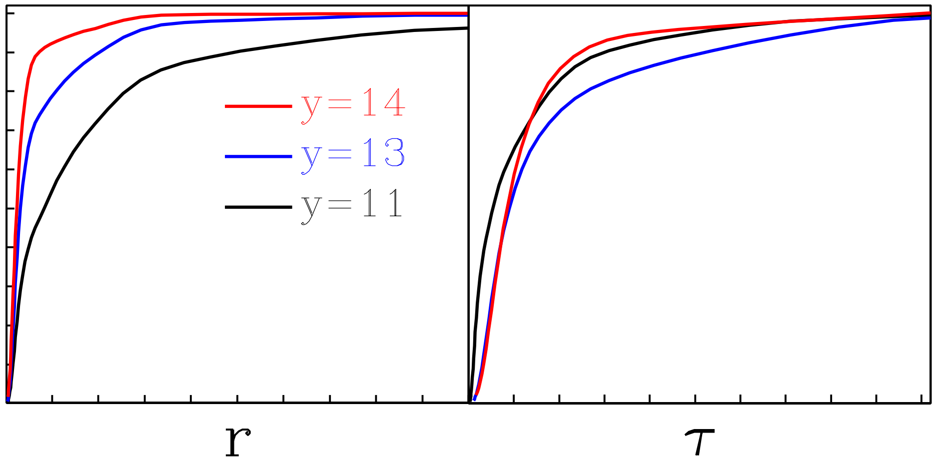
for fixed α_s:

$$N(y,r,b) \quad \Longrightarrow \quad N(\tau)$$

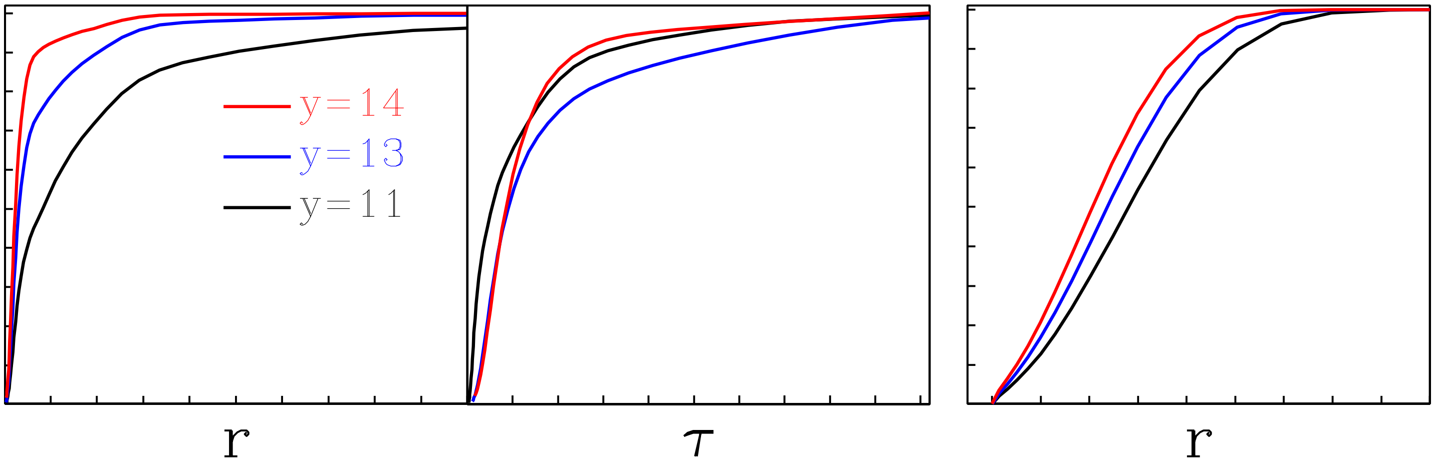


Geometrical Scaling

In the saturation region, the amplitude is a function of one variable $\tau \equiv rQ_s(y,b)$.

for fixed α_s: $N(y,r,b) \implies N(\tau)$

for running α_s: $N(y,r,b)$

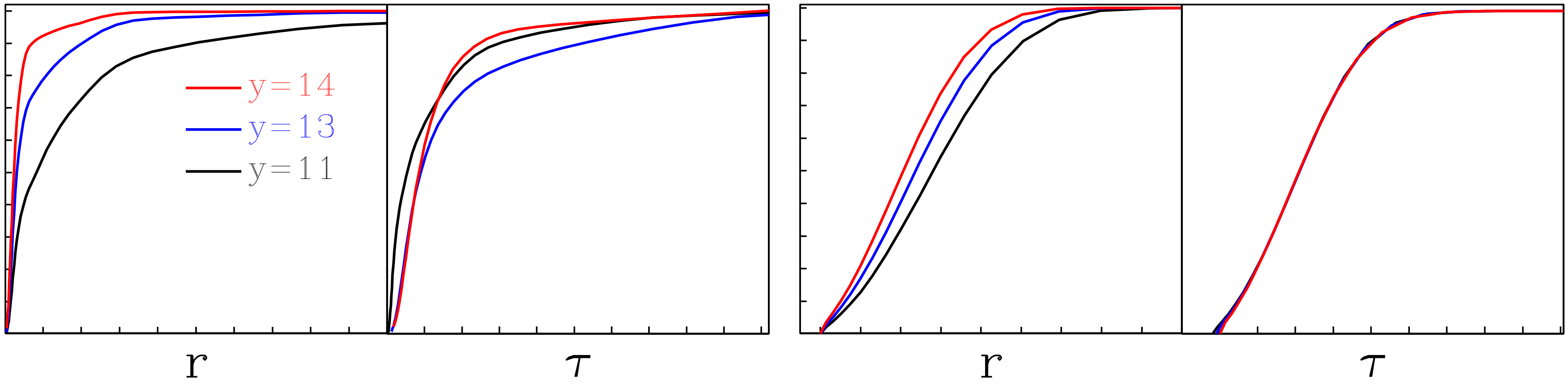


Geometrical Scaling

In the saturation region, the amplitude is a function of one variable $\tau \equiv rQ_s(y,b)$.

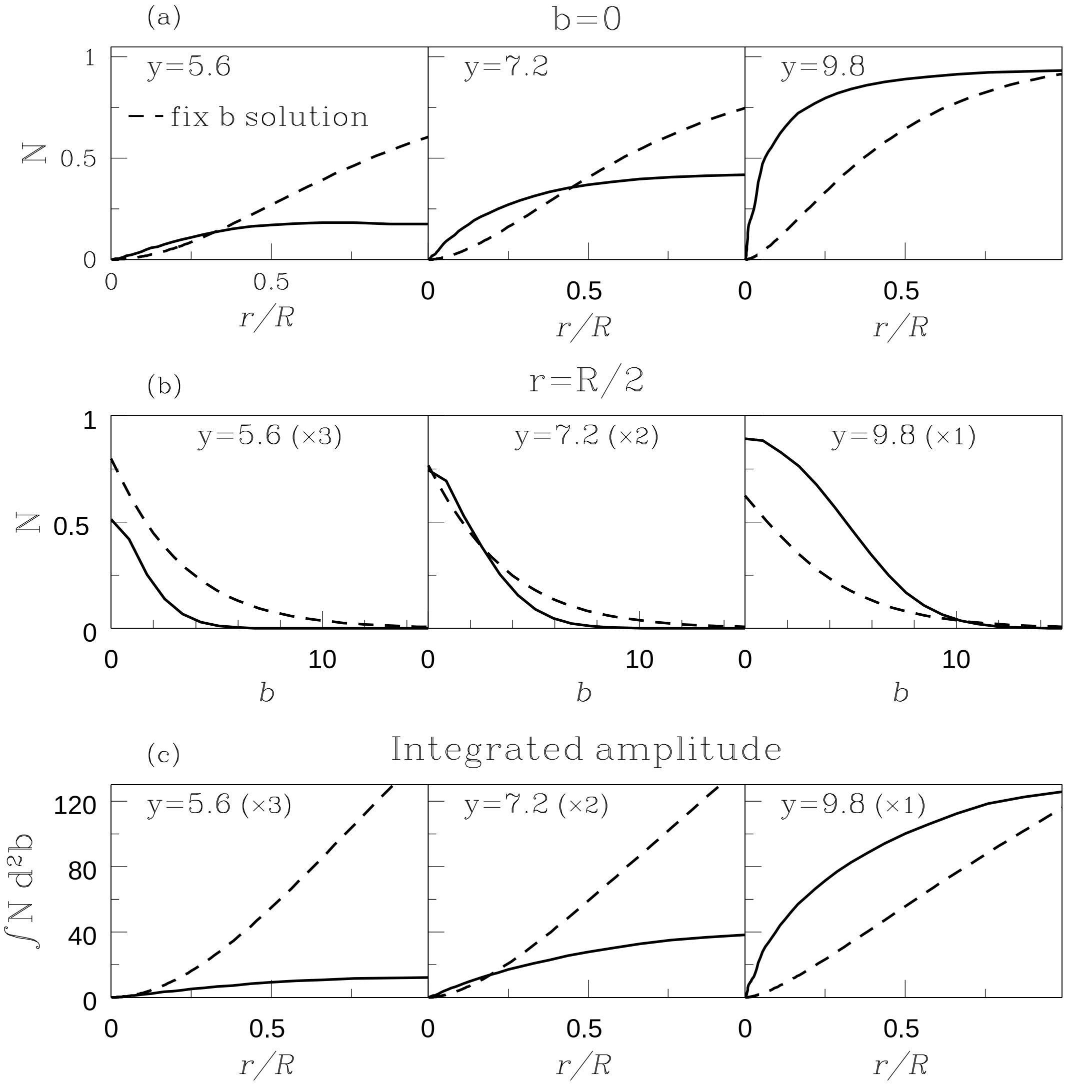
for fixed α_s: for running α_s:

$N(y,r,b) \Longrightarrow N(\tau)$ $N(y,r,b) \Longrightarrow N(\tau)$

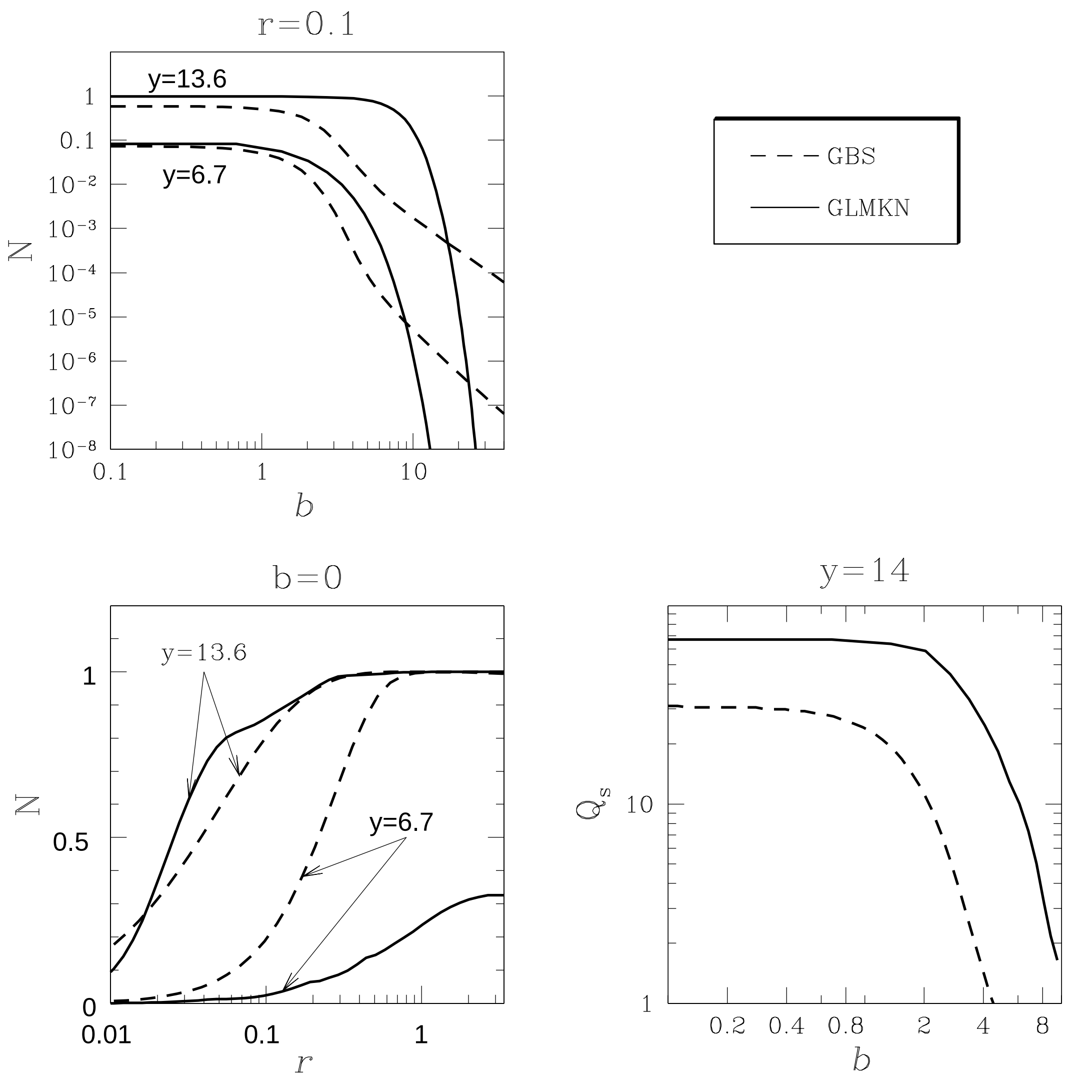


- Better for running α_s

Comparison with the Fixed b Solution



Comparison with Golec-Biernat & Stasto



Summary

- We obtained a numerical solution of the BK nonlinear evolution equation for arbitrary b, by imposing a restriction on the size of the interacting dipoles.
- For the initial conditions, we used the fixed b solution, with the modification $b \longrightarrow \sqrt{b^2 + r^2}$.
- Our solution does not exhibit a power-like tail in the b-dependence. We conclude that the power-like behavior is an artifact of using the BK equation in the kinematic region in which this equation is not valid.
- The geometrical scaling behavior, $N = N(\tau, b)$, where $\tau = r\, Q_s(y, b)$, is more striking for running α_s, than for fixed α_s. An explanation for such behavior can be related to the idea that the running QCD coupling is frozen at the saturation momentum.