

# Unintegrated pdfs in CCFM

H. Jung, DESY

DIS, Strbské Pleso, 2004

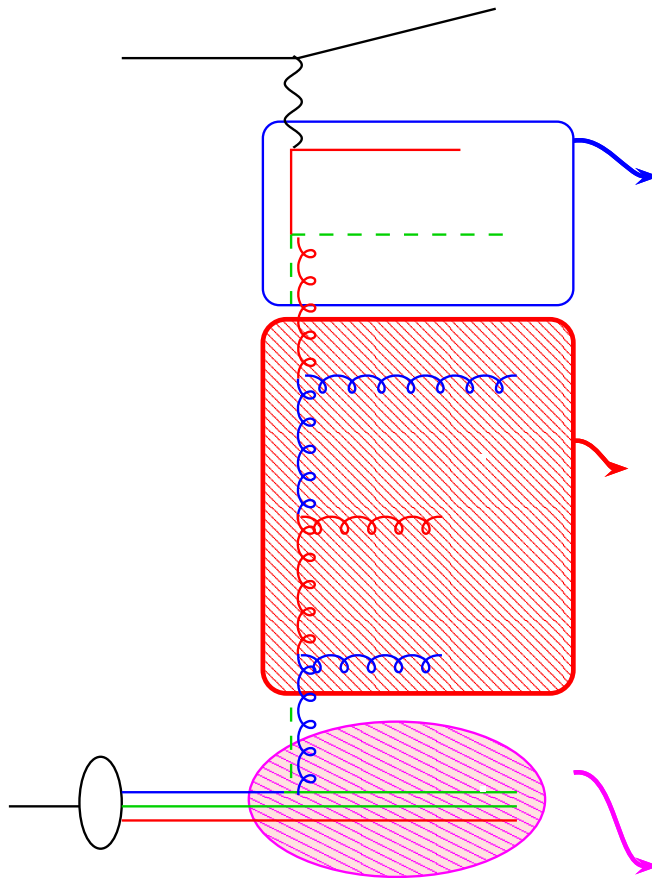
- CCFM equation
  - one-loop — all-loops
  - collinear- vrs  $k_t$ -factorisation
- fits to  $F_2$ 
  - fit initial condition for one-loop — all-loops
  - sensitivity to starting scale
  - choice of factorisation scale
  - small  $k_t$ -behavior
- application to LHC
- conclusion

# Basic idea - Collinear factorization

## CCFM

### CCFM (one loop)

● angular ordering

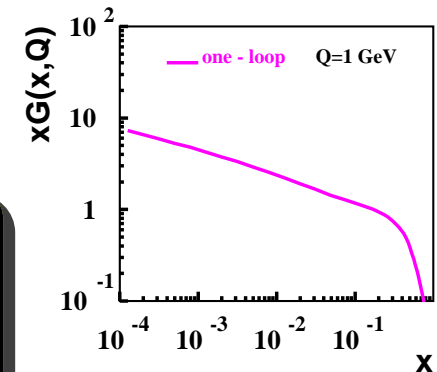
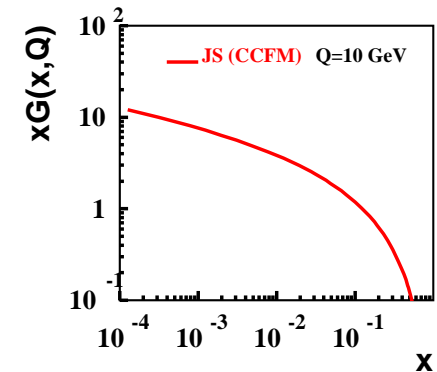


BGF matrix element  
on mass shell

evolution of parton cascade  
with DGLAP splitting fct.

$$\tilde{P} = \bar{\alpha}_s \left( \frac{1}{1-z} + \frac{1}{z} \right)$$

initial distribution: steep

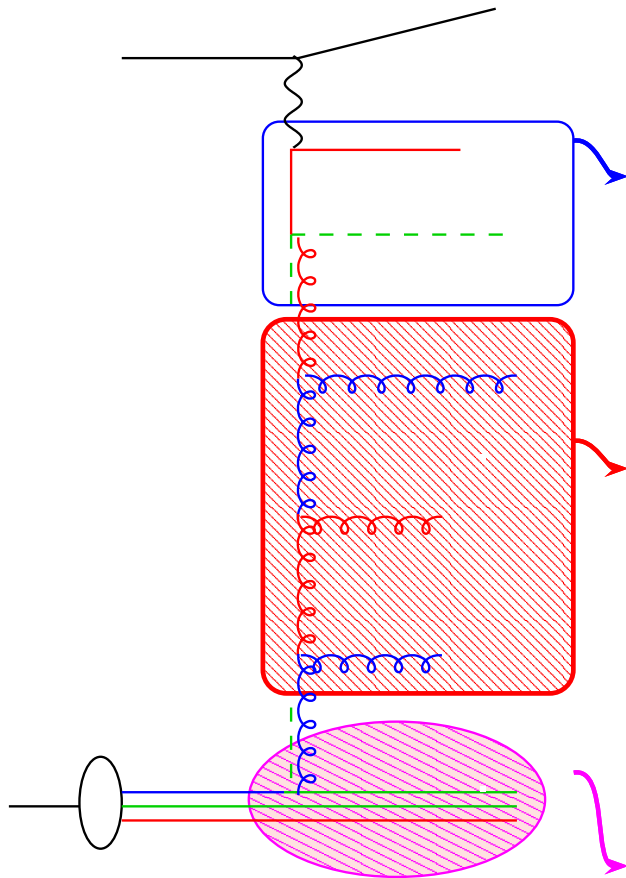


$$\sigma(ep \rightarrow e' q \bar{q}) = \int \frac{dy}{y} d^2 Q \frac{dx_g}{x_g} \hat{\sigma}(\hat{s}, 0, Q) \int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with  $\int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) = x_g G(x_g, Q^2)$

# Basic idea - $k_t$ factorization

## CCFM



BGF matrix element  
off mass shell

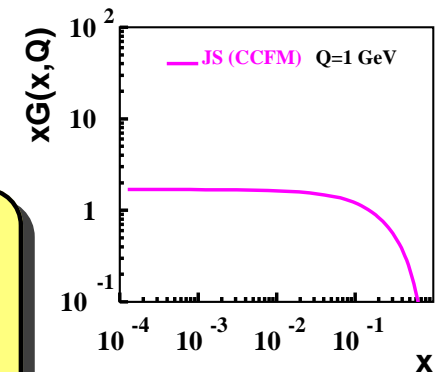
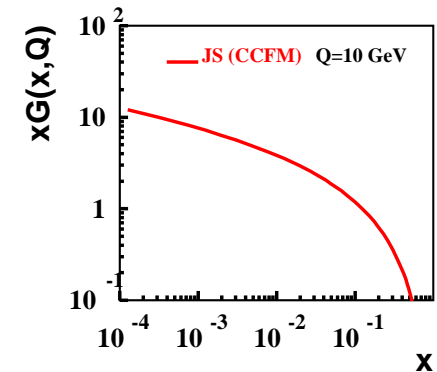
evolution of parton cascade  
with CCFM splitting fct.

$$\tilde{P} = \bar{\alpha}_s \left( \frac{1}{1-z} + \frac{1}{z} \Delta_{ns} \right)$$

initial distribution: flat

## CCFM (all loops)

- angular ordering  
(instead of  $q_t$  ordering)
- $\Delta_{ns}$  (non - Sudakov)



$$\sigma(ep \rightarrow e' q \bar{q}) = \int \frac{dy}{y} d^2 Q \frac{dx_g}{x_g} \int d^2 k_t \hat{\sigma}(\hat{s}, k_t, Q) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with  $\int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$

# CCFM equation: one loop — all loops

$$\mathcal{A}(x, k_t, \bar{q}) = \mathcal{A}_0(x, k_t) \Delta_s(\bar{q}, Q_0) + \int \frac{dz}{z} \int \frac{d^2q}{\pi q^2} \Theta(\bar{q} - zq) \cdot \Delta_s(\bar{q}, zq) \tilde{P}(z, q, k_t) \mathcal{A}\left(\frac{x}{z}, k'_t, q\right)$$

**CCFM Splitting fct:**  $\tilde{P}(z, q, k_t) = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\text{ns}}(z, q, k_t)$

**Sudakov  $\Delta_s(a, b)$ :** **probability for no radiation in  $[a, b]$**

**angular ordering:**  $\bar{q} > z_n q_n, q_n > z_{n-1} q_{n-1}, \dots, q_1 > Q_0$

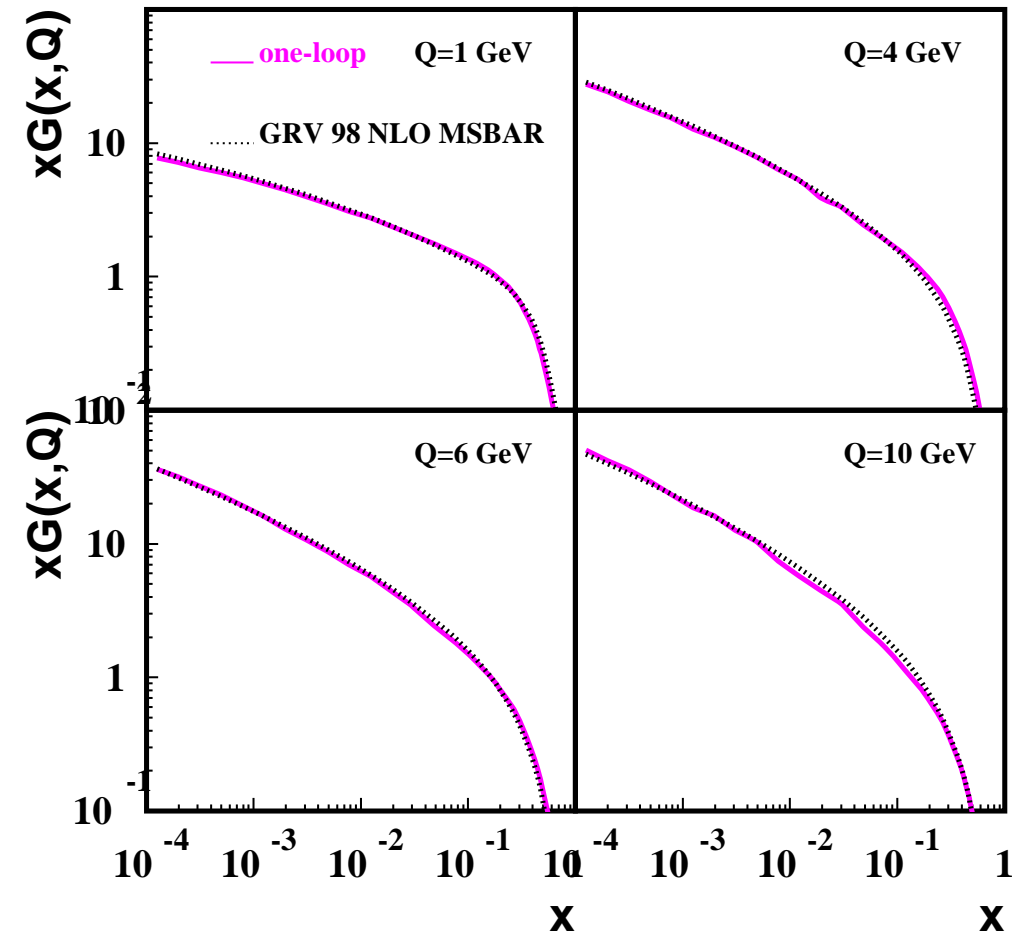
**small  $x$  (all loops)**

- ➔ **BFKL limit ( $z \rightarrow 0$ )**
- ➔ **angular ordering**
- ➔ **no restriction on  $q_i$**

**large  $x$  (one loop)**

- ➔ **DGLAP limit ( $z \gg 0$ )**
- ➔ **DGLAP splitting fct  $\tilde{P}$  with  $\Delta_{\text{ns}} = 1$**
- ➔ **angular ordering  $\rightarrow q_i$  ordering**

# DGLAP unintegrated gluon density - integrated -



one-loop gluon integrated over  $k_t$   

$$\int_0^{\bar{q}} dk_t^2 x \mathcal{A}(x, k_t, \bar{q}) = xG(x, \bar{q})$$
 ➤ compare to evolved DGLAP gluon

one-loop gluon:

- at starting scale use GRV
- full treatment of kinematics
- good agreement with full splitting fct  
but also with  $\tilde{P} = \bar{\alpha}_s \left( \frac{1}{1-z} + \frac{1}{z} \right)$
- evolution machinery works perfectly

# Non-Sudakov and all - loop resummation

**Splitting Fct:**  $\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\text{ns}}(z, q, k_t)$

**Non - Sudakov form factor**  $\blacktriangleright$  **all loop resummation:**

$$\Delta_{\text{ns}} = \exp \left[ -\bar{\alpha}_s(k_t^2) \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_t - q) \Theta(q - z' q_t) \right]$$

$$\Delta_{\text{ns}} = 1 + \left( -\bar{\alpha}_s(k_t^2) \int \frac{dz'}{z'} \int \frac{dq^2}{q^2} \right)^1 + \frac{1}{2!} \left( -\bar{\alpha}_s(k_t^2) \int \frac{dz'}{z'} \int \frac{dq^2}{q^2} \right)^2 \dots$$

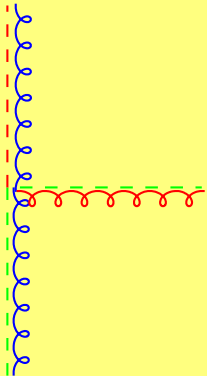
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$$\bar{\alpha}_s(k_t) \frac{1}{z} \left[ 1 \right]$$

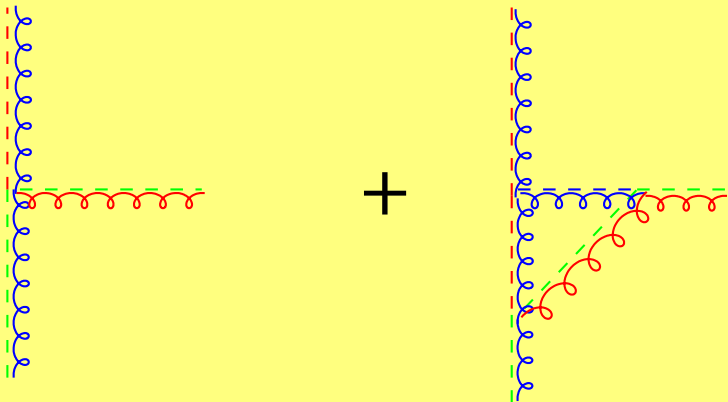
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$$\bar{\alpha}_s(k_t) \frac{1}{z} \left[ 1 + \bar{\alpha}_s \log \left( \frac{z}{z_0} \right) \log \left( \frac{k_t^2}{z_0 z q^2} \right) \right]$$

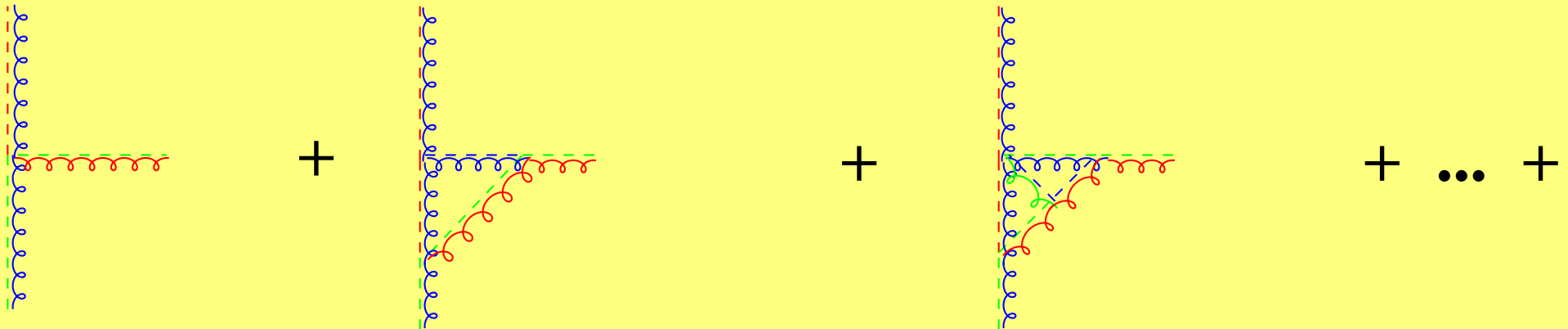
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$$\bar{\alpha}_s(k_t) \frac{1}{z} \left[ 1 + \bar{\alpha}_s \log \left( \frac{z}{z_0} \right) \log \left( \frac{k_t^2}{z_0 z q^2} \right) + \frac{1}{2!} \left( \bar{\alpha}_s \log \left( \frac{z}{z_0} \right) \log \left( \frac{k_t^2}{z_0 z q^2} \right) \right)^2 \dots \right]$$

# Structure Function $F_2(x, Q^2)$

together with G.P. Salam, EPJC 19, 351 (2001)

With  $\sigma = \int dk_t^2 dx_g \mathcal{A}(x_g, k_t^2, \bar{q}) \sigma(\gamma^* g^* \rightarrow q\bar{q})$  fit  $F_2(x, Q^2)$

(data from H1 Coll, NPB 470 (1996) 3.)

## Parameters in fit

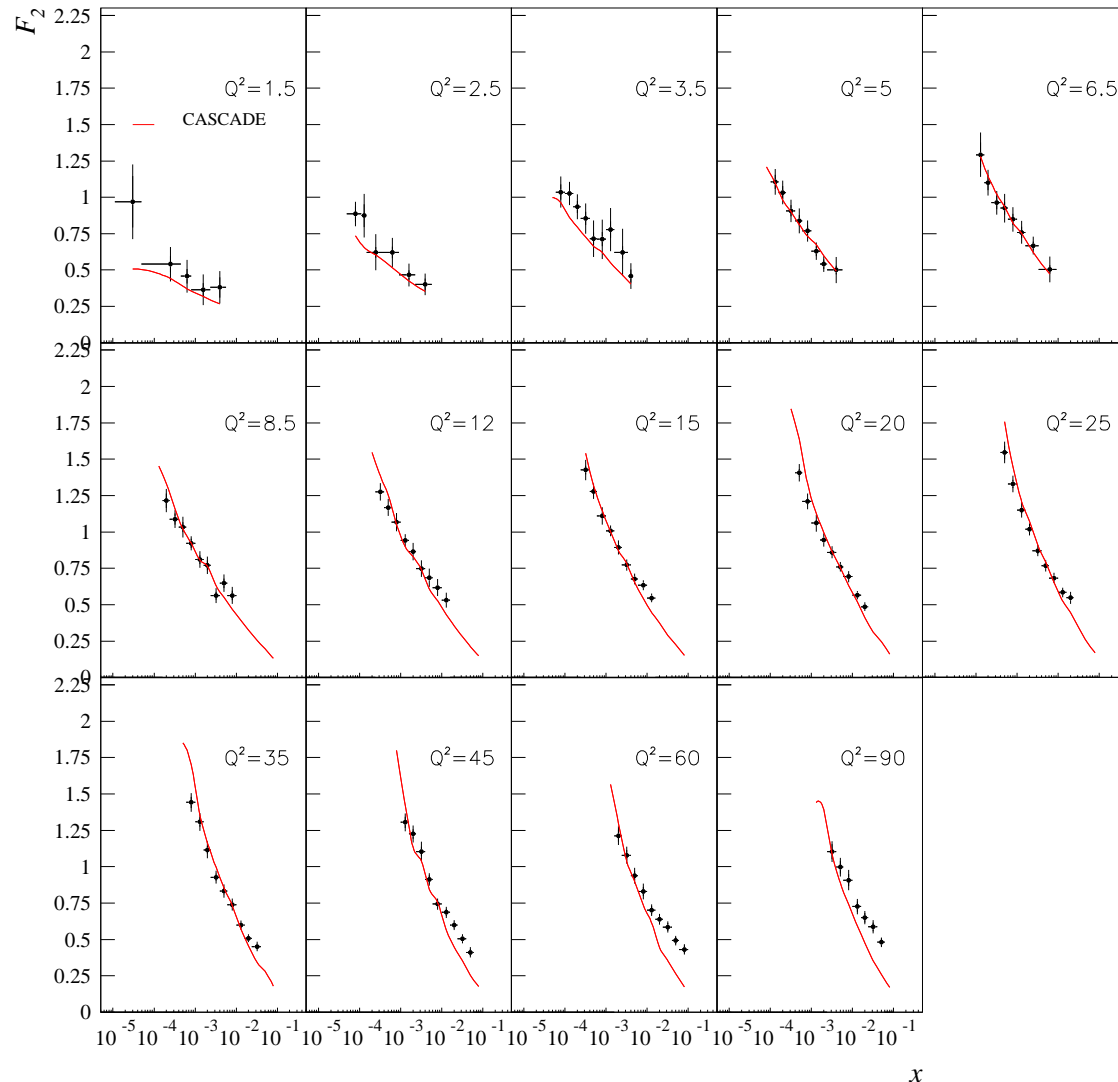
(fitted for  $Q^2 > 5 \text{ GeV}^2$ ,  $x < 10^{-2}$ )

- collinear cut-off  
 $Q_0 = 1.4 \text{ GeV}$
- initial gluon  $x\mathcal{A}_0(x, k_{t0}^2)$
- freezing of  $\alpha_s(k_t)$  for  
 $k_t \rightarrow 0$   
 $k_t$  not constrained ...
- light quark masses:  
 $m_q = 0.250 \text{ GeV}$ ,  
 $m_c = 1.5 \text{ GeV}$

unintegrated gluon density

$$x\mathcal{A}(x, k_t^2, \bar{q})$$

obtained from fit to  $F_2$



# New fit: full splitting function

- improve splitting function

- $P_{gg} \sim \bar{\alpha}_s \left( \frac{1}{z} \Delta_{ns} + \frac{1}{1-z} \right)$

- ➔ to include non-singular terms

$$\frac{1}{z} \Delta_{ns} - 2 + z(1-z) + \frac{1}{1-z}$$

???

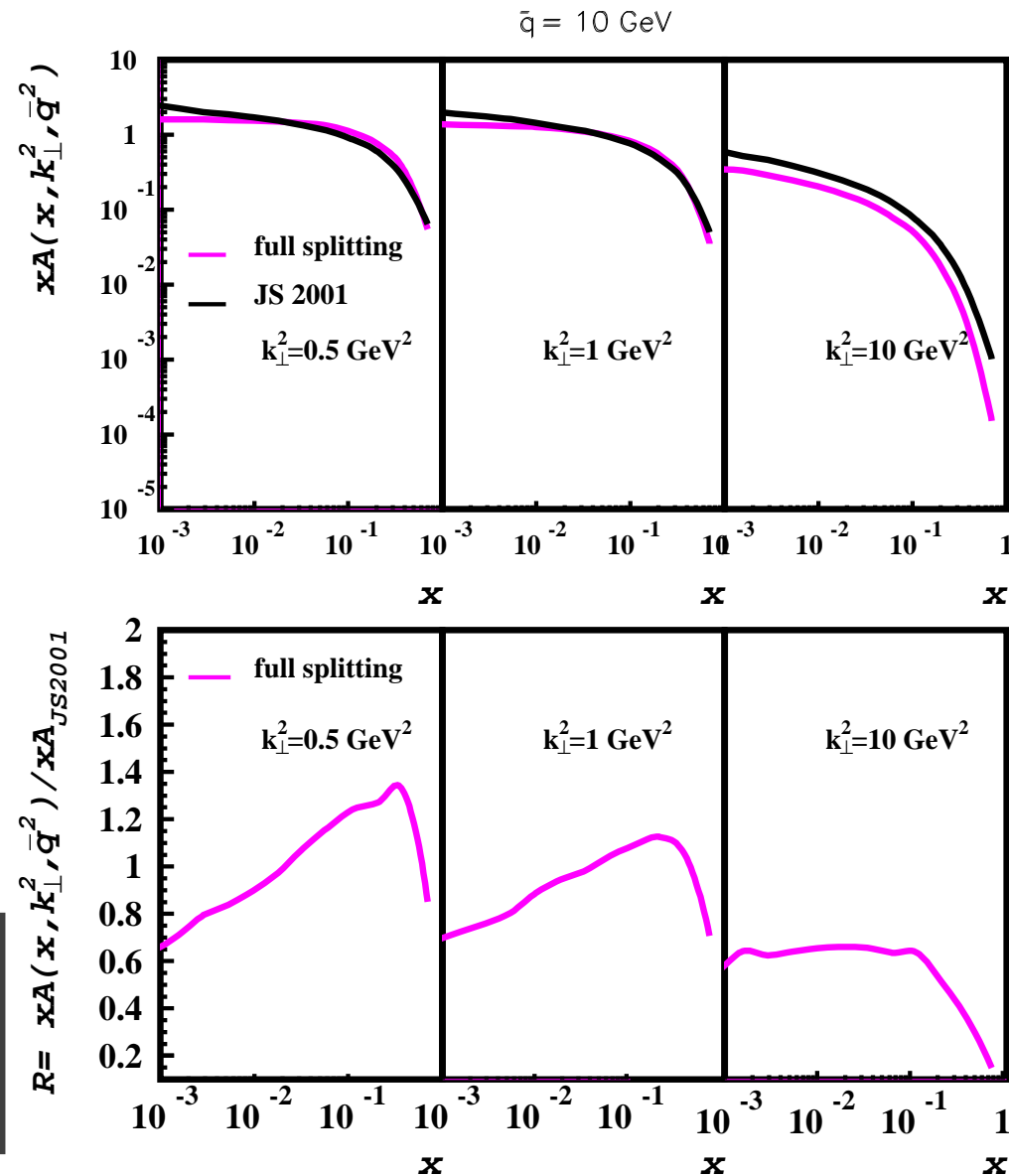
- new

$$P = \bar{\alpha}_s \left( \frac{(1-z)}{z} + \frac{z(1-z)}{2} \right) \Delta_{ns} + \bar{\alpha}_s \left( \frac{z}{1-z} + \frac{z(1-z)}{2} \right)$$

- need also new Sudakov

- new non-Sudakov

- ➔ gluon pdfs are different
- ➔ effect of non-sing. terms visible

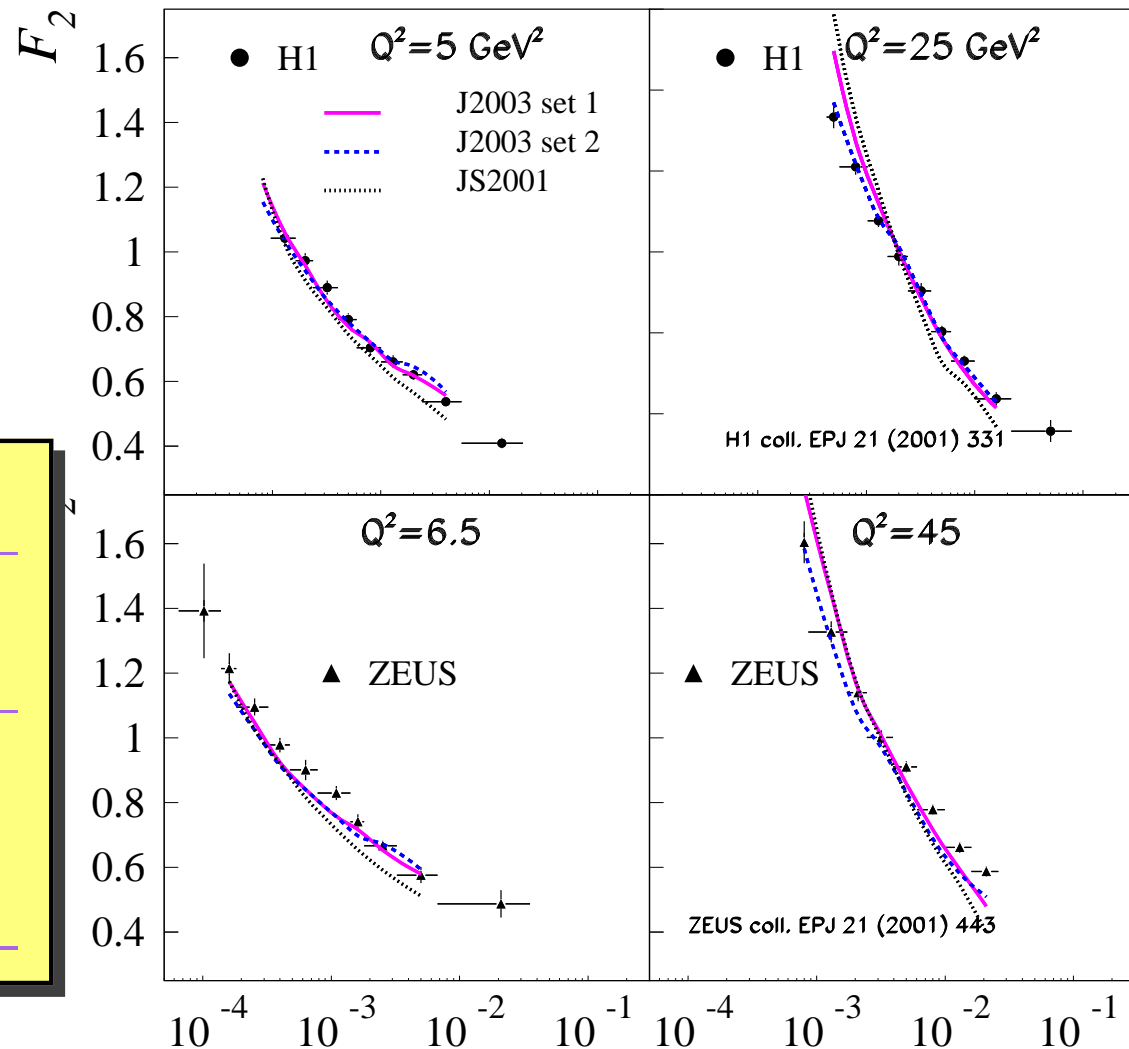


# Precision fits to $F_2(x, Q^2)$

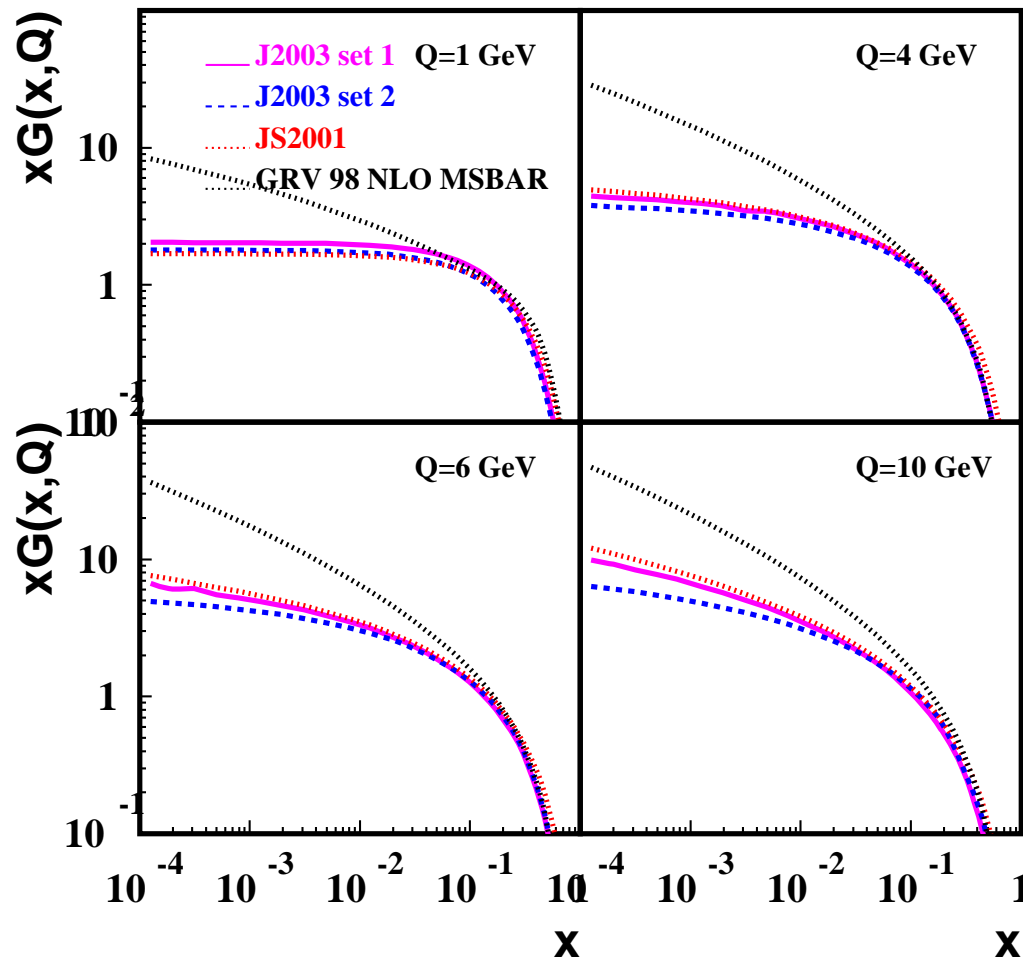
With  $\sigma = \int dk_t^2 dx_g \mathcal{A}(x_g, k_t^2, \bar{q}) \sigma(\gamma^* g^* \rightarrow q\bar{q})$  fit  $F_2(x, Q^2)$

- more precise data:
  - H1** NPB 470 (1996) 3., EPJ 21 (2001) 331.
  - ZEUS** ZPC 72 (1996) 399., EPJ 21 (2001) 443.
- fit  $Q^2 > 4.5 \text{ GeV}^2, x < 0.005$
- small  $k_t$  - region ?
- full splitting function ?

Fits to $F_2(x, Q^2)$		
set	$k_t^{cut}$ (GeV)	$\chi^2/ndf$ ndf = 248
$k_t^{cut} = Q_0$	<b>1.33</b>	<b>1.29</b>
full splitting	<b>1.18</b>	<b>1.18</b>
JS2001	<b>0.25</b>	<b>4.8</b>



# CCFM unintegrated gluon density - integrated -



**CCFM gluon integrated over  $k_t$**   

$$\int_0^{\bar{q}} dk_t^2 x \mathcal{A}(x, k_t, \bar{q}) = xG(x, \bar{q})$$

**CCFM gluon:**

- at starting scale  $q = 1$  GeV  
**flat !!!**
- small  $x$  rise  
generated perturbatively

**DGLAP gluon:**

- rise at small  $x$  at low scales

**Remember: gluon density is no observable, only cross sections !!!**

# One loop fits to $F_2(x, Q^2)$

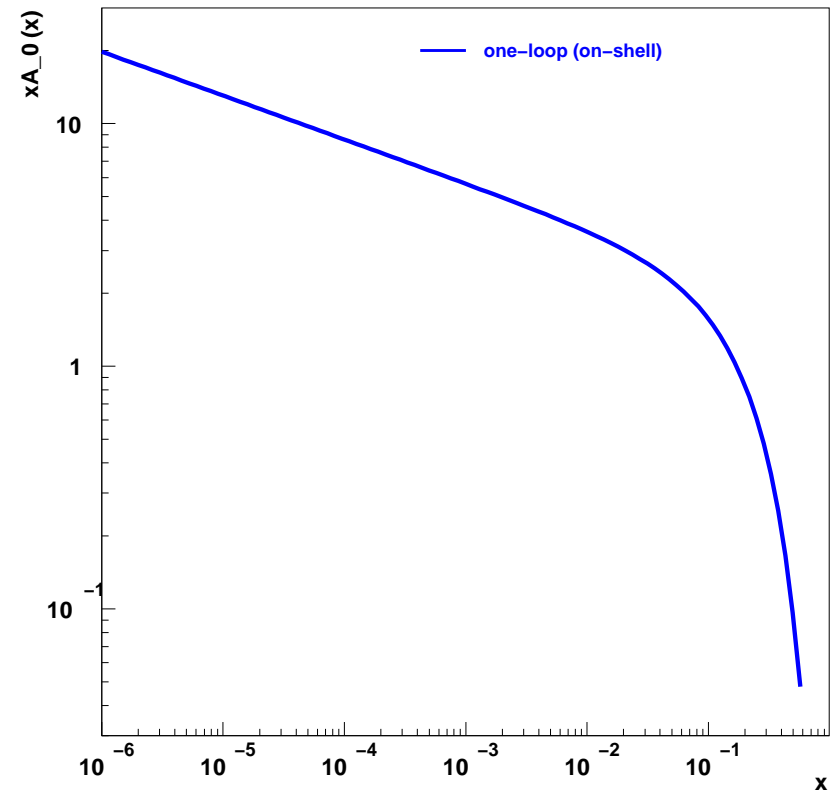
use one-loop (DGLAP) splitting fct.  
replace angular ordering by  $q$  ordering  
use only gluons

$m_q = 0.250$  GeV,  $m_c = 1.5$  GeV

fit  $Q^2 > 4.5$  GeV<sup>2</sup>,  $x < 0.005$

use standard on-shell BGF ME

initial gluon density



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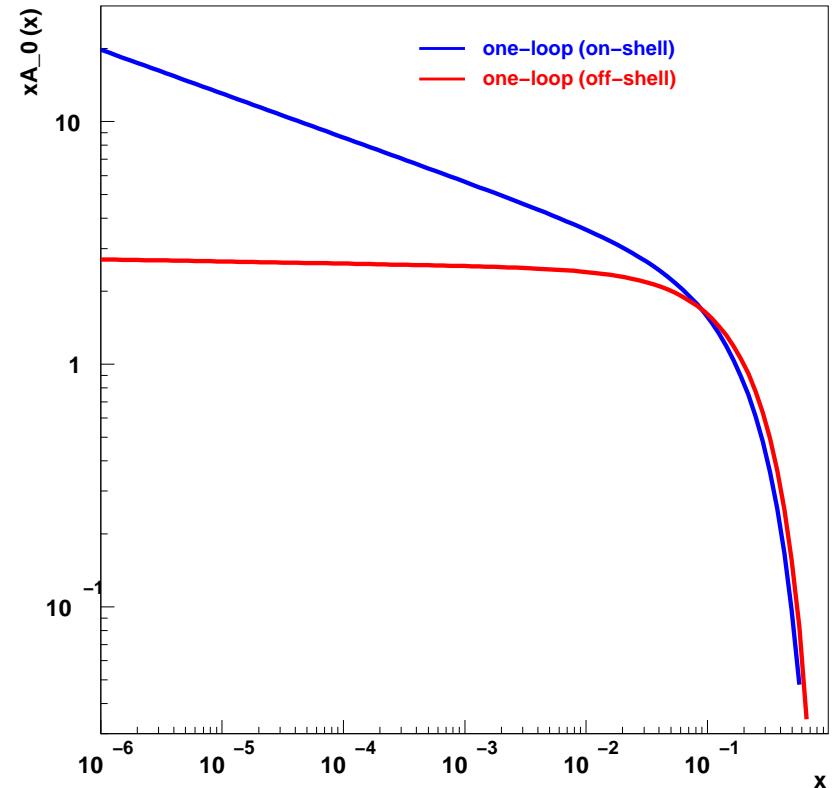
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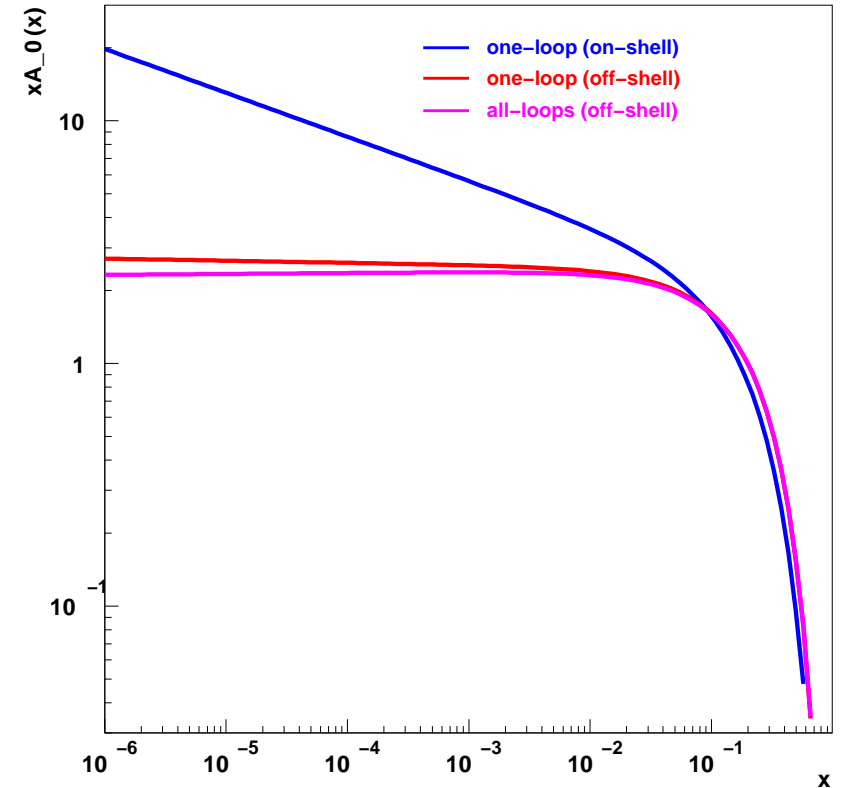
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use standard on-shell BGF ME

use off-shell BGF ME

use off-shell BGF ME and all loops  
with angular ordering  
and CCFM splitting fct.

initial gluon density



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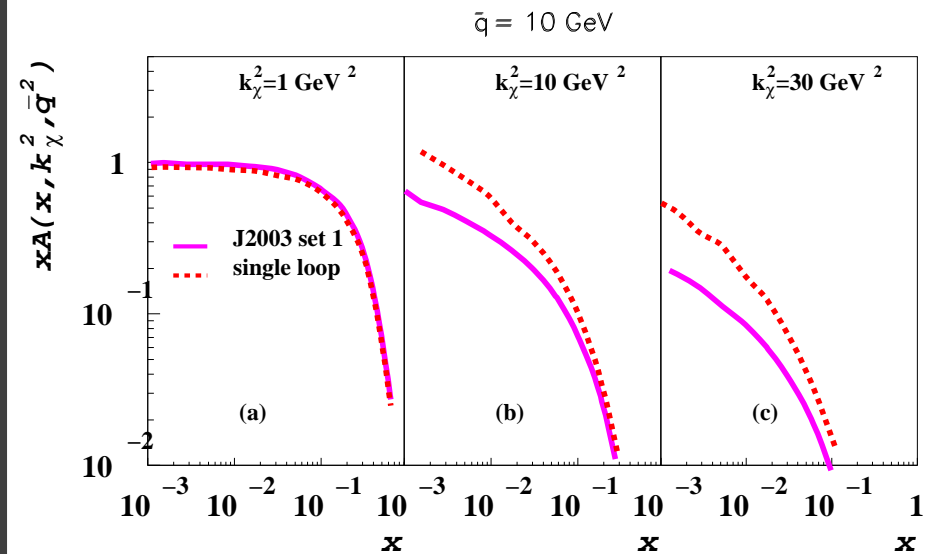
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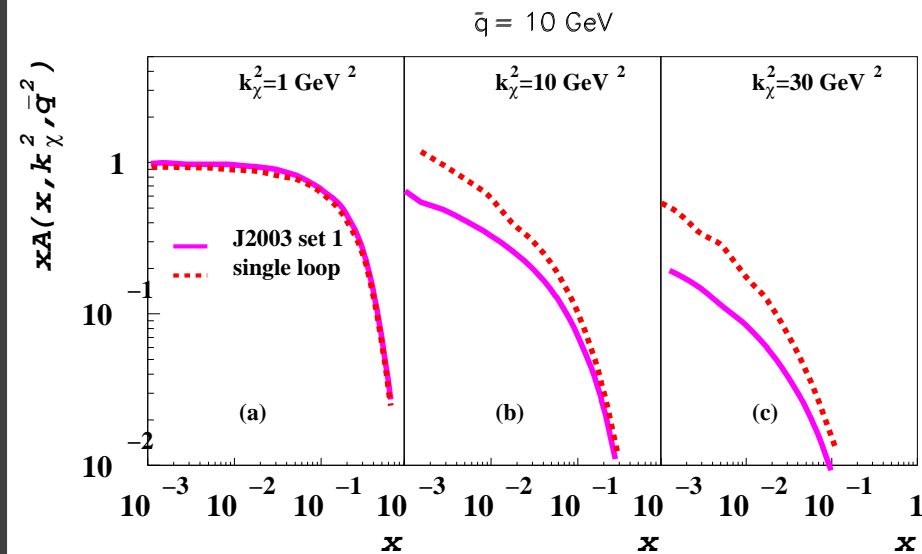
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with angular ordering  
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● off-shell ME has large effect

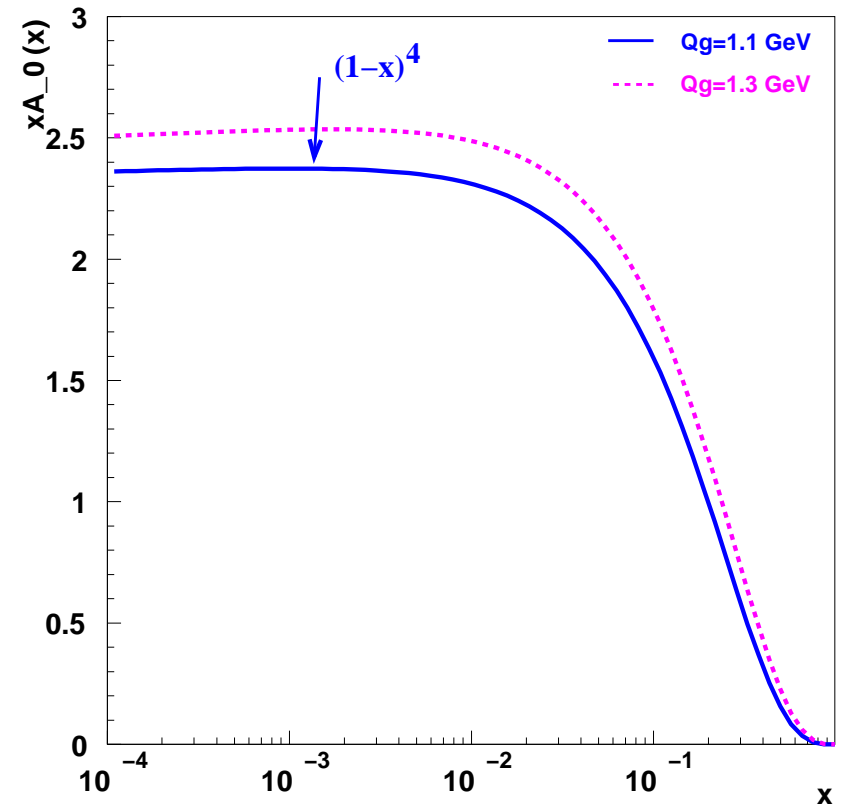
● ↪ simulates NNLO for  $F_2$



# All loop fits to $F_2(x, Q^2)$

- all-loop splitting function
- full angular ordering
- use off-shell BGF ME, only gluons
- $m_q = 0.250$  GeV,  $m_c = 1.5$  GeV
- fit  $F_2$  for  $Q^2 > 4.5$  GeV<sup>2</sup>,  $x < 0.005$
- similar  $\chi^2$  for different intrinsic  $k_t$   
Gaussian: 0.3 - 0.9 GeV
- sensitivity on starting scale

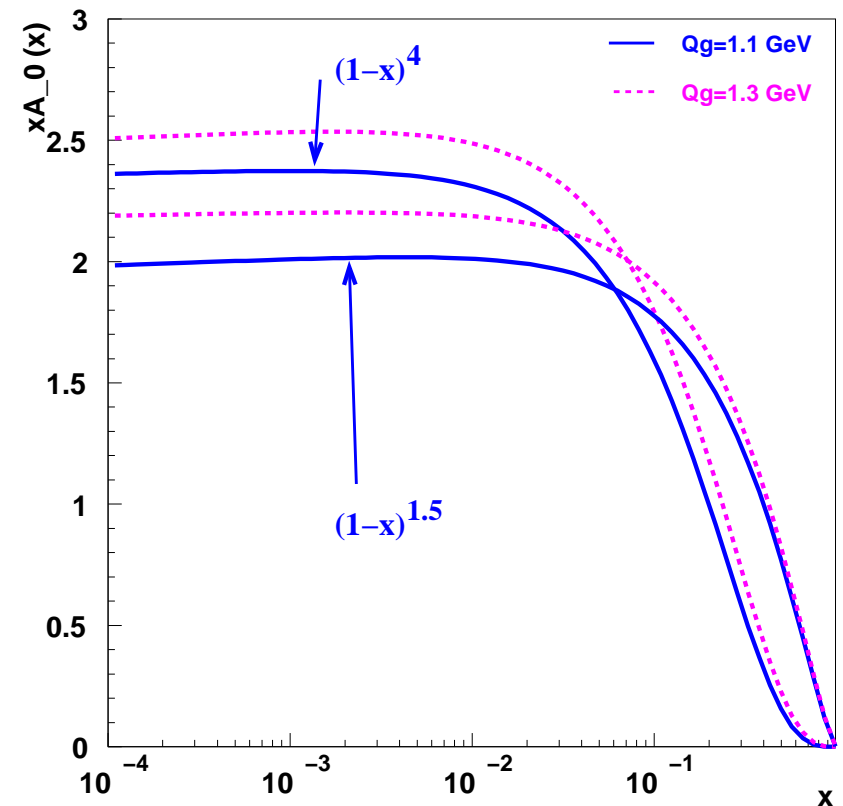
initial gluon density



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- similar  $\chi^2$  for different intrinsic  $k_t$   
Gaussian: 0.3 - 0.9 GeV
- sensitivity on starting scale
- find 2 solutions (at least)...
- large  $x$  not really constrained
- small  $x$  similar

initial gluon density

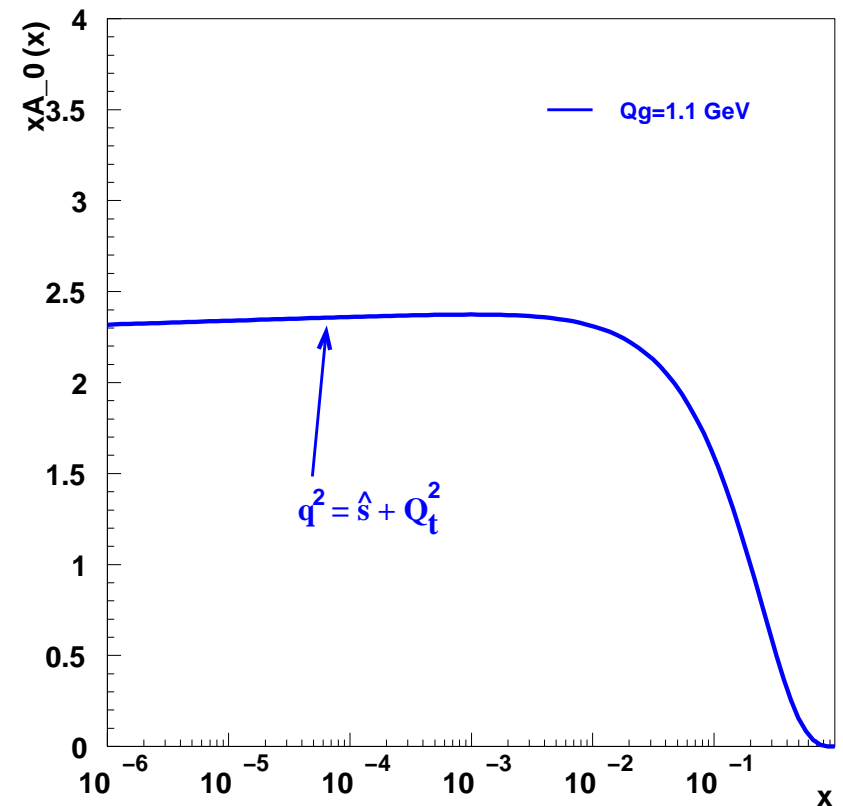


# All loop fits to $F_2(x, Q^2)$ choice of factorization scale ...

- **CCFM: ordering in rapidity of emitted gluons**
- $z_{i-1}q_{i-1} < q_i < \bar{q}$  with  

$$q_i = x_{i-1} \sqrt{s \xi_i} = \frac{p_{ti}}{1-z_i}$$
- **what is factorization scale  $\bar{q}$ ?**
- $\bar{q}^2 = x_g^{(2)} \Xi s = \hat{s} + Q_t^2$
- **or related to  $p_t$  of quarks ?**  $\frac{p_{ti}}{1-z_i} \ll \hat{s}$
- **fit  $F_2$  for  $Q^2 > 4.5 \text{ GeV}^2$ ,  $x < 0.005$**

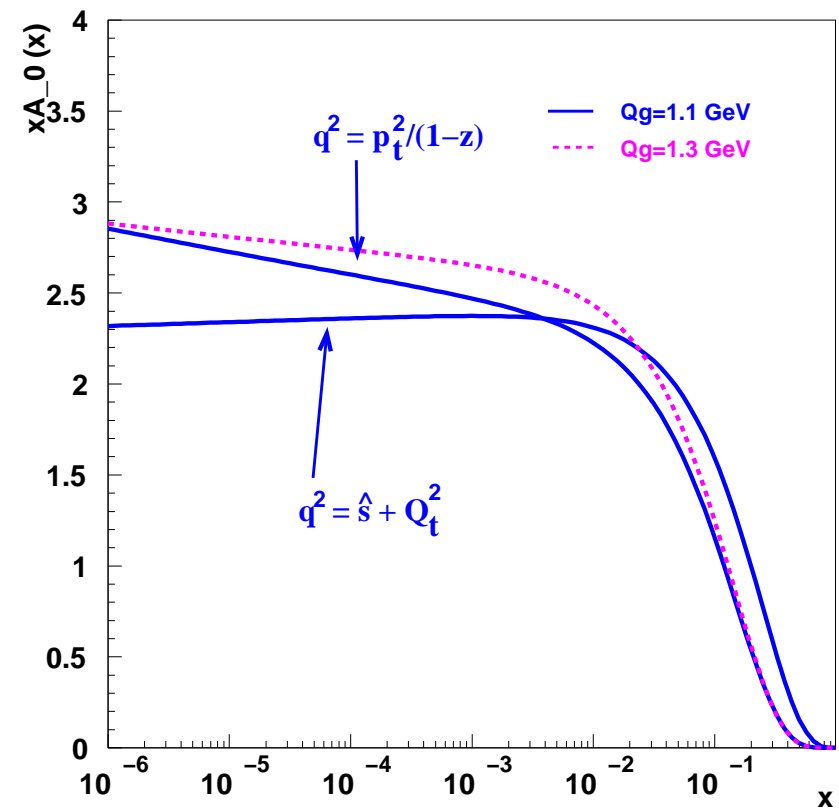
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- **or related to  $p_t$  of quarks ?  $\frac{p_{ti}}{1-z_i} \ll \hat{s}$**
- **fit  $F_2$  for  $Q^2 > 4.5 \text{ GeV}^2$ ,  $x < 0.005$**
- **change of small  $x$  behavior...**
- **shorter evolution ladder**

initial gluon density



# Effect of initial condition — small $k_t$ - region

$$\begin{aligned} \mathcal{A}(x, k_t, \bar{q}) &= \mathcal{A}_0(x, k_t) \Delta_s(\bar{q}, Q_0) + \\ &\int \frac{dz}{z} \int \frac{d^2 q}{\pi q^2} \Theta(\bar{q} - zq) \Delta_s(\bar{q}, zq) \cdot \\ &\tilde{P}(z, q, k_t) \mathcal{A}\left(\frac{x}{z}, k'_t, q\right) \end{aligned}$$

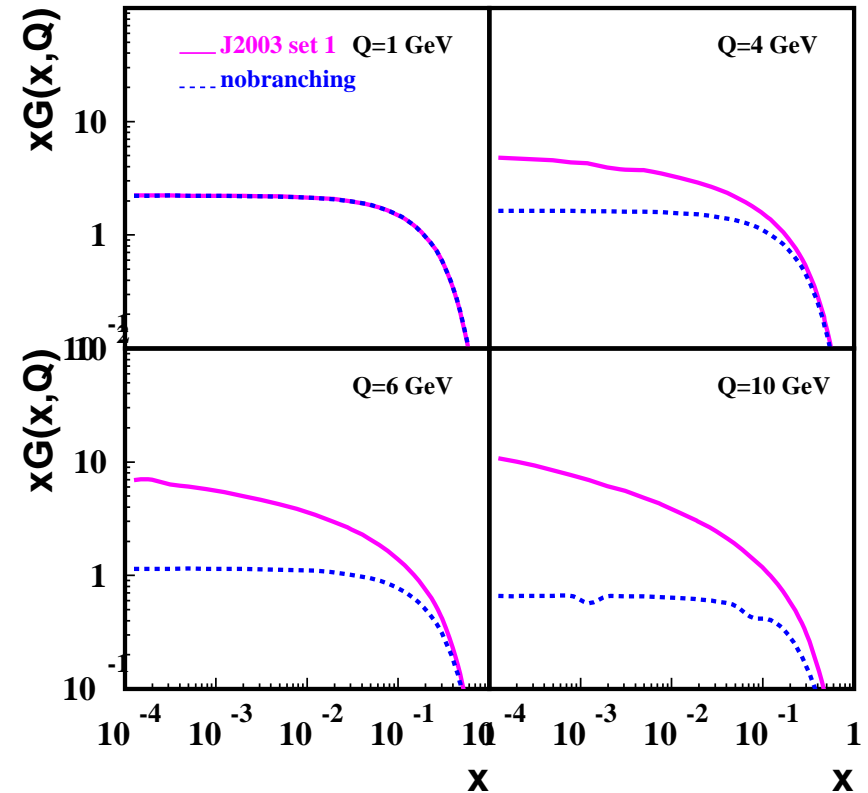
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integrated pdf:  
effect of evolution and initial condition  
not clearly separated ...

where is:

- small  $k_t$  region ?
- saturation region ?

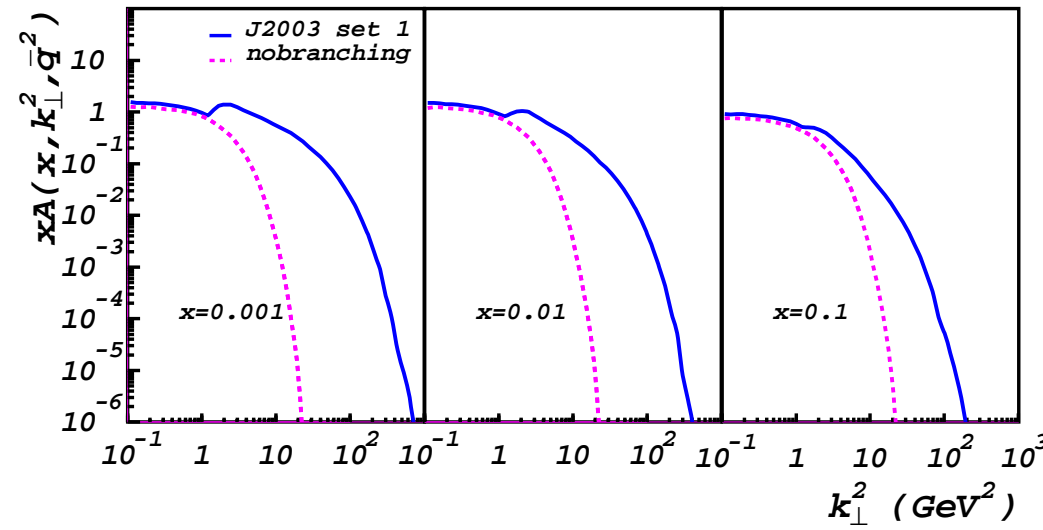
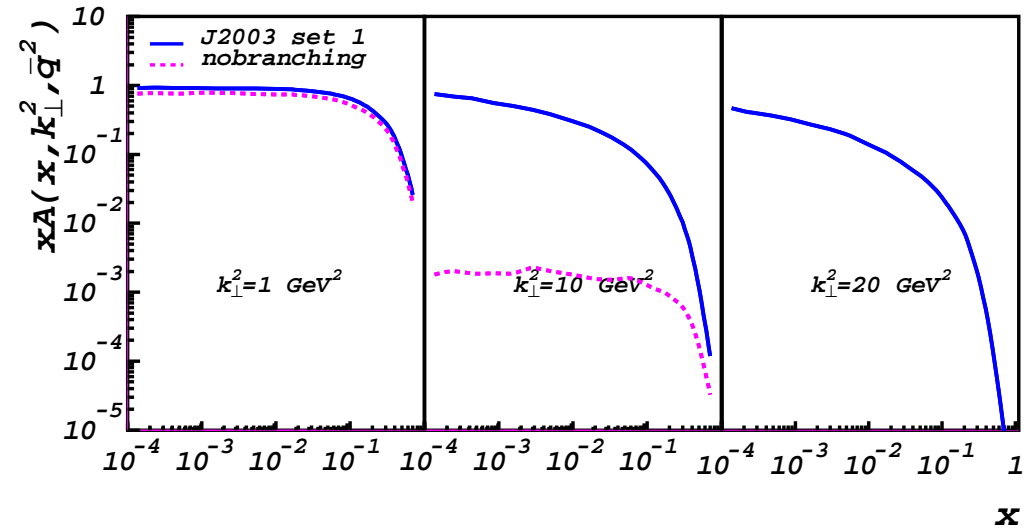


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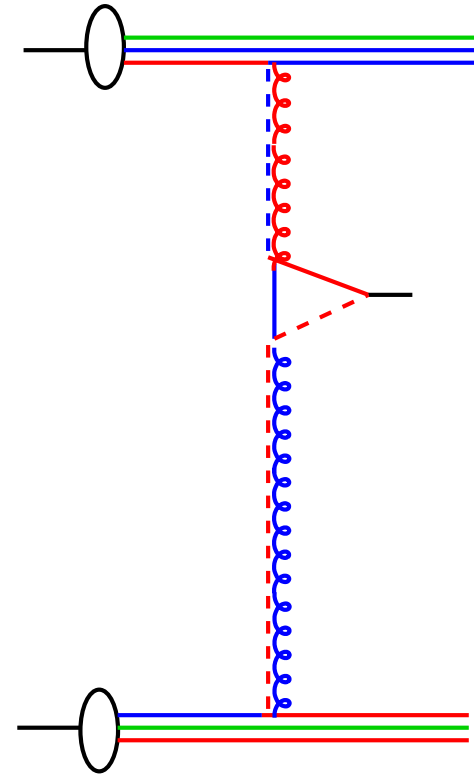
**Advantage of uPDF:**  
initial condition clearly seen  
in small  $k_t$  region  
even at large scales  $\bar{q}$

$\bar{q} = 10 \text{ GeV}$



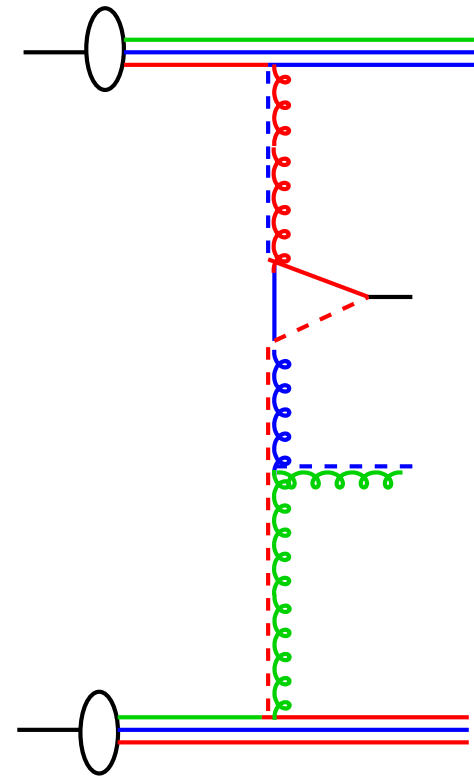
# Higgs production at LHC

- search for Higgs ...
- basic process:  
LO  $\mathcal{O}(\alpha_s^2)$   $gg \rightarrow \text{Higgs}$



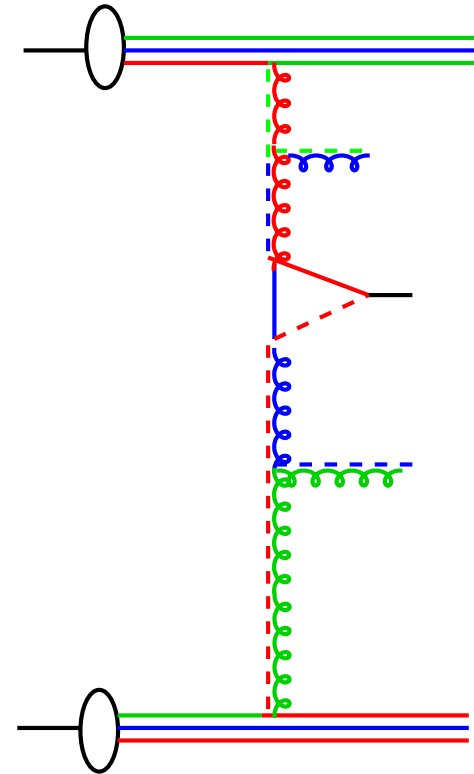
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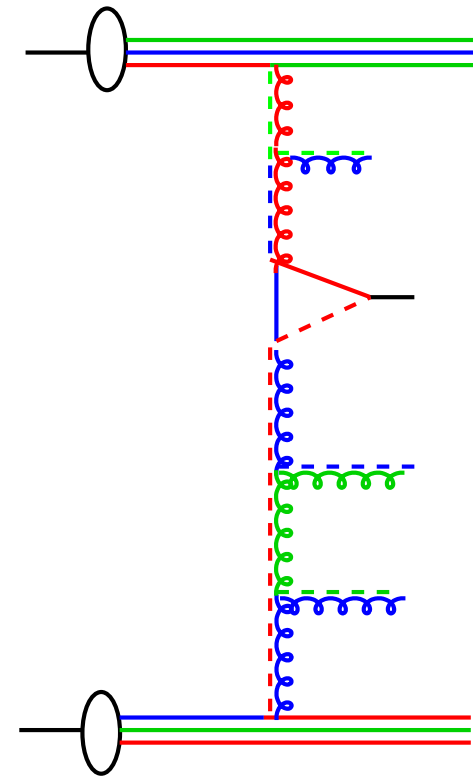
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- NNLO  $\mathcal{O}(\alpha_s^4)$  not yet calculated



# Higgs production at LHC

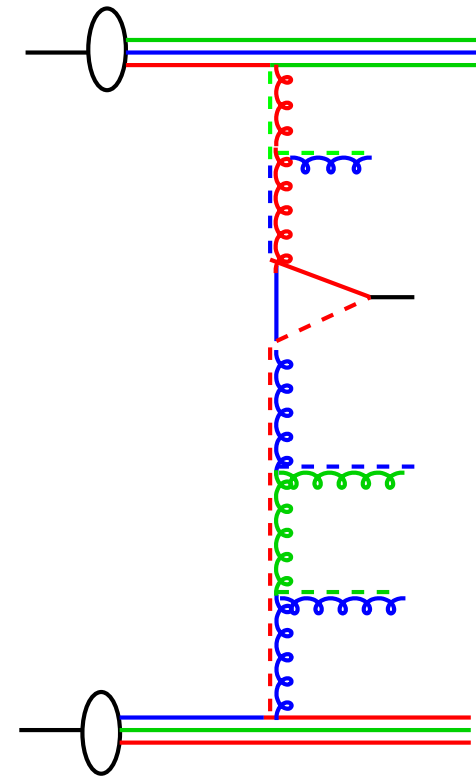
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- NNNLO  $\mathcal{O}(\alpha_s^5)$  not yet calculated
- available only: NLO + NNLL resummation....

Bozzi et al (PLB 564 (2003) 65, hep-ph/0302104)



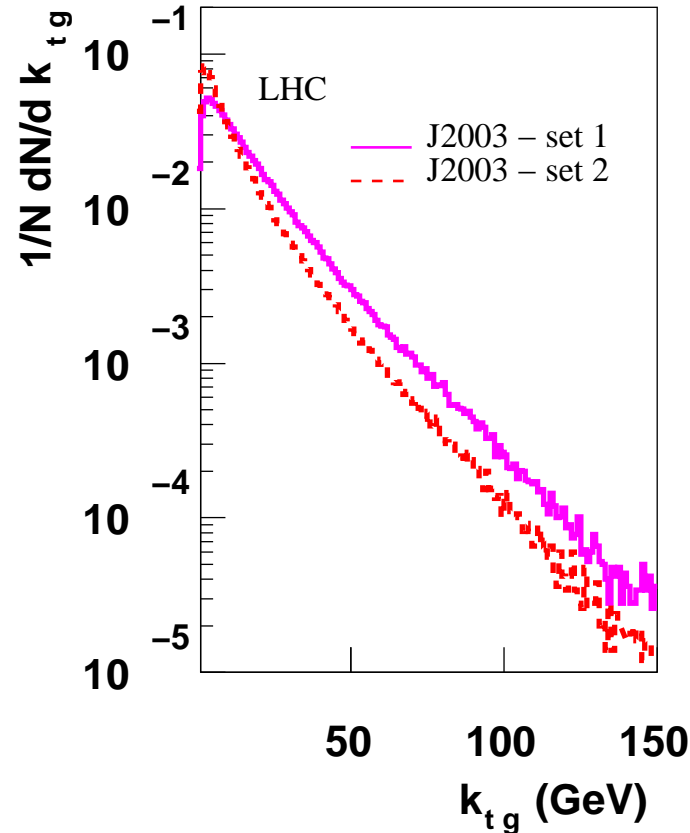
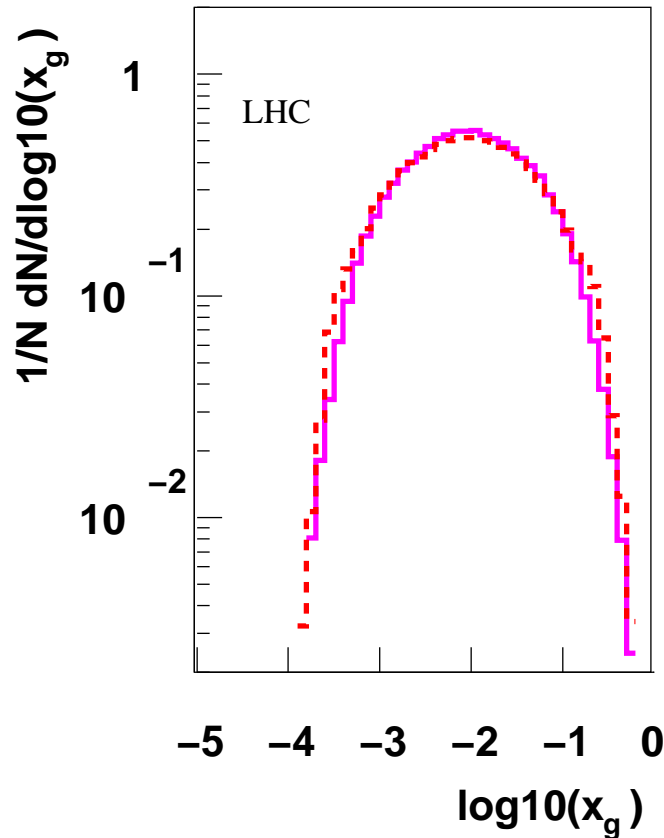
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Bozzi et al (PLB 564 (2003) 65, hep-ph/0302104)
- calculate  $gg \rightarrow \text{Higgs}$  in  $k_t$  factorisation
- small  $x$  approximation and for  $m_t \rightarrow \infty$   
F. Hautmann, PLB 535 (2002) 159
- obtain NNLO correction to gluon-gluon  
x-section for  $x \ll 1$
- estimate higher order corrections ...
- get resummation to all orders



# Higgs production at LHC - a typical $k_t$ -factorization process ??????

- $k_t$ -factorization:  $E_{\text{gluon}} \sim k_t$

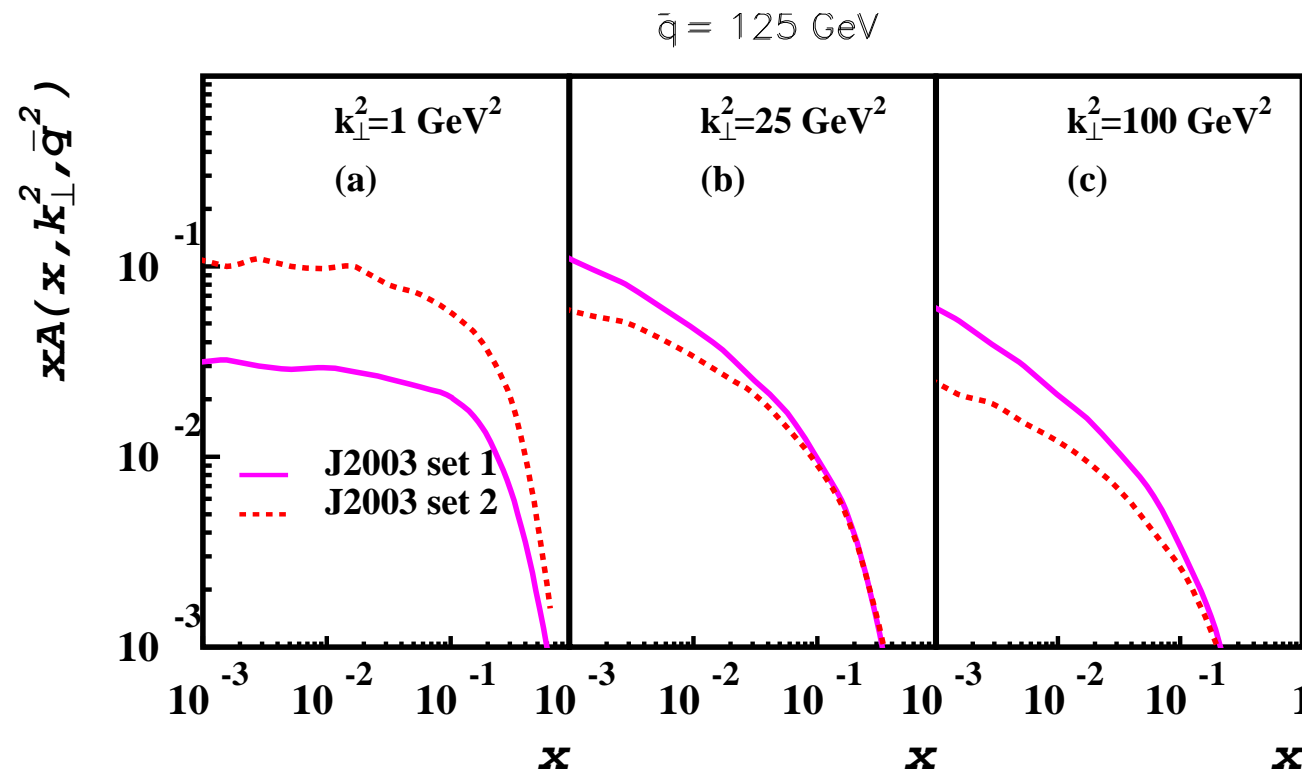


- two sets of unintegrated gluon distr.
- both work at HERA
- different  $k_t$ 's

- $E_{\text{gluon}} \sim 10^{-2} \dots 10^{-3} \cdot 7000 \text{ GeV}$  compared to mean  $k_t \sim \mathcal{O}(15 \text{ GeV})$
- $k_t$  cannot be neglected .... as usually done in DGLAP...

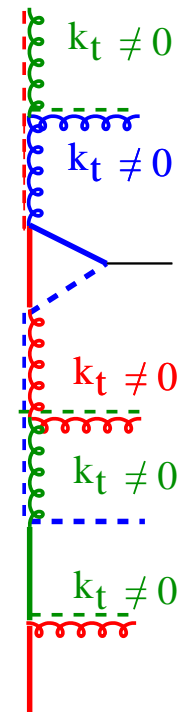
# Higgs production at LHC

- gluon density at  $\bar{q} = m_{\text{higgs}}$
- on gluon chains included



- gluon densities different at large scales ....
- include also quark chains ???

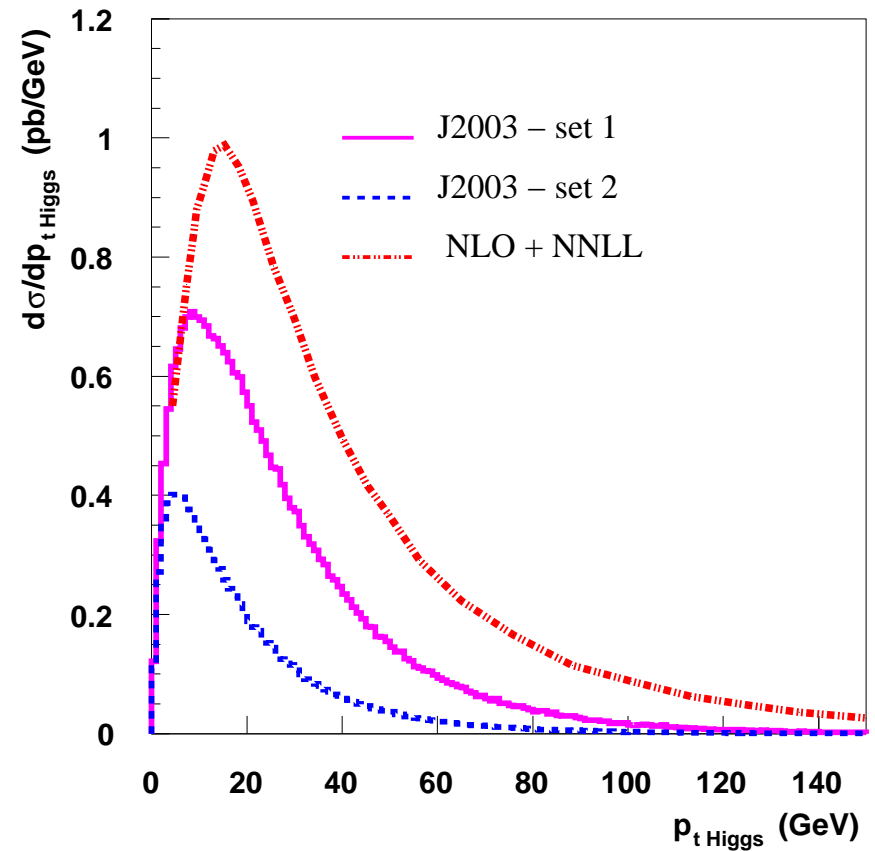
● what about ?



● unintegrated quarks ?

# Higgs production at LHC

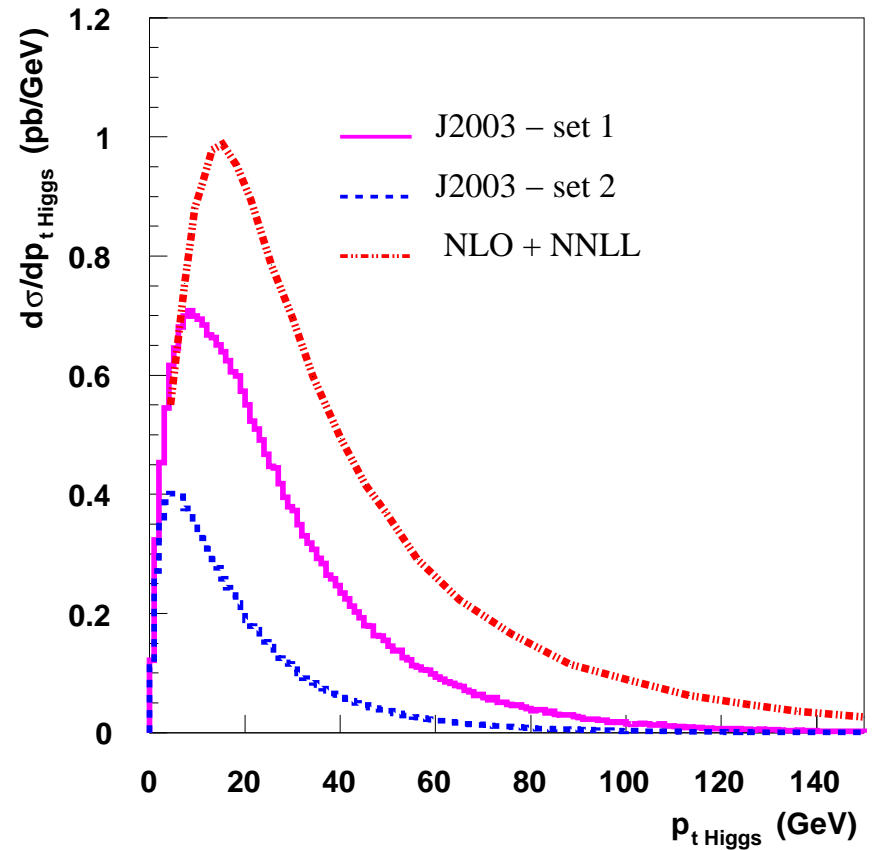
- use **new** matrix element (off-shell)  
F. Hautmann, PLB 535 (2002) 159
- calculate  $q_T$  spectrum with CCFM unintegrated gluon:  
two sets, both determined from HERA
- sensitive to trans. mom. of gluons



- new approach to calculate Higgs prod. at LHC
- important for x-section estimate
- different result than NLO ...
- better constrain unintegrated gluon ...

# Higgs production at LHC

- use **new** matrix element (off-shell)  
F. Hautmann, PLB 535 (2002) 159
- calculate  $q_T$  spectrum with CCFM unintegrated gluon:  
two sets, both determined from HERA
- sensitive to trans. mom. of gluons
- up to now only gluon initiated cascades
- **BUT**, what about quark initiated cascades?

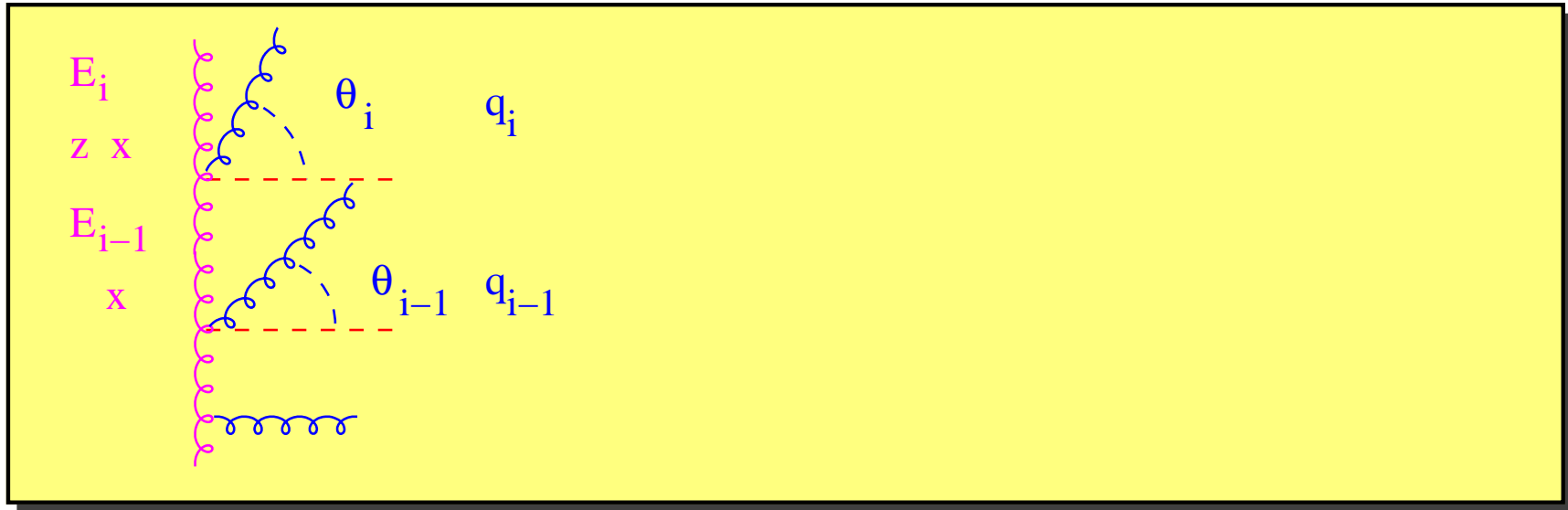


- **new approach to calculate Higgs prod. at LHC**
- **important for x-section estimate**
- **different result than NLO ...**
- **better constrain unintegrated gluon ...**

# Conclusions

- different limits of CCFM equation
  - ✚ one-loop – all loops
- treatment of matrix element important (on-shell–off-shell)
  - ✚ different small  $x$  behavior of starting distribution
- choice of factorization scale ✚ small  $x$  behavior
- uPDF important for study of small  $k_t$  effects...
  - ✚ starting distribution visible
  - ✚ search for saturation
- use uPDF for detailed study
- perform global fits (including quarks)
- $k_t$  - factorization:
  - ✚ applications also for LHC: Higgs

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:



- ordering in  $q$  (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL



**WOW**

for small  $z$  no restriction in  $q$ :  random walk in  $q$

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:

$$p_{ti} = |q_i^0| \sin \Theta_i, z = \frac{E_i}{E_{i-1}}$$

$$E_{i-1} = E_i + q_i^0 = z E_{i-1} + q_i^0, \leftarrow q_i^0 = (1 - z) E_{i-1}$$

$$p_{ti} = q_i^0 \sin \Theta_i \simeq (1 - z) E_{i-1} \Theta_i$$

$$\frac{p_{ti}}{1 - z} \simeq E_{i-1} \Theta_i$$

with:  $q_i = \frac{p_{ti}}{1 - z_i} \leftarrow \Theta_i = \frac{q_i}{E_{i-1}}$  and  $\Theta_{i+1} = \frac{q_{i+1}}{E_i}$

- ordering in  $q$  (DGLAP) implies also angular ordering
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**WOW**

for small  $z$  no restriction in  $q$ : random walk in  $q$

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:

**in lab. frame**

$$\Theta_{i+1} > \Theta_i$$

$$q_{i+1} > z_i q_i$$

with  $q = \frac{p_t}{1-z}$

- ordering in  $q$  (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL

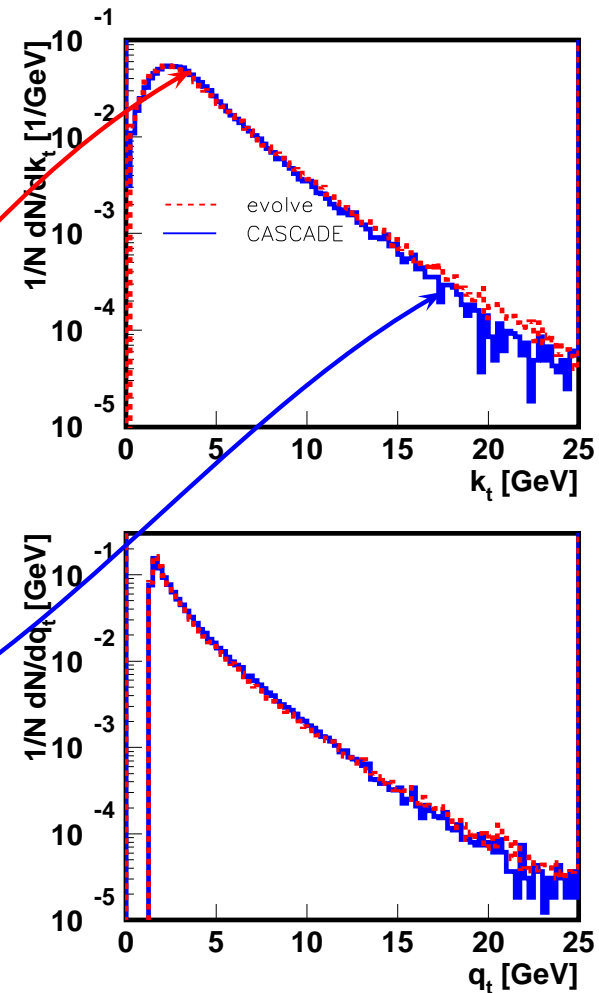


**WOW**

for small  $z$  no restriction in  $q$ :  random walk in  $q$

# Advantage of CCFM: parton emissions

- DGLAP or BFKL
- ☞ only inclusive predictions
- ☞ no info on emitted partons !!!
- CCFM treats explicitly:
  - partons emitted during cascade
  - color coherence
  - energy momentum conservation
- best to implement in MC generator
- ☞ compare **evolution** and MC
- CASCADE MC generator



**evolution** - MC parton shower comparison  
**never** shown for DGLAP type MC's!!!

# The Monte Carlo Generator CASCADE

- CCFM backward evolution implemented in MC generator **CASCADE** (<http://www.quark.lu.se/hannes/cascade>)
- initial state CCFM cascade with strict angular ordering
- off-shell hard scattering processes:
  - ☞  $\gamma g^* \rightarrow q\bar{q}, \gamma^* g^* \rightarrow Q\bar{Q}, \gamma g^* \rightarrow J/\psi g, \gamma\gamma \rightarrow Q\bar{Q}$
  - ☞  $g^* g^* \rightarrow q\bar{q}, g^* g^* \rightarrow Q\bar{Q}, g^* g^* \rightarrow h$
- *P*-remnant treatment like in PYTHIA (*q*-di-*q*, primordial  $k_t$ )
- final state parton showers added to quarks hadronization via JETSET/PYTHIA

**CASCADE** is MC implementation of CCFM  
for  $ep, ee, \gamma\gamma$  and also for  $p\bar{p}$

# New fit: small $k_t$ - region

- use H1 + ZEUS  $F_2$  data (from 94 and 96-97)
- fit for  $x < 0.005$   $Q^2 > 4.5 \text{ GeV}^2$
- fit  $Q_0$  and normalization in initial pdf  $x\mathcal{A}_0 = N(1-x)^4$

## Treatment of soft region

no  $k_t$  ordering  $\rightarrow$  diffusion into soft

- what about  $\alpha_s$  for  $k_t < k_t^{cut}$  in
- $\rightarrow$  saturation of x-section for  $k_t < k_t^{cut}$

## What is soft?

- JS2001  $k_t^{cut} > 0.25 \text{ GeV}$
- now  $k_t^{cut} > Q_0$
- similar to saturation scale  $Q_s \sim 1 \text{ GeV}$

