The QCD S–matrix in the high–energy limit

Based on E.I., A.H. Mueller, hep-ph/0309044 & hep-ph/0309276

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Motivation

- What is the high energy limit of the S–matrix for elastic dipole–dipole scattering in QCD ?

  The answer seems a priori obvious ! $S_Y \to 0$ when $Y \to \infty$

- What is the precise approach towards the black disk limit ?

- Which is/are the right formalism(s) to study this approach ?
  - *Leading–order formalism: "BFKL + unitarity corrections", fixed α_s*
  - *Color Dipole Picture (Mueller, 94)*
  - *Color Glass Condensate (MV, JIMWLK, BK)*

- What is the relation between CGC and color dipoles ?

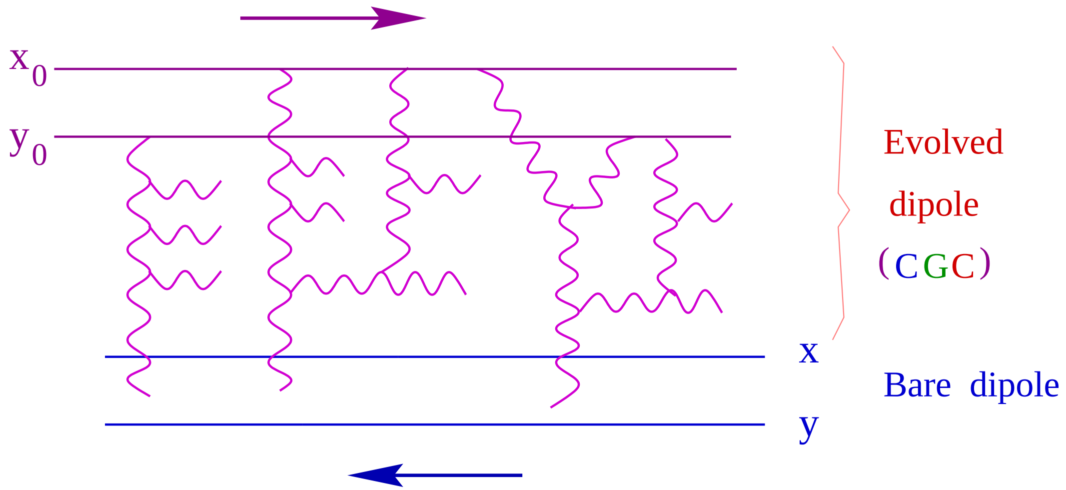
- How to go beyond the Kochegov equation ?

- What are the gluon configurations in the dipole wavefunction which control the high energy limit ?

- How do these configurations look like in various frames ?

Dipole–dipole scattering in the CGC

- In principle, the problem is well posed !
- CGC : an effective theory for the light–cone wavefunction of an energetic 'hadron' (here, dipole)
- Asymmetric ('dipole') frame : one of the dipole looks 'bare'
  $\Longrightarrow$ (elementary) dipole — color glass scattering:



$$S_Y(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{N_c}\Big\langle \mathrm{tr}\big(V^\dagger(\boldsymbol{x})V(\boldsymbol{y})\big)\Big\rangle_Y \equiv \int \mathrm{D}[\alpha]\, W_Y[\alpha]\, \frac{1}{N_c}\,\mathrm{tr}\Big(V^\dagger_{\boldsymbol{x}}[\alpha]\, V_{\boldsymbol{y}}[\alpha]\Big)$$

$$V^\dagger_{\boldsymbol{x}}[\alpha] \equiv \mathrm{P}\exp\Big(ig\int dx^-\alpha^a(x^-,\boldsymbol{x})t^a\Big)$$ ($\alpha_a = A^+_a$ for a left mover)

An elementary dipole as a color glass

▷ $W_Y[\alpha]$: solution to JIMWLK eq. with dipolar initial conditions

$Y=0$: $\alpha^a(\boldsymbol{z}) =$ the color field of an elementary dipole $(\boldsymbol{x}_0, \boldsymbol{y}_0)$

- Transverse positions are fixed: $\boldsymbol{x}_0$ for q and $\boldsymbol{y}_0$ for $\bar{q}$
- Color degrees of freedom must be averaged over (singlet)

$$W_0[\alpha] \,=\, \int \prod_a dQ^a \, \exp\left\{-\frac{Q^aQ^a}{2\lambda}\right\} \delta\left[\alpha^a(\boldsymbol{z}) - Q^a \mathcal{G}(\boldsymbol{z}|\boldsymbol{x}_0, \boldsymbol{y}_0)\right]$$

$Q^a \longleftrightarrow gt^a$: classical color charges

$$\langle Q^a\, Q^b\rangle \,=\, \delta^{ab}\lambda \longleftrightarrow\, (1/N_c)\mathrm{tr}(gt^a\, gt^b) \,\Longrightarrow\, \lambda \,\equiv\, g^2/2N_c$$

$$\mathcal{G}(\boldsymbol{z}|\boldsymbol{x}_0, \boldsymbol{y}_0) \,\equiv \Delta(\boldsymbol{z}-\boldsymbol{x}_0) - \Delta(\boldsymbol{z}-\boldsymbol{y}_0) \,=\, \frac{1}{4\pi}\,\ln\frac{(\boldsymbol{z}-\boldsymbol{y}_0)^2}{(\boldsymbol{z}-\boldsymbol{x}_0)^2}$$

▷ Check: dipole–dipole scattering at $Y=0$

$$T_0(\boldsymbol{x},\boldsymbol{y}|\boldsymbol{x}_0,\boldsymbol{y}_0) \,\equiv\, 1 - S_0(\boldsymbol{x},\boldsymbol{y}|\boldsymbol{x}_0,\boldsymbol{y}_0) \,\approx\, \#\,\alpha_s^2\left[\mathcal{G}(\boldsymbol{x}|\boldsymbol{x}_0,\boldsymbol{y}_0) - \mathcal{G}(\boldsymbol{y}|\boldsymbol{x}_0,\boldsymbol{y}_0)\right]^2$$

Non–linear evolution of the color glass

▷ $W_Y[\alpha]$: solution to JIMWLK eq. with dipolar initial conditions

$Y = 0$: the 'color glass' itself is an elementary dipole

$$\frac{\partial W_Y[\alpha]}{\partial Y} = \frac{1}{2}\int_{\boldsymbol{x},\boldsymbol{y}} \frac{\delta}{\delta\alpha_Y^a(\boldsymbol{x})}\,\eta^{ab}(\boldsymbol{x},\boldsymbol{y})[\alpha]\,\frac{\delta W_Y}{\delta\alpha_Y^b(\boldsymbol{y})}$$

(Jalilian-Marian, E.I., McLerran, Weigert, Leonidov, Kovner, 97 – 00)

▷ A functional Fokker–Planck eq. with "diffusion coefficient" :

$$\eta^{ab}(\boldsymbol{x},\boldsymbol{y})[\alpha] = \int \frac{d^2\boldsymbol{z}}{4\pi^3} \underbrace{\frac{(\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{y}-\boldsymbol{z})}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}}_{\text{BFKL-like}} \underbrace{(1-\tilde{V}_{\boldsymbol{z}}^\dagger \tilde{V}_{\boldsymbol{x}})^{fa}(1-\tilde{V}_{\boldsymbol{z}}^\dagger \tilde{V}_{\boldsymbol{y}})^{fb}}_{\text{adjoint repres.}}$$

▷ But how to solve the JIMWLK equation ?

▷ Recent lattice implementation *(Rummukainen, Weigert, 03)*

$\Longrightarrow$ Difficulties in achieving the continuum limit.

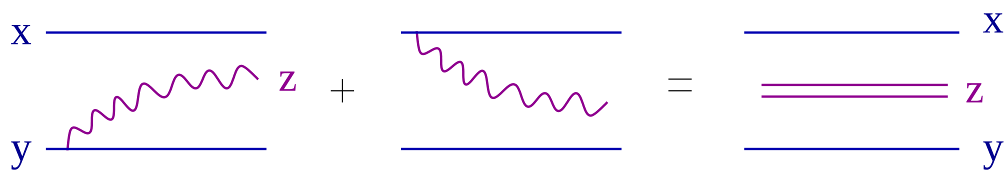
▷ Can one make analytic progress ?

▷ No closed evolution equation for the S–matrix !

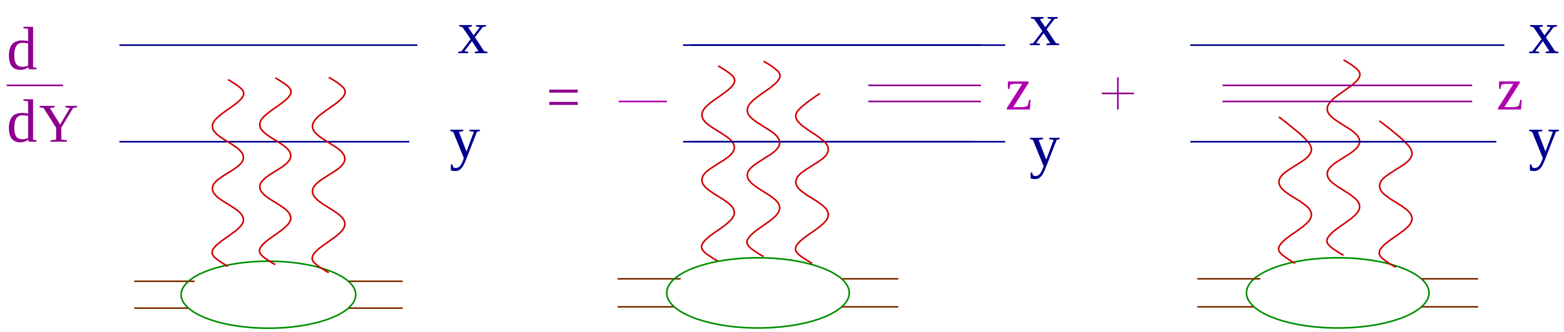
Balitsky–Kovchegov equation (1)

▷ An equation can be deduced for $S_Y = \mathrm{tr}(V_{\boldsymbol{x}}^{\dagger} V_{\boldsymbol{y}})/N_c$

Use the increment dY to accelerate the 'bare' dipole:



$$\frac{\partial}{\partial Y}\langle \mathrm{tr}(V_{\boldsymbol{x}}^{\dagger} V_{\boldsymbol{y}})\rangle = \alpha_s \int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}\Big\langle -N_c \underbrace{\mathrm{tr}(V_{\boldsymbol{x}}^{\dagger} V_{\boldsymbol{y}})}_{\text{2-point ftion}} + \underbrace{\mathrm{tr}(V_{\boldsymbol{x}}^{\dagger} V_{\boldsymbol{z}})\,\mathrm{tr}(V_{\boldsymbol{z}}^{\dagger} V_{\boldsymbol{y}})}_{\text{3-point ftion}}\Big\rangle$$



▷ But this is only the first equation in an infinite hierarchy ! *(Balitsky, 96)*

Balitsky–Kovchegov equation (2)

▷ Assume : Uncorrelated color fields + Large $N_c \Longrightarrow$

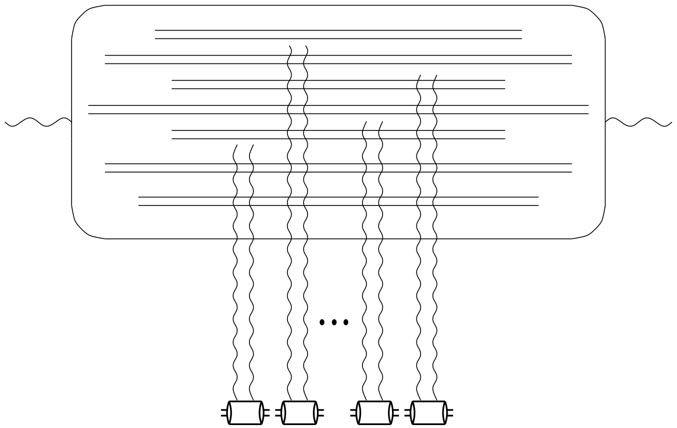
$$\langle \mathrm{tr}(V_{\boldsymbol{x}}^\dagger V_{\boldsymbol{z}})\, \mathrm{tr}(V_{\boldsymbol{z}}^\dagger V_{\boldsymbol{y}})\rangle \approx \langle \mathrm{tr}(V_{\boldsymbol{x}}^\dagger V_{\boldsymbol{z}})\rangle \langle \mathrm{tr}(V_{\boldsymbol{z}}^\dagger V_{\boldsymbol{y}})\rangle \Longrightarrow$$ Closed eq. *(Kovchegov, 99)*

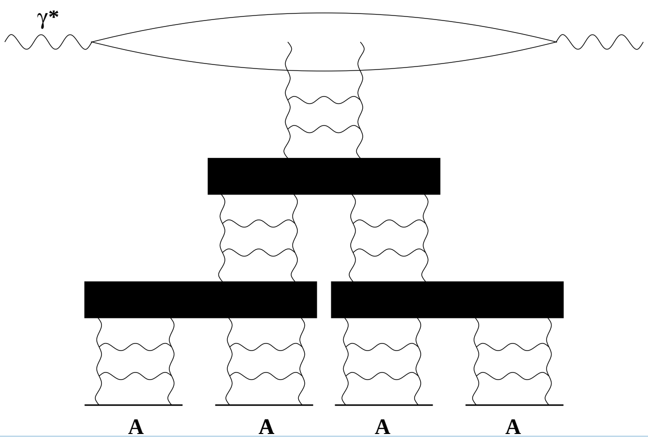
▷ Bad approximation for dipole–dipole scattering !

▷ Kovchegov' original derivation :

- Dipole–nucleus ($A \gg 1$) scattering, in the nucleus rest frame
- Dipole picture *(BFKL + large N_c)* for the dipole wavefunction

  $\Longrightarrow$ Valid so long as $\alpha_s^2\, n(Y) \sim \alpha_s^2\, \mathrm{e}^{(\alpha_{\mathbb{P}}-1)Y} \ll 1$
- High density of uncorrelated color charges in the nucleus

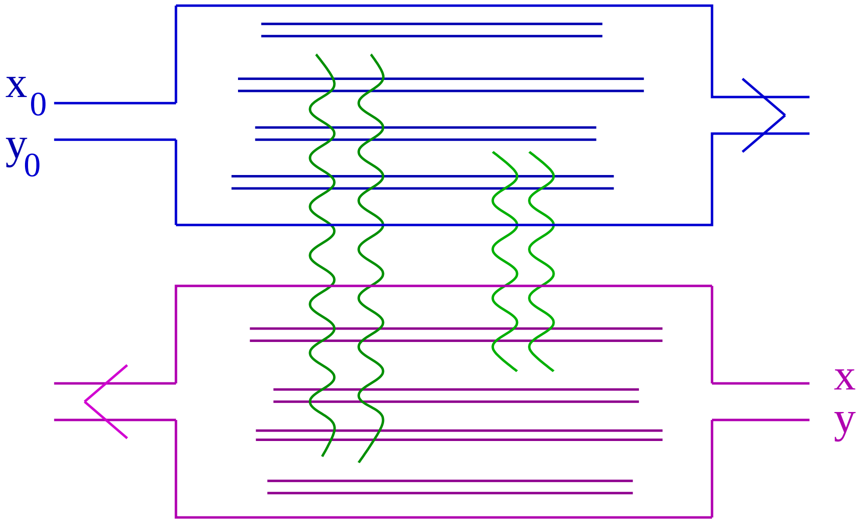




Color Dipole Picture *(Mueller, 94)*

▷ Dipole–dipole scattering in the center–of–mass frame

- Dipole (BFKL) picture for the wavefunctions: no saturation
- Simultaneous scattering of several pairs of dipoles



▷ Valid so long as $\alpha_s^2\, n(Y/2) \;\sim\; \alpha_s^2\, \mathrm{e}^{(\alpha_{\mathbb{P}}-1)Y/2} \;\ll\; 1$, or $Y \ll Y_c$

▷ Unitarity corrections $\sim \mathcal{O}(1)$ when $(\alpha_s\, n(Y/2))^2 \sim 1$, or $Y \gtrsim Y_c/2$

$$Y_c/2 \lesssim Y \;\ll Y_c = \frac{2}{\alpha_{\mathbb{P}}-1}\,\ln\frac{1}{\alpha_s^2}$$

▷ No closed equation, but simple numerical implementation *(Salam, 96)*

A new factorization in the framework of CGC

▷ Is this consistent/equivalent with the CGC ?

▷ Center–of–mass scattering between two non–saturated color glasses

$$S_Y = \underbrace{\int \mathrm{D}[\alpha_R]\, W_{Y/2}[\alpha_R]}_{\text{right mover}} \underbrace{\int \mathrm{D}[\alpha_L]\, W_{Y/2}[\alpha_L]}_{\text{left mover}} \underbrace{\exp\left\{\mathrm{i}\int d^2\boldsymbol{z} \nabla^i \alpha_L^a(\boldsymbol{z}) \nabla^i \alpha_R^a(\boldsymbol{z})\right\}}_{\text{eikonal interaction}}$$

▷ $W_Y[\alpha_R]$: the color–glass wavefunction in the BFKL approx. :

Solution to the weak field & large–N_c version of JIMWLK eq.

$$1 - \tilde{V}_{\boldsymbol{z}}^{\dagger} \tilde{V}_{\boldsymbol{x}} \approx ig\big(\alpha(\boldsymbol{x}) - \alpha(\boldsymbol{z})\big)$$

$$S_Y = \sum_{N,N'=1}^{\infty} \int d\Gamma_N P_N(Y/2) \int d\Gamma_{N'} P_{N'}(Y/2) \exp\left\{-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N'} \big[T_0(i|j)\big]^2\right\}$$

plus (ordinary) evolution equations for $P_N(\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_{N-1}|\boldsymbol{x}_0, \boldsymbol{y}_0, Y)$

▷ The same result as in the Color Dipole Picture

▷ Unveils some of the complexity of the CGC wavefunction

What is the high–energy limit of the S–matrix ?

▷ Numerical simulations within the color dipole picture (COM frame) *(Salam, 96; Mueller & Salam, 96)*

$$S_Y(r_0, r_0) \sim \mathrm{e}^{-\kappa \bar{\alpha}_s^2 Y^2}, \qquad \bar{\alpha}_s = \alpha_s N_c/\pi, \qquad \kappa \approx 0.72$$

Very different from incoherent multiple scattering !

▷ Typical configurations would give a much smaller result !

$$S_Y \sim \exp\left\{-\kappa_0 \bar{\alpha}_s^2 n^2(Y/2)\right\} \quad \text{with} \quad n^2(Y/2) \sim \mathrm{e}^{(\alpha_{\mathbb{P}}-1)Y}$$

▷ The BK equation is almost right ! *(Levin & Tuchin, 01)*

$$S_Y \simeq \mathrm{e}^{-\frac{c}{2}\bar{\alpha}_s^2 (Y-Y_0)^2} S_{Y_0}, \qquad c \simeq 4.883 > 2\kappa$$

Dipole frame: BK equation picks up the typical configurations !

c : the saturation exponent: $Q_s^2(Y) \simeq \frac{1}{r_0^2}\, \mathrm{e}^{c\bar{\alpha}_s(Y-Y_0)}$

Typical vs. rare configurations

▷ Typical configurations : $P_N(Y) \sim 1$ for $N \sim n(Y) \sim \ \mathrm{e}^{(\alpha_{\mathbb{P}}-1)Y}$

▷ Rare configurations :

- Fewer gluons : $N \ll n(Y) \Longrightarrow P_N(Y) \ll 1$
- ... but also less scattering $\Longrightarrow$ larger contributions to S !

▷ The same physical configurations can be “typical" or “rare" depending upon the frame ! E.g. : BK equation

▷ The optimal configurations are rare in any frame !

▷ No systematic way to construct the optimal configurations

- Minimize the # of gluons (to have a larger S)
  $\Longrightarrow$ suppress the evolution !
- but maximize the probability of the ensuing configuration
  $\Longrightarrow$ minimize the suppression !

Our best configurations: $c \to c/2 \approx 2.44 \ > \ 2\kappa \approx 1.44$?

▷ See also Levin & Kozlov (04) and the next talk by Kozlov

Optimal configuration in the dipole frame

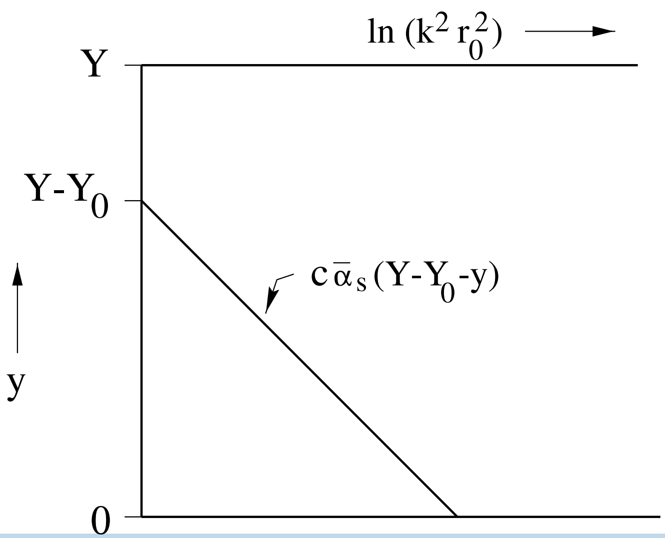
▷ Y_0 : rapidity gap for the onset of 'blackness' : $Q_s^2(Y_0) = \frac{1}{r_0^2}$

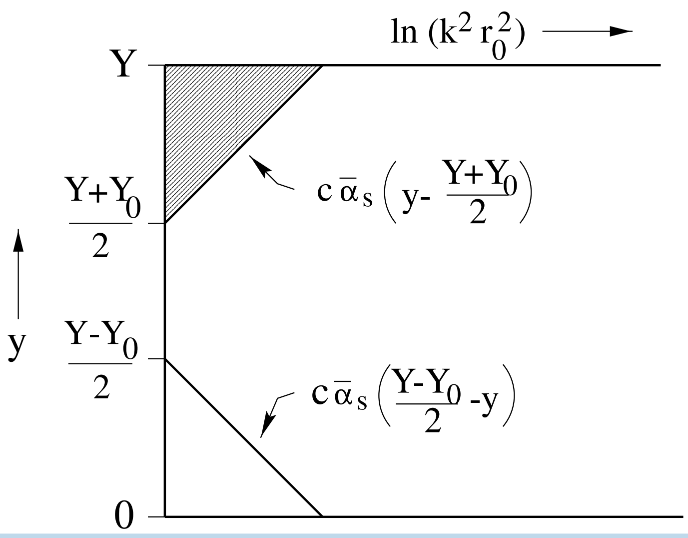
▷ Typical configurations (as retained by the BK equation) :

- Linear (BFKL) evolution from $y = Y$ down to $y = Y - Y_0$
- Non–linear (JIMWLK) evolution from $Y - Y_0$ down to 0

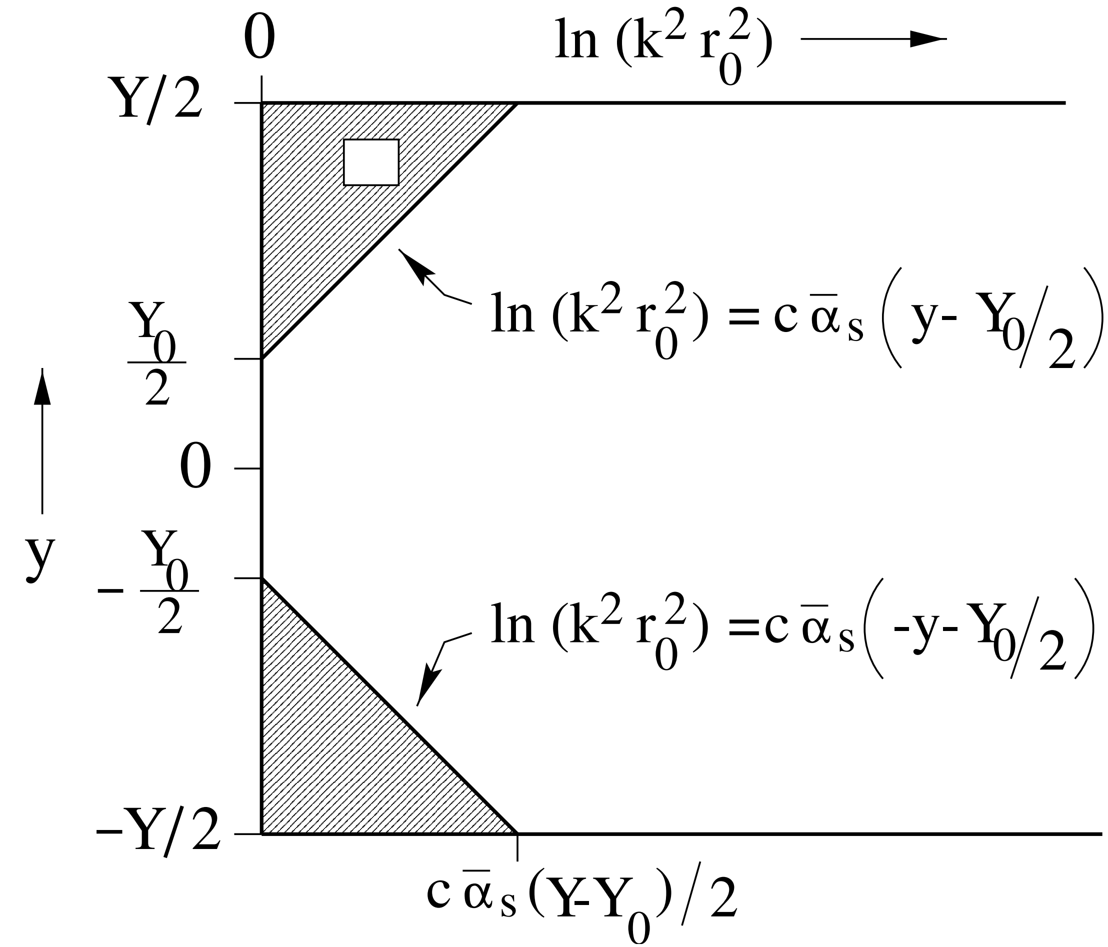
▷ Rare configurations : $S_Y = \exp\{-\bar{\alpha}_s \sum \triangle \text{ areas}\}$

- Evolution is suppressed down to $y = (Y + Y_0)/2$:
  $\Longrightarrow$ only one dipole of size $\geq r_0$
- Normal evolution (BFKL + JIMWLK) down to $y = 0$





Optimal configuration in COM frame



$$S_Y(r_0) \simeq \exp\left\{-\frac{c}{4}\bar{\alpha}_s^2 (Y-Y_0)^2\right\} S_{Y_0}(r_0)\,, \qquad c/4 \simeq 1.22 \;>\; \kappa \simeq 0.72$$