Saturation scale from the Balitsky–Kovchegov equation

Krzysztof Golec-Biernat
Institute of Nuclear Physics, Kraków

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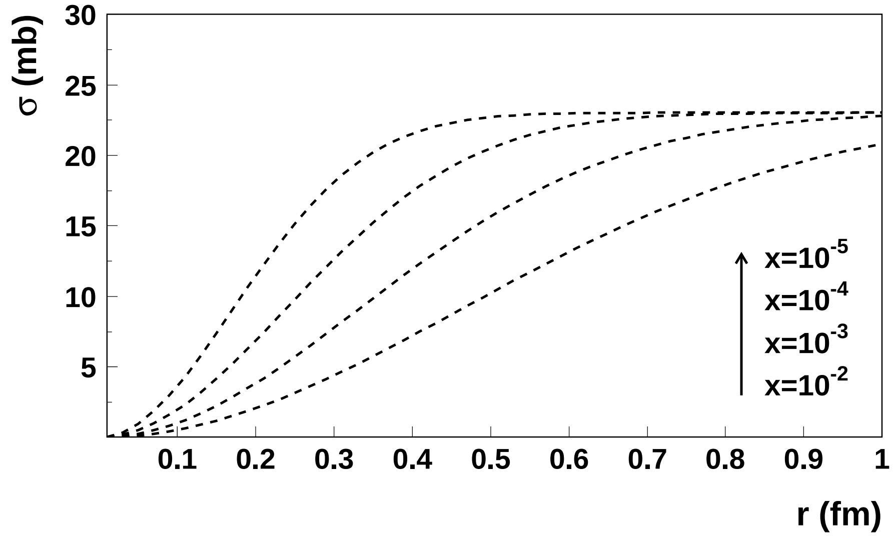
Motivation

- Saturation model for $q\bar{q}$ dipole scattering amplitude

$$\hat{N}(rQ_s(Y)) = 1 - \exp\{-r^2 Q_s^2(Y)\} \qquad Y = \ln(x_0/x)$$

Saturation scale: $\quad Q_s^2(Y) \sim \exp\{\lambda Y\} \qquad \lambda \approx 0.3$



- Balitsky–Kovchegov equation gives saturation: $\hat{N} \leq 1$ with saturation scale

- Munier–Peschanski travelling wave solution leading to saturation scale.

Balitsky-Kovchegov equation

Introduce distribution of gluons:

$$\mathcal{N}(k,Y) = \int d^2r \; \mathrm{e}^{ik\cdot r} \; \frac{\hat{\mathrm{N}}(r,Y)}{r^2}$$

BK equation:

$$\frac{\partial \mathcal{N}}{\partial Y} = \bar{\alpha}_s \, \chi(-\partial_L) \, \mathcal{N} \; - \; \bar{\alpha}_s \, \mathcal{N}^2 \qquad\qquad \partial_L = \frac{\partial}{\partial \ln k^2}$$

where BFKL kernel

$$\chi(\gamma) = \; 2\psi(1) - \; \psi(\gamma) - \; \psi(1-\gamma) \qquad\qquad \psi(\gamma) = \frac{d \ln \Gamma(\gamma)}{d\gamma}$$

Munier–Peschanski travelling wave solution

Expansion around saddle point $\gamma_0 = 1/2$:

$$\chi(\gamma) \;=\; \chi_0 \;+\; \frac{\chi_0''}{2}\,(\gamma - 1/2)^2 \qquad\qquad \gamma \rightarrow (-\partial_L)$$

KPP equation

$$x = a\ln(k/k_0) + b\,Y \qquad\qquad t = c\,Y \qquad\qquad u(x,t) \;=\; d\,\mathcal{N}(k,Y)$$

$$\frac{\partial u}{\partial t} \;=\; \frac{\partial^2 u}{\partial x^2} \;+\; u(1-u)$$

Travelling wave solution for large t gives saturation scale:

$$u \;=\; u(x - m(t)) \qquad\qquad \longleftrightarrow \qquad\qquad \mathcal{N} \;=\; \mathcal{N}\,(k/Q_s(Y))$$

$$\ln Q_s(Y) \;=\; \;\bar{\alpha}_s\,\frac{\chi_0''}{4}\,(c_\beta\,\gamma_c - 1)\;Y \;-\; \frac{d_\beta}{2\gamma_c}\,\ln Y$$

Critical point γ_c solves the equation: $\gamma_c\,\chi'(\gamma_c) \;=\; \chi(\gamma_c)$

c_β and d_β depend on the behaviour of initial condition at $x \to +\infty \quad (k^2 \to \infty)$

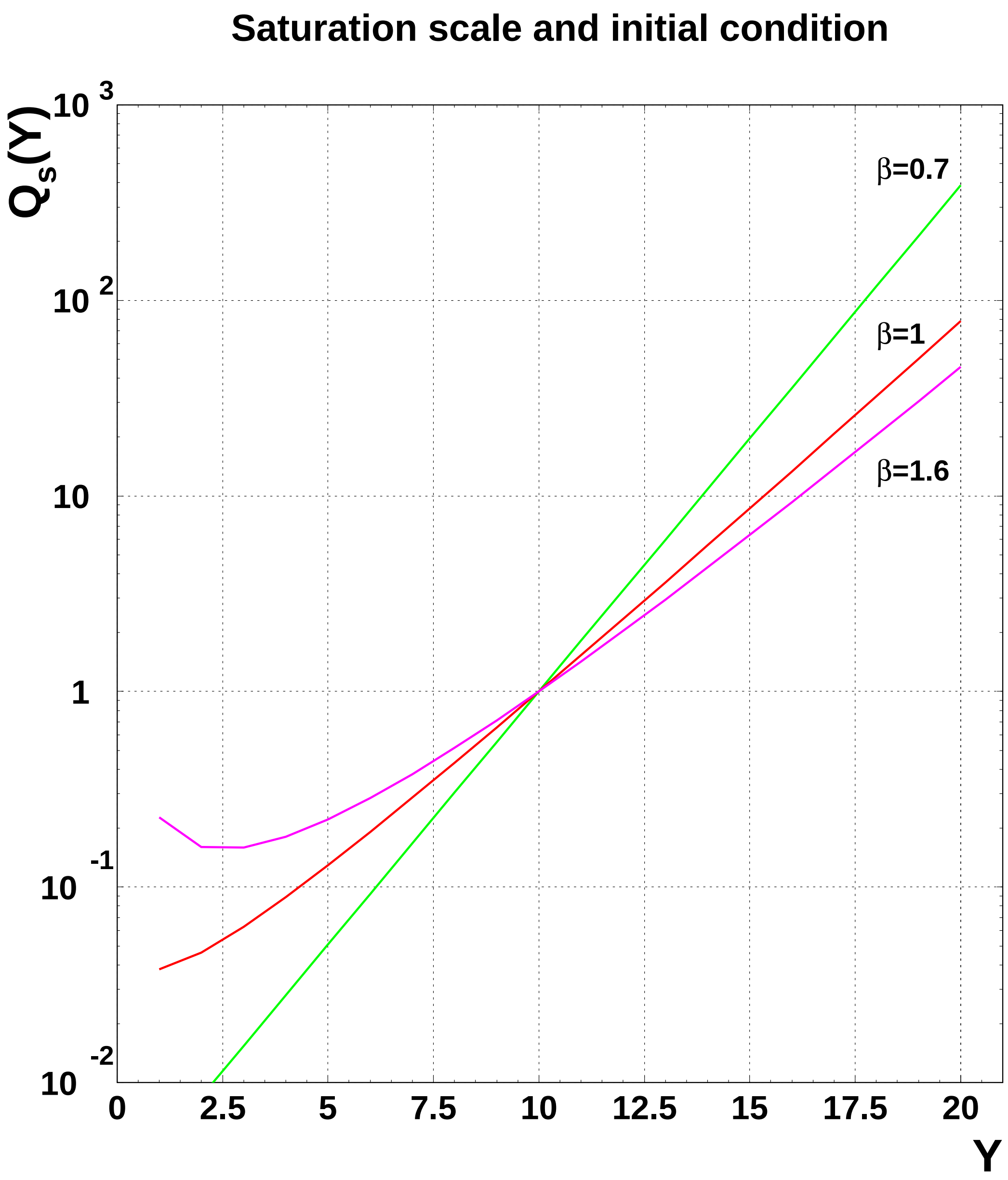
$$u_0(x) \;\sim\; \mathrm{e}^{-\beta x} \qquad \longleftrightarrow \qquad \mathcal{N}_0(k) \;\sim\; \left(\frac{1}{k^2}\right)^{0.63\,\beta}$$

In particular

$$c_\beta = \begin{cases} \beta + 1/\beta & \text{for} \quad \beta < 1 \\ 2 & \text{for} \quad \beta = 1 \\ 2 & \text{for} \quad \beta > 1 \end{cases} \qquad\qquad d_\beta = \begin{cases} 0 \\ 1/2 \\ 3/2 \end{cases}$$

Saturation scale $Q_s(Y)$ depends on initial condition.

Does the exact BK equation preserve these results ?



Saturation scale and initial condition
$Q_s(Y)$
10^{3}
10^{2}
10
1
10^{-1}
10^{-2}
0
2.5
5
7.5
10
12.5
15
17.5
20
Y
β=0.7
β=1
β=1.6

Solving BK equation

Numerical solution to

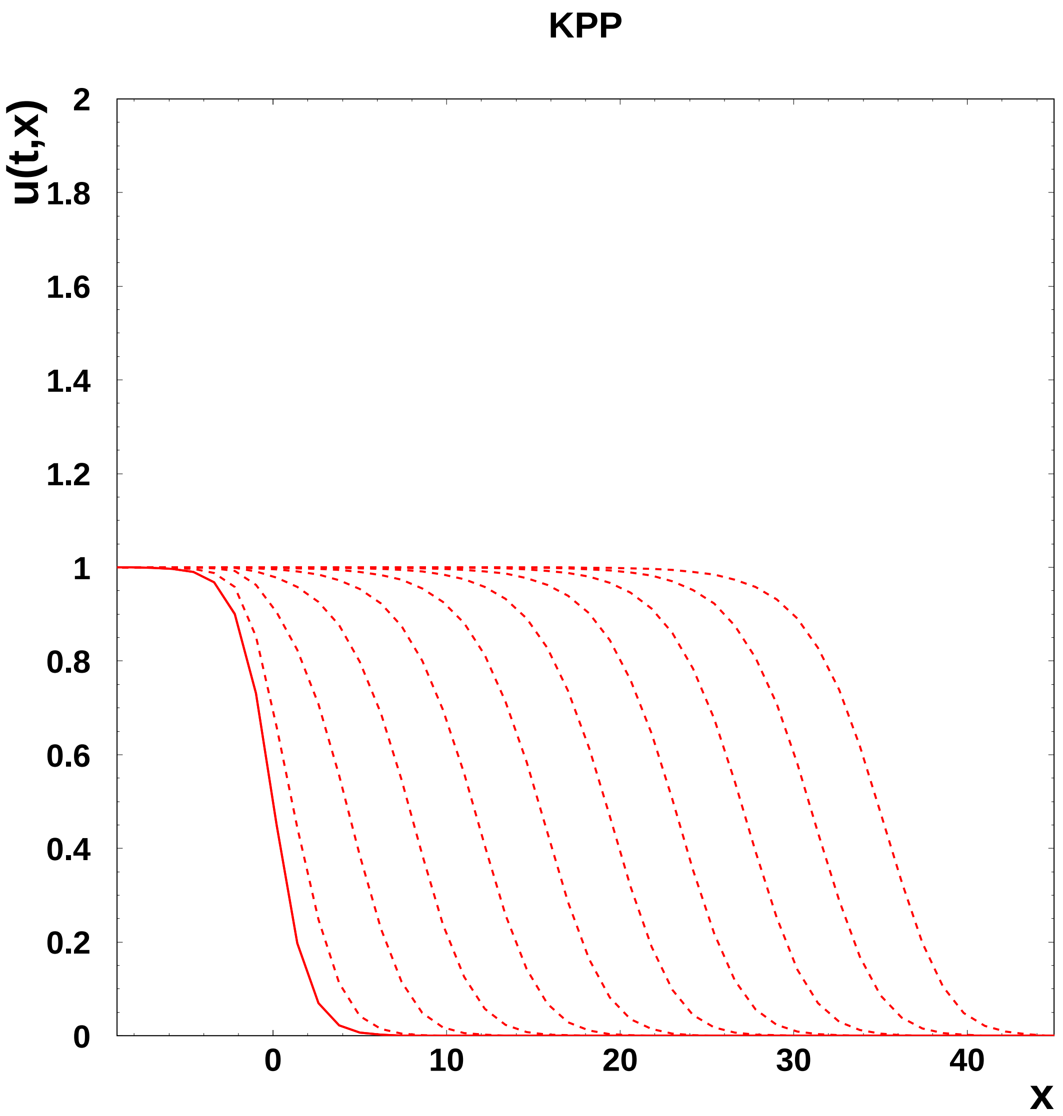
$$\frac{\partial \mathcal{N}}{\partial Y} = \bar{\alpha}_s \underbrace{\int_0^\infty \frac{dq^2}{q^2} \left\{ \frac{q^2 \mathcal{N}(q^2) - k^2 \mathcal{N}(k^2)}{|q^2 - k^2|} + \frac{k^2 \mathcal{N}(k^2)}{\sqrt{4q^2 + k^2}} \right\}}_{\chi(-\partial_L)\,\mathcal{N}} - \bar{\alpha}_s \mathcal{N}^2(k)$$

using Chebyshev polynomial expansion.

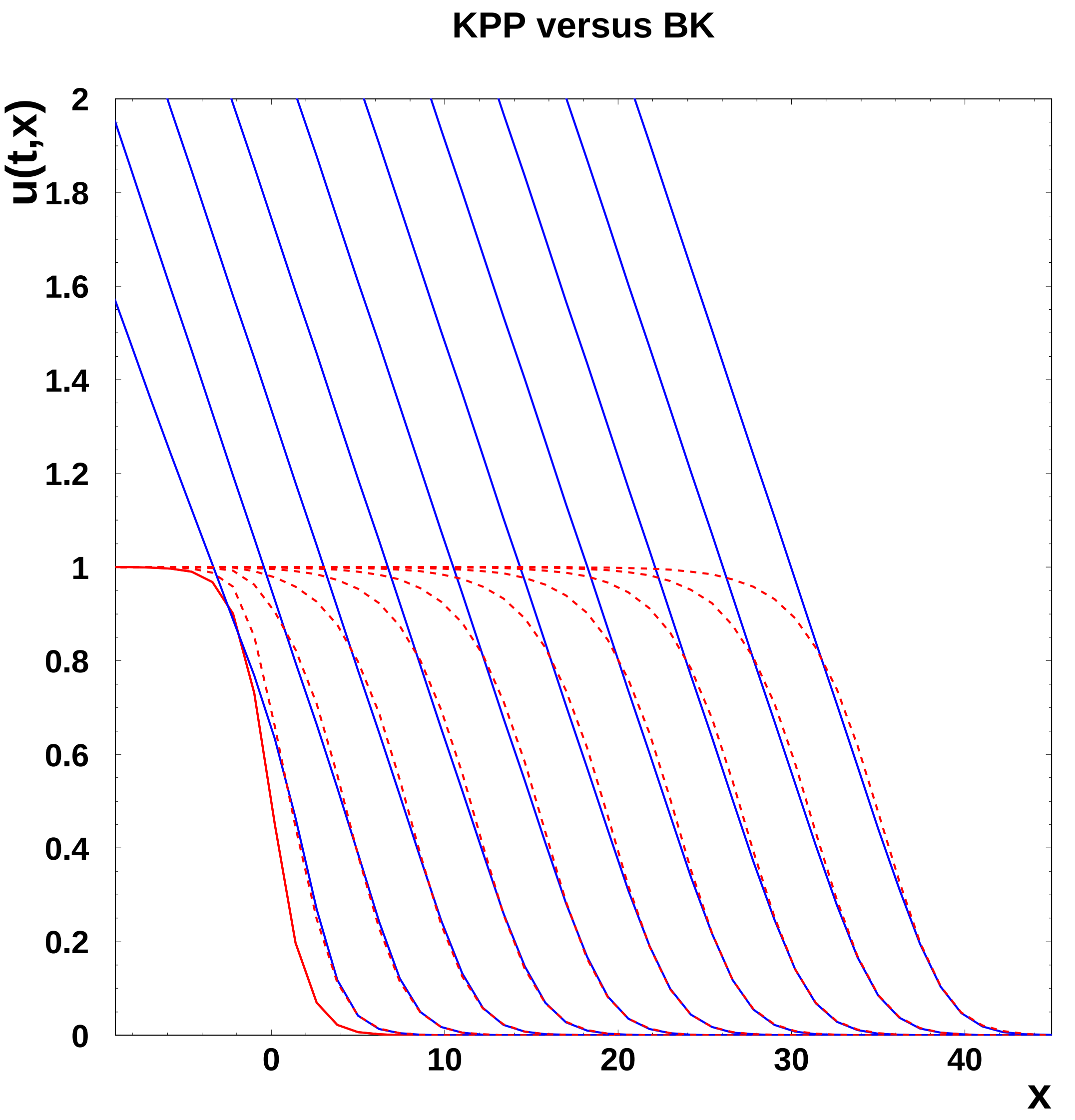
Initial condition:

$$u_0(x) = \frac{\exp\{-\beta x\}}{1 + \exp\{-\beta x\}} \qquad \longrightarrow \qquad \mathcal{N}_0(k)$$

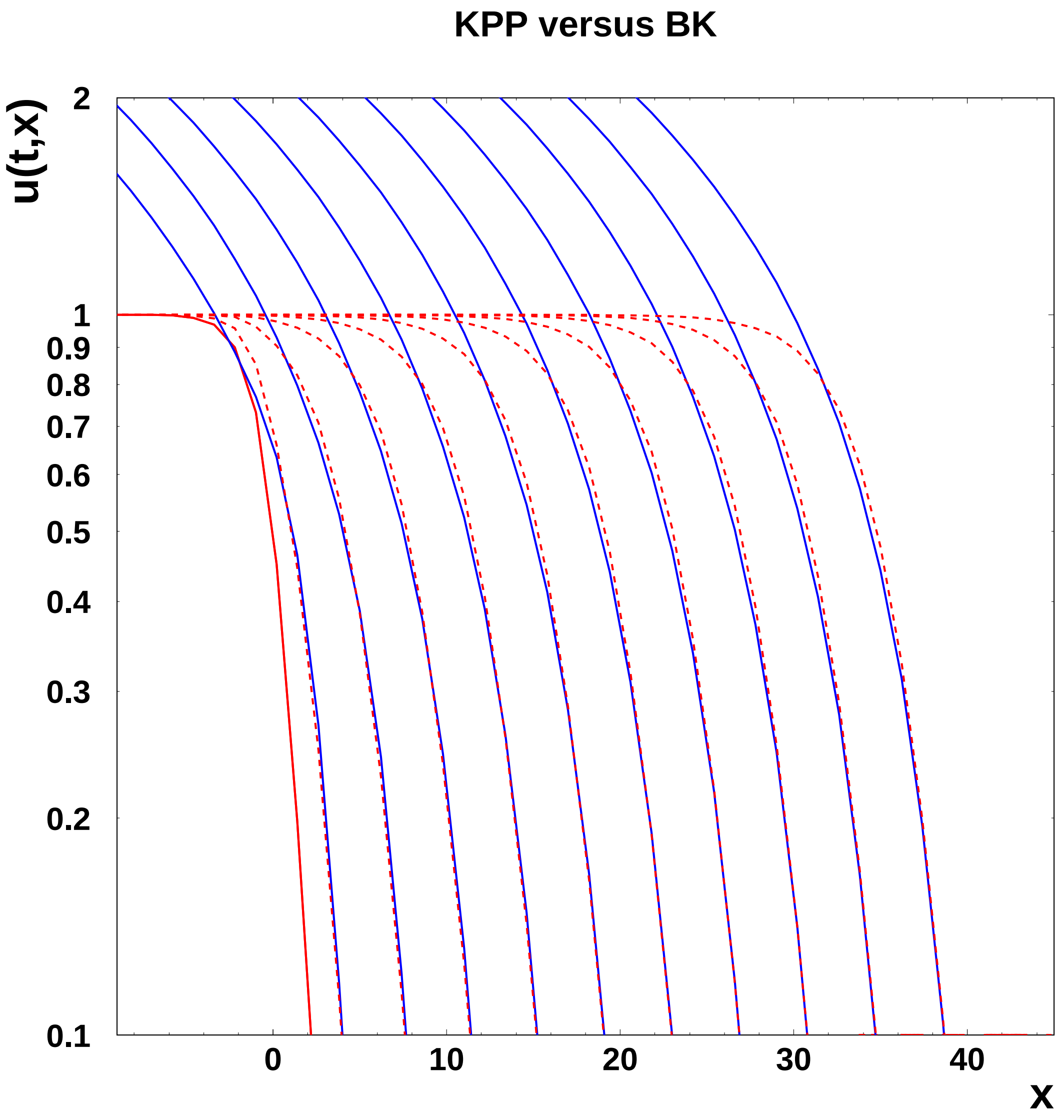
Solution to KPP equation



KPP versus BK equation

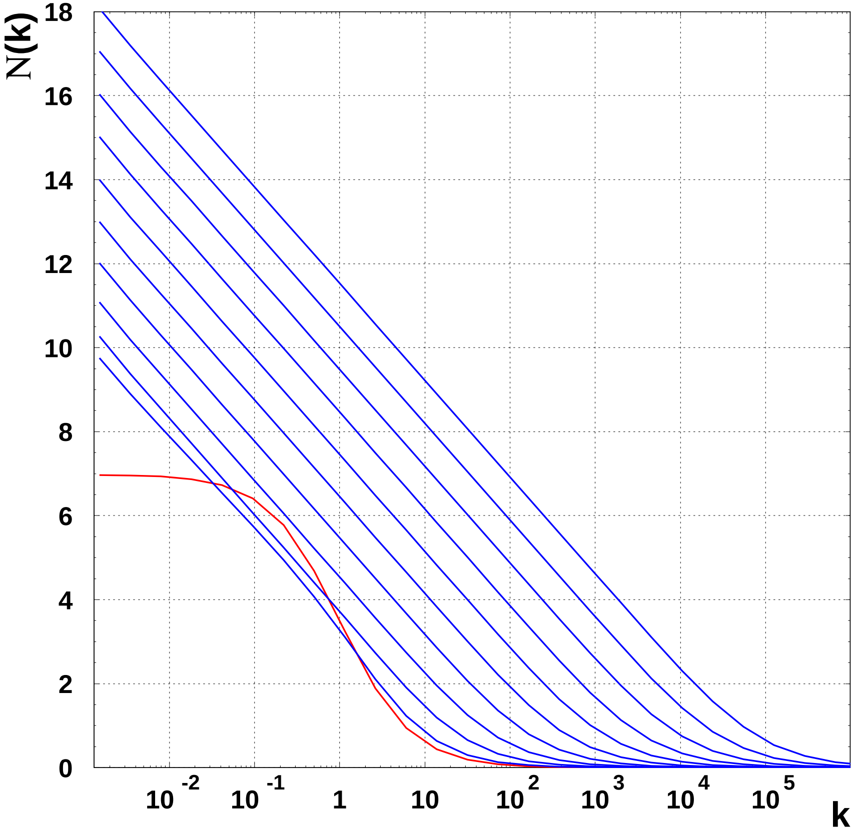


KPP versus BK equation (log)

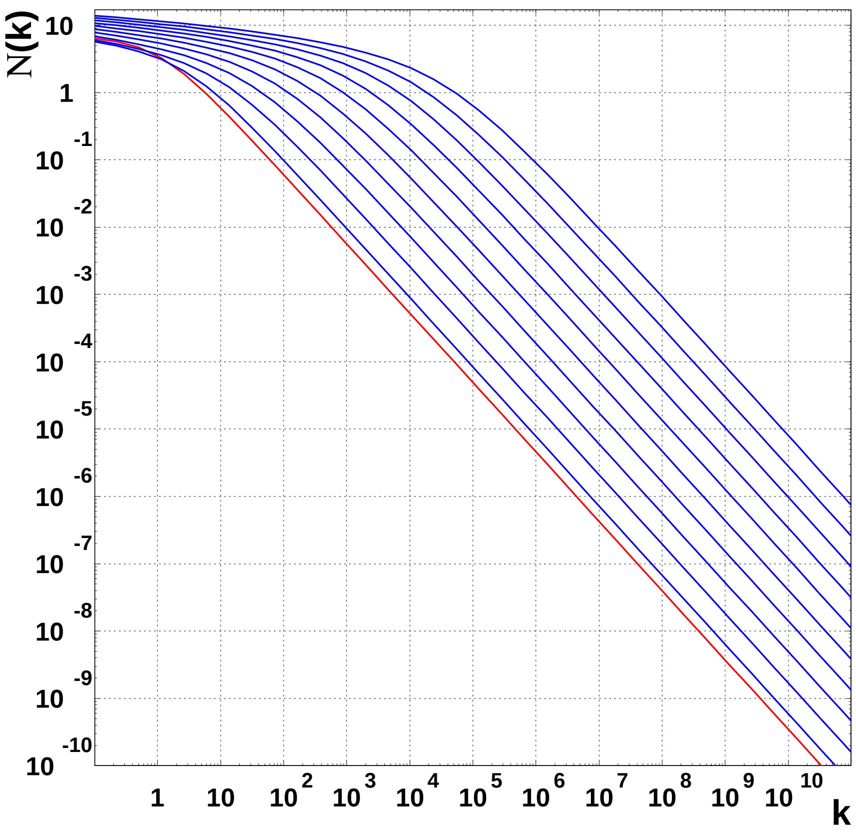


Saturation scale from BK equation

Saturation



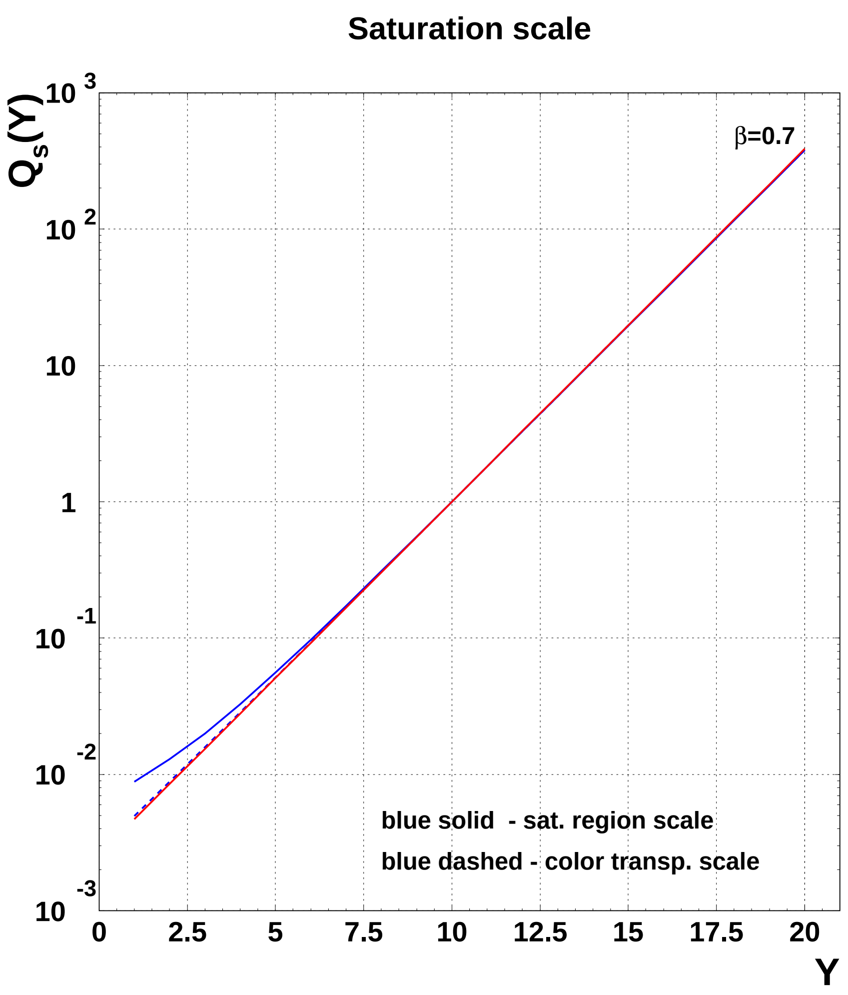
Color transparency

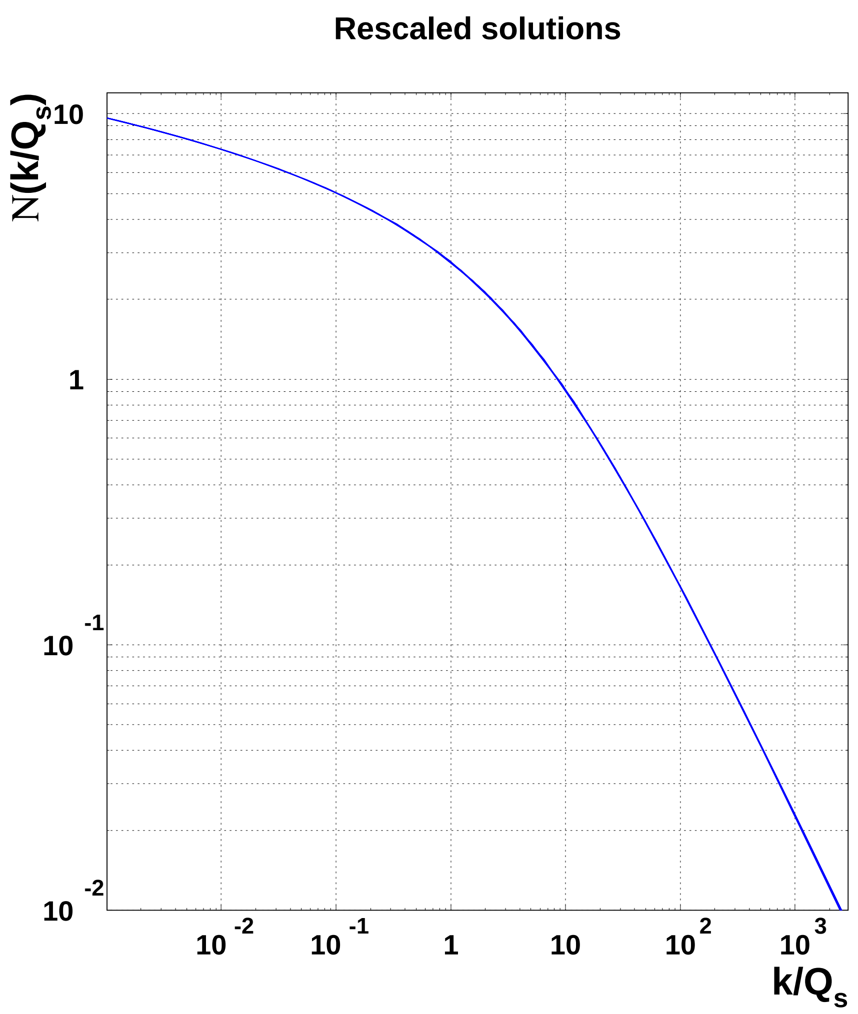


$$\mathcal{N}(k,Y) = \ln(Q_s(Y)/k)$$

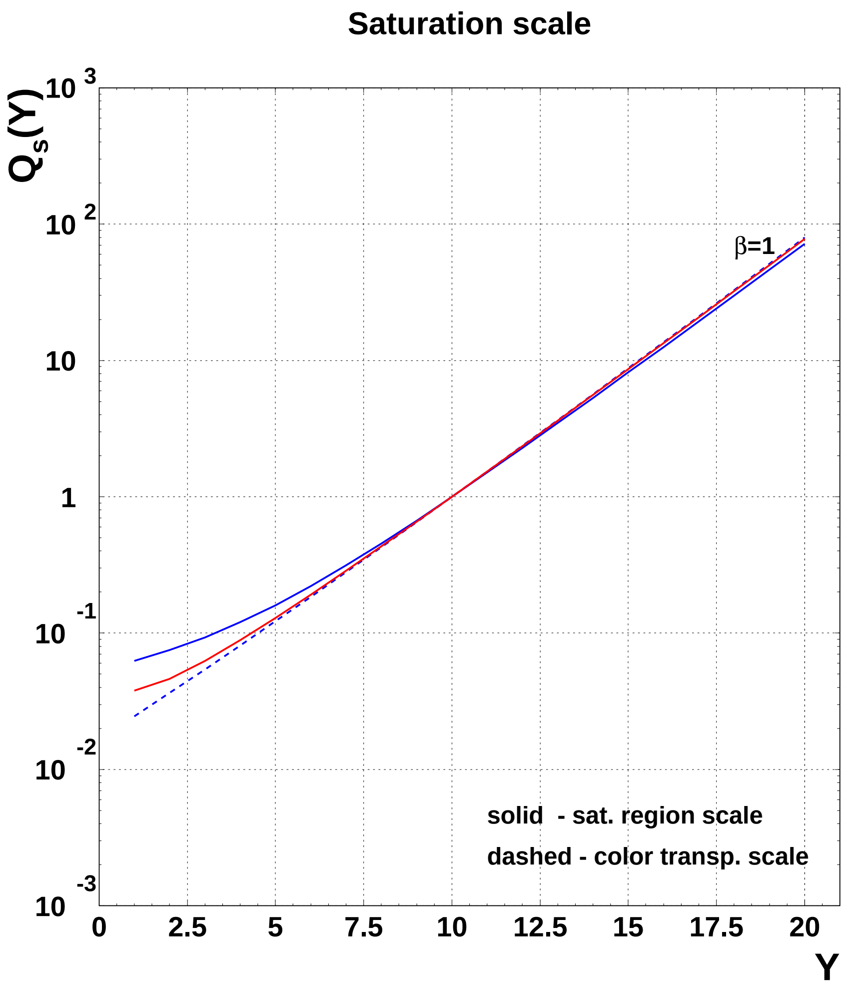
$$\mathcal{N}(Q_s(Y),Y) = \mathcal{N}_0 < 1$$

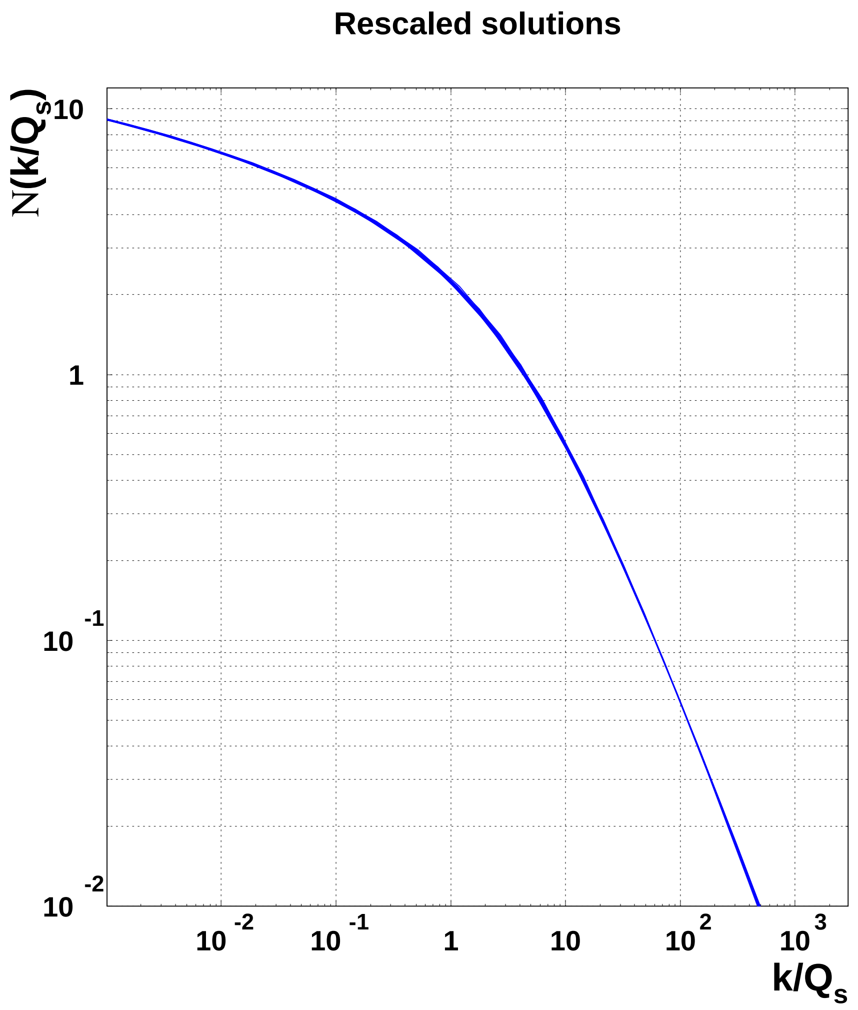
Results for $\beta = 0.7$



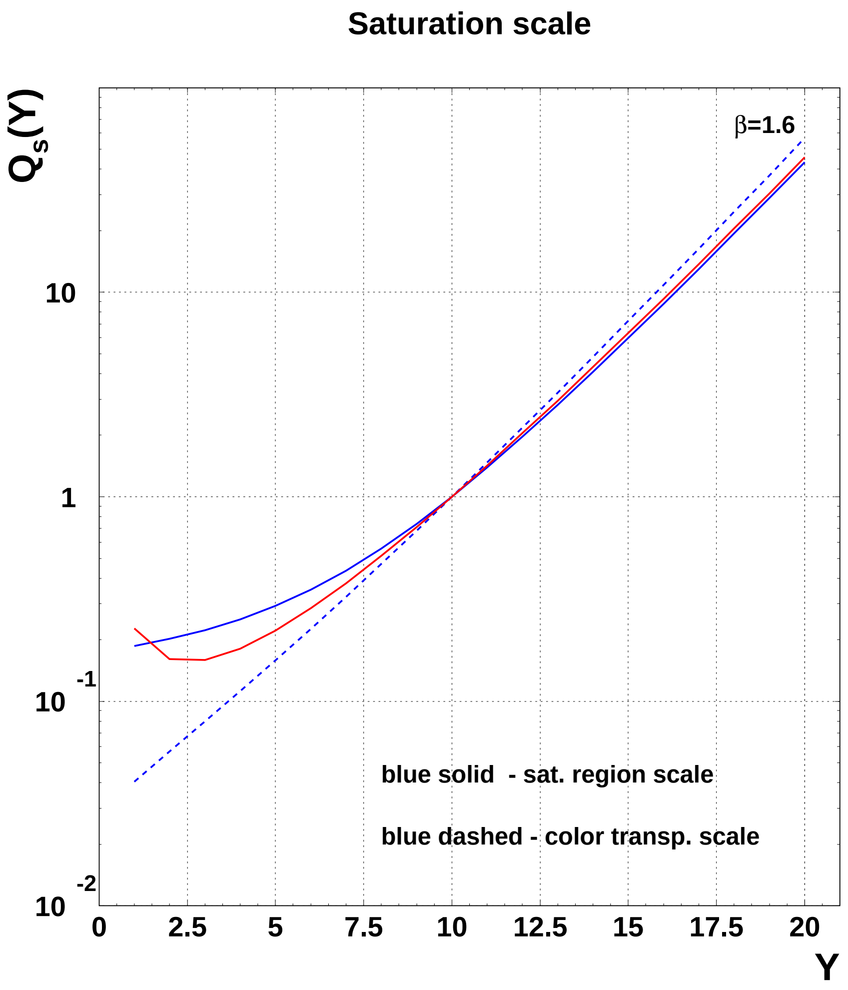


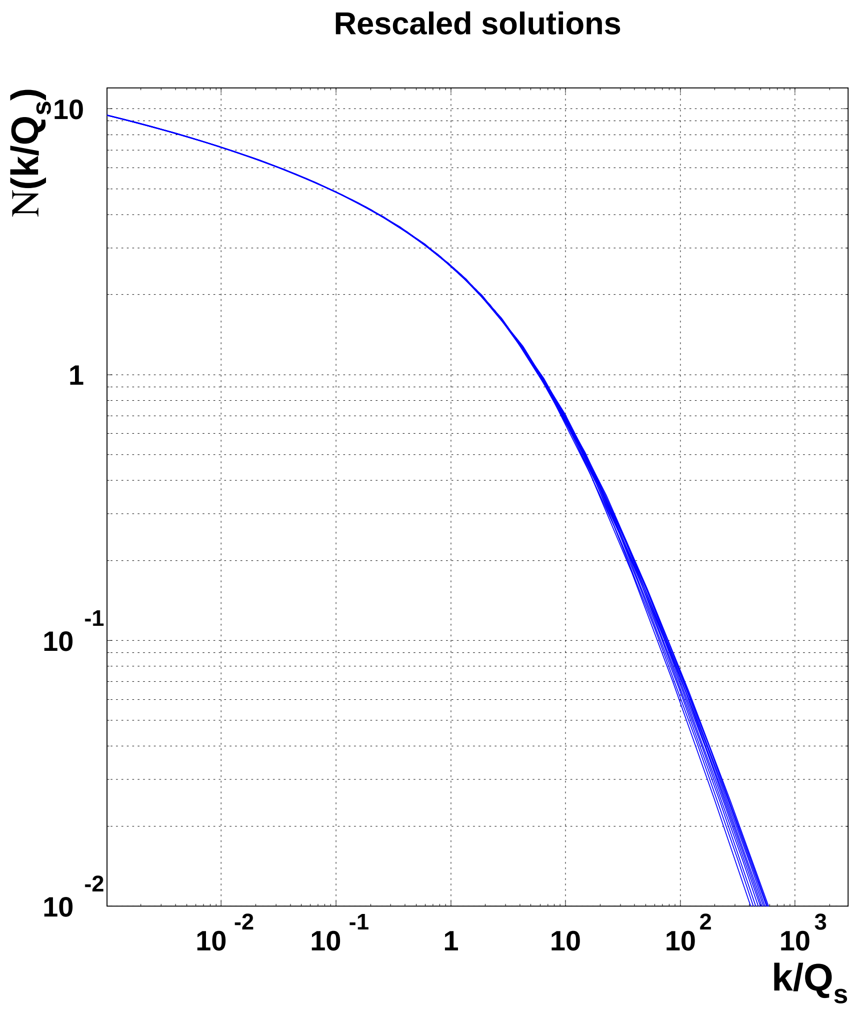
Results for $\beta = 1$





Results for $\beta = 1.6$





Munier–Peschanski again

Expansion of BFKL function around critical point $\gamma_c = 0.627$

$$\chi(\gamma) = \chi_c + \chi_c' (\gamma - \gamma_c) + \frac{\chi_c''}{2} (\gamma - \gamma_c)^2 \qquad\qquad \gamma \to (-\partial_L)$$

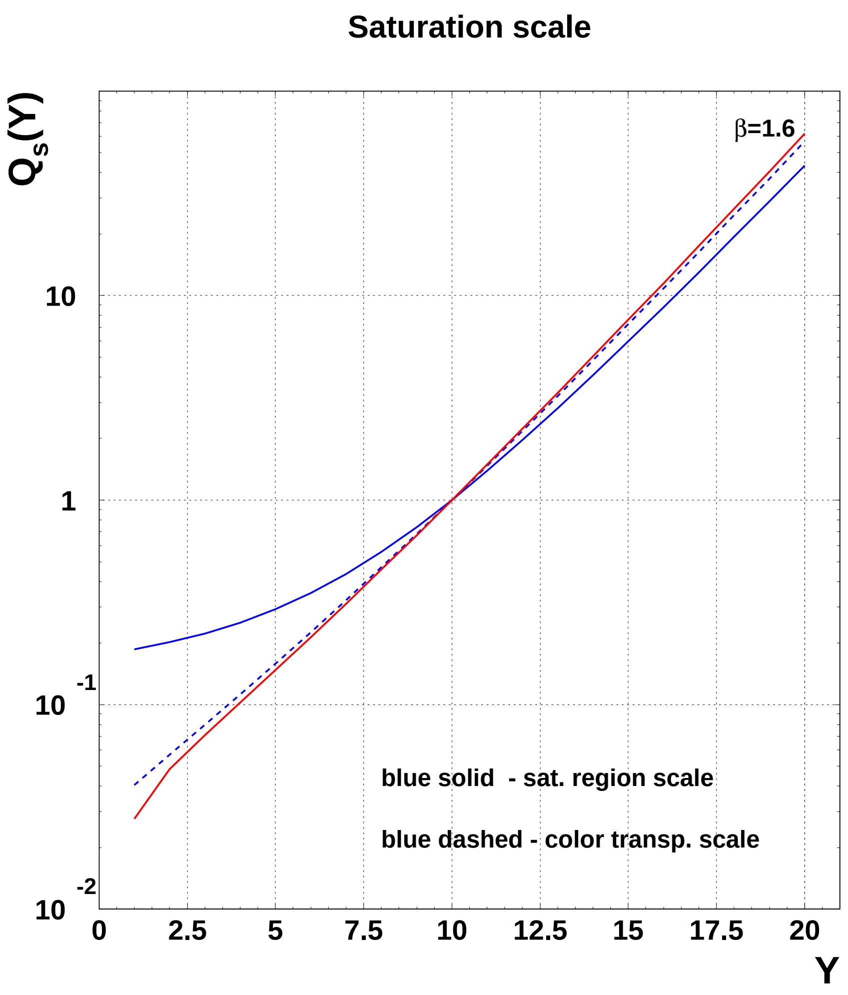
For $\beta > 1$ saturation scale:

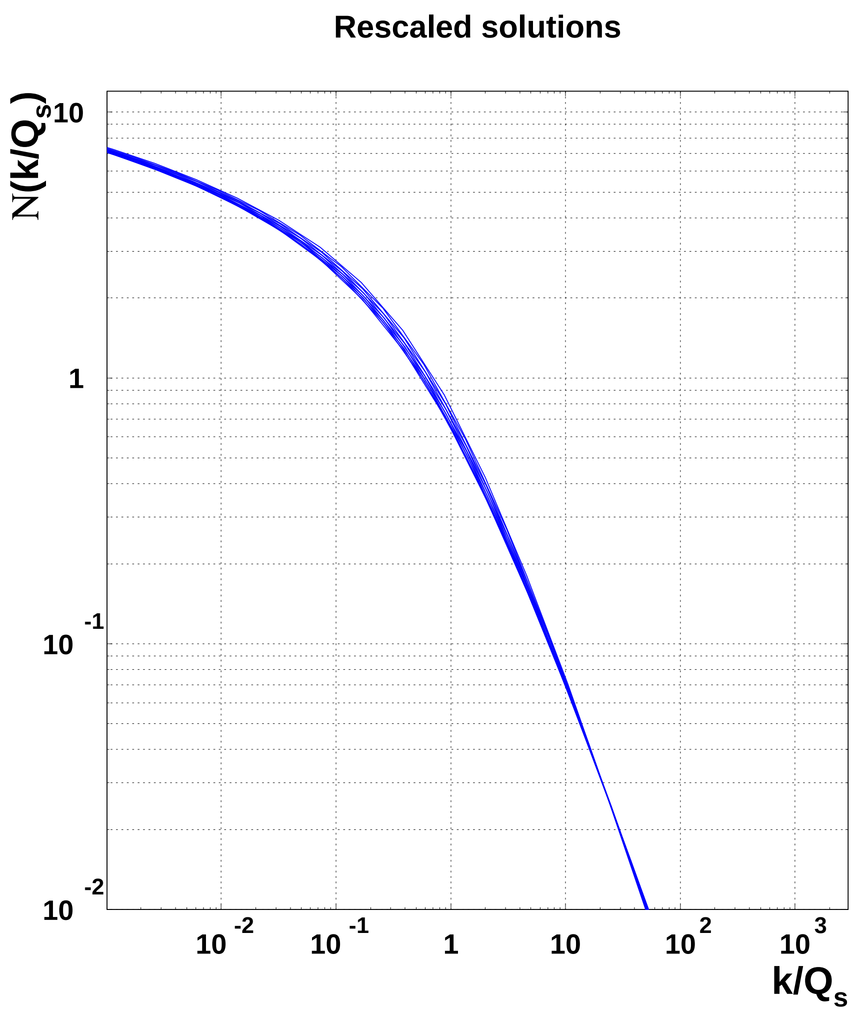
$$\ln Q_s(Y) = \bar{\alpha}_s \frac{\chi_c}{2\gamma_c} Y - \frac{3}{4\gamma_c} \ln Y - \frac{3}{2\gamma_c^2} \sqrt{\frac{2\pi}{\bar{\alpha}_s \chi_c''}} \frac{1}{\sqrt{Y}}$$

With this scale agreement for the scale computed form the condition:

$$\mathcal{N}(Q_s(Y), Y) = 0.1.$$

Results for $\beta = 1.6$





Conclusions

- The full BK equation contains saturation scale which depends on an initial condition.
- The analysis of saturation scales based on the KPP equation confirmed.
- The case with the color transparency initial condition $\mathcal{N} \sim 1/k^2$ should be carefully reexamined.