HIGH ENERGY AMPLITUDE AS AN ADMIXTURE OF SOFT AND HARD POMERONS

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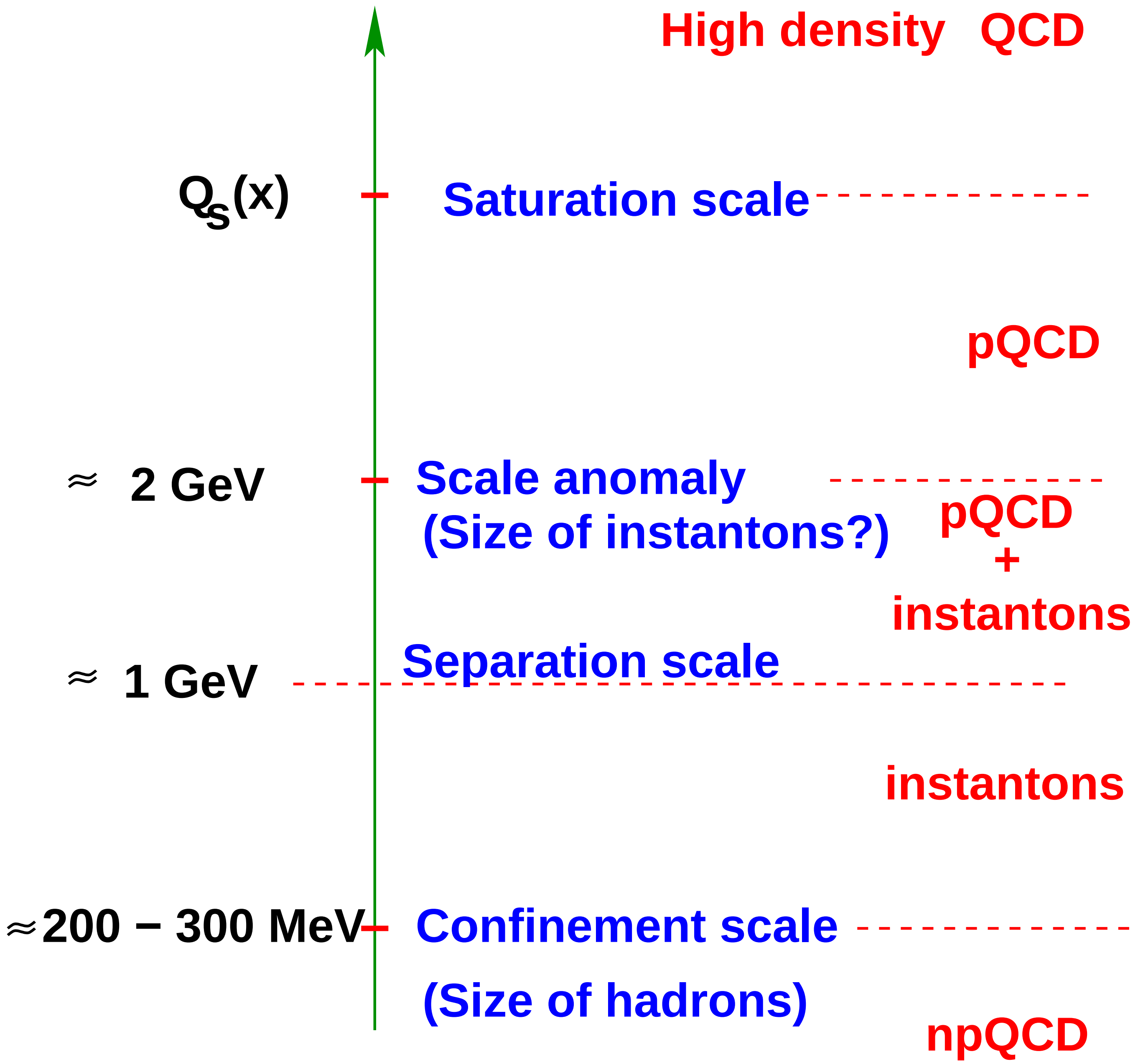
Key idea:

"Soft" Pomeron $\longrightarrow$ nonperturbative QCD but at sufficiently short distances

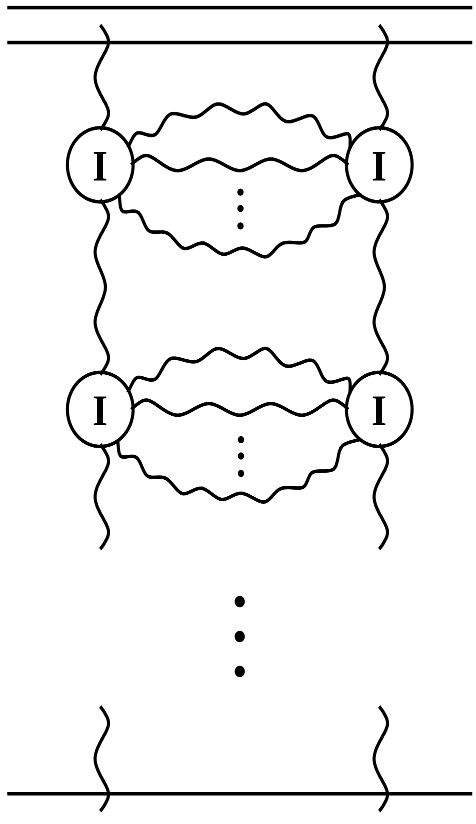
$$r_\perp(Pomeron) \quad = \quad 1/M_0 \quad \gg \quad 1/\Lambda$$

$$\alpha_S(\,M_0\,) << 1$$

Scales of QCD



Soft Pomeron



Our approach:

- $K(q^2, q'^2) =$

  $$\Delta_S\, \phi(q^2)\, \phi(q'^2)\,;$$

- $\phi(q^2) = e^{-q^2/q_S^2}\;;$

- $q_S^2 = M_0^2 = 4\,GeV^2\,;$

- $K(q^2, q'^2) = \Delta_S \phi(q^2)\, \phi(q'^2) \longrightarrow s^{\Delta_S}$;

- $K(q^2, q'^2) = \Delta_S \phi(q^2)\, \phi(q'^2) \longrightarrow$ diffusion in impact parameters (b_t) ;

- $K(q^2, q'^2) = \Delta_S \phi(q^2)\, \phi(q'^2) \longrightarrow R = \alpha'_P \ln s$ where R is the radius of interaction ;

- $K(q^2, q'^2) = \Delta_S \phi(q^2)\, \phi(q'^2) \longrightarrow \alpha'_P \propto 1/q_0^2$;

Hard Pomeron

• • Hard Pomeron = BFKL Pomeron in NLO + running α_S • •

BFKL Pomeron

- The BFKL equation:

$$\omega\, G^{H}_{\omega}(\vec{q}_f, \vec{q}_i) =$$
$$= \delta^{(2)}(\vec{q}_f - \vec{q}_i) + \int d^2q'\, K_H(\vec{q}_f, \vec{q}\,')\, G^{H}_{\omega}(\vec{q}\,', \vec{q}_i)$$

- Eigenvalue equation:

$$\alpha(q^2) \int dq'^2\, \tilde{K}\left(\frac{q^2}{q'^2}\right)\, \phi_f(q'^2) = \frac{r_H}{r}\omega(f)\, \phi_f(q^2)$$

where

$$K_H(\vec{q}_f, \vec{q}\,') = \alpha(q_f^2)\, \tilde{K}\left(\frac{q_f^2}{q'^2}\right)$$

and

$$r = \ln(q^2/\Lambda^2) - \frac{1}{2}\ln\alpha(q^2)\,,\, r_H = \ln(q_H^2/\Lambda^2)$$

BFKL Pomeron

- Diffusion approach

$$\omega(f) \;=\; \omega_L \left(1 \;+\; D\, f^2 \;+\; O(f^4) \right)$$

- BFKL Green's function in the diffusion model

$$G^H_\omega(r_f, r_i) = \sqrt{\frac{r_i^2}{16\,\pi\,\omega\,\omega_L\,r_H\,D}} \left(\frac{4\omega}{\omega_L\,r_H\,D}\right)^{\frac{1}{6}} \tilde{A}i(\xi;\zeta)$$

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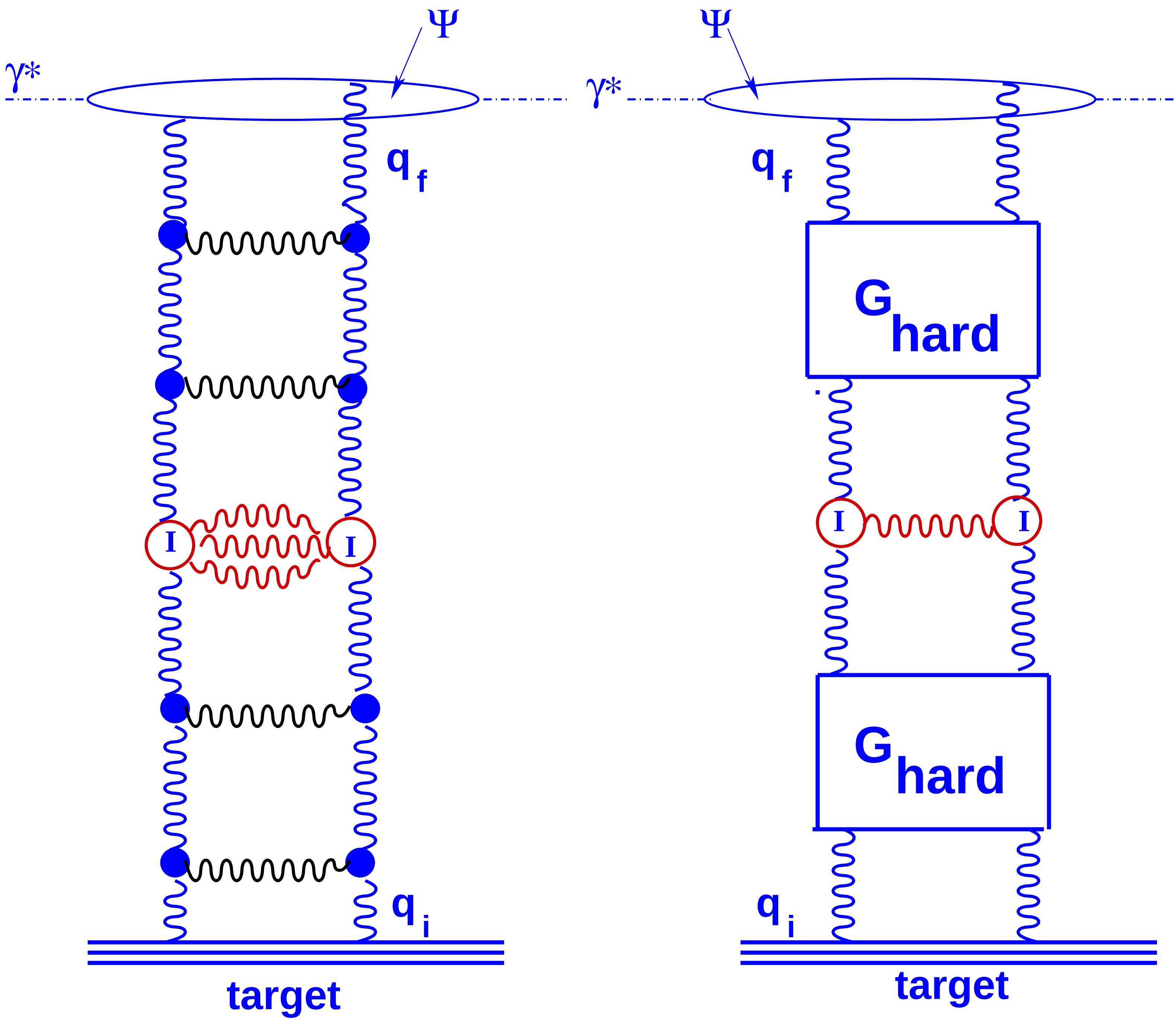
$$\tilde{A}i(\xi;\zeta) = \left(\int_{C_1} + \int_{C_2} \right) \frac{d\,\nu}{\sqrt{\nu}}\, exp\{ -\xi\,\nu \;+\; \frac{\nu^3}{3} \;-\; \frac{\zeta^2}{\nu} \}$$

$$\xi \;=\; \left(\frac{D\,\omega_L\,r_H}{4\omega}\right)^{-\frac{1}{3}} \{ \frac{r_f \;+\; r_i}{2} \;-\; \frac{\omega_L\,r_H}{\omega} \}$$

$$\zeta = \frac{\delta r}{2} \left(\frac{\omega}{2\,D\,\omega_L\,r_H}\right)^{\frac{1}{3}} \;\; with\;\; \delta r = r_f \;-\; r_i$$

"Effective" Pomeron in DIS



"Effective" Pomeron in DIS

- "Effective" Pomeron equation

$$G_\omega^{S-H}(r_f, r_i) = D(\omega; r_f, r_i) + D \bigotimes (K_S + K_H) \bigotimes G_\omega^{S-H}$$

$$G_\omega^{H}(r_f, r_i) = D(\omega; r_f, r_i) + D \bigotimes K_H \bigotimes G_\omega^{H}$$

where

$$D(\omega; r_f, r_i) = \delta(r_f - r_i)/\omega$$

- Solution

$$G_\omega^{S-H}(r_f, r_i) = G_\omega^{H}(r_f, r_i) +$$

$$+\Delta_S \frac{\left(G_\omega^H(r_f, r'') \otimes \phi(r'')\right)\left(\phi(r') \otimes G_\omega^H(r', r_i)\right)}{1 - \Delta_S\, \phi(r') \otimes G_\omega^H(r', r'') \otimes \phi(r'')}$$

- "Effective" pole equation

$$1 - \Delta_S \int dr' \int dr'\, \phi(r')\, G_\omega^H(r', r'')\, \phi(r'') = 0$$

- Resulting pole

$$\omega = \Delta_{S-H} = \Delta_S + \tilde{\omega}_S + \frac{D\, \tilde{\omega}_S\, \Delta_S}{96\, r_S^2\, (\tilde{\omega}_S + \Delta_S)}$$

where

$$\tilde{\omega}_S = \frac{r_H\, \omega_L}{r_S}$$

“Effective” Pomeron in DIS

- Restriction for “Effective” Pomeron

$$\Delta_{S-H} \geq \tilde{\omega} \;\; where \;\; \tilde{\omega} = \frac{r_H\,\omega_L}{\frac{r_f + r_i}{2}}$$

1. Small coupling constant

$$0 < Y < \frac{r_f^2}{\omega_L\, r_H}, \quad \bar{\alpha}_S < 0.1$$

2. Large virtualities

$$0 < Y < \frac{r_f^2}{\omega_L\, r_H}, \quad \frac{r_f + r_i}{2} > \frac{\omega_L\, r_H}{\Delta_{S-H}} \sim r_S$$

3. Large rapidity

$$Y \geq \frac{\omega_L\, r_H}{D\,\Delta_S^2}$$

- Final answer for Green’s function

$$G^{DIS}(Y, r_f, r_i) = \frac{\Delta_S}{q_f\, q_i}\left(\frac{s}{q_f\, q_i}\right)^{\Delta_{S-H}} \tilde{g}\,(\omega = \Delta_{S-H}, r_f, r_i)$$

Conclusion

- The high energy asymptotic behavior is due to the exchange of the resulting “effective” Pomeron;
- The “effective” Pomeron intercept is the sum of the contributions from pQCD and non-perturbative processes;
- The non-perturbative contribution is essential even for DIS process;
- The result depends on the value of the soft Pomeron intercept;
- The result depends on the values of the soft Pomeron scale and separation scale;

"Effective" Pomeron at large b

- Dipole-dipole BFKL amplitude

$$N^{BFKL}(y, r_{1,t}, r_{2,t}; b_t) = \int \frac{d\nu}{2\pi} \phi_{in} 2^{4-8i\nu} e^{\omega(\nu)y} \left(\frac{r_{1,t}^2 \, r_{2,t}^2}{b^4} \right)^{\frac{1}{2}+i\nu}$$

- Dipole-dipole BFKL amplitude

$$N(y, r_{1,t}, r_{2,t}; b_t) = \int \frac{d\nu}{2\pi} \phi_{in} 2^{4-8i\nu} e^{\omega(\nu)y} \left(\frac{r_{1,t}^2 r_{2,t}^2}{(b^2 + r_{2,t}^2/4)^2} \right)^{\frac{1}{2}+i\nu}$$

- Correct Born term

$$\phi_{in} = \pi \, \alpha_s^2 \frac{N_c^2 - 1}{N_c^2} \frac{1}{1/2 - i\,\nu}$$

- Dipole-dipole BFKL amplitude

$$N(y, r_{1,t}, r_{2,t}; q) = \alpha_s^2 \frac{N_c^2 - 1}{N_c^2} \left(r_{1,t} \, r_{2,t} \right)$$

$$\int \frac{d\,\omega}{2\pi\, i} \int \frac{d\,\nu}{2\,\pi} \frac{e^{\omega\, y} \left(r_{1,t}^2 \, q^2 \right)^{i\,\nu}}{(\omega - \omega(\nu))\,(1/2 - i\,\nu\,)} \frac{K_{2\,i\,\nu}\left(|q|\,|r_{2,t}|/2\right)}{\Gamma(1 + 2i\nu)}$$

“Effective” Pomeron at large b

- “Effective” pomeron

$$N^{\omega}_{S-H}\,(r_{1,t},\,r_{2,t};\,q)\;=$$

$$=\frac{\tilde{N}^{\omega}(r_{1,t},\,r_{2,t};\,q)}{1\;-\;A\int\;d^2\,k_1\;\int\;d^2\,k_2\,\phi(k_1,\,q)\,k_1^2\,N_0^{\omega}(k_1,\,k_2;\,q)\,k_2^2\,\phi(k_2,\,q)}$$

here

- $$N_0^{\omega}(r_{1,t},\,r_{2,t};\,q)\;=$$

$$(\,r_{1,t}r_{2,t})\int\frac{d\,\nu}{2\pi}\frac{\left(r_{1,t}^2q^2\right)^{i\nu}\;2^{4-8i\nu}}{(\omega-\omega(\nu))(1/2-i\nu)}\frac{K_{2i\nu}(|q||r_{2,t}|/2)}{\Gamma(1+2i\nu)}$$

- $$\tilde{N}^{\omega}(r_{1,t},\,r_{2,t};\,q)\;=$$

$$\int\,d^2k_1\int\,d^2k_2e^{i\vec{k}_1\vec{r}_{1,t}+i\vec{k}_2\vec{r}_{2,t}}\tilde{N}^{\omega}(k_1,\,k_2;\,q)\,=$$

$$=\frac{\Delta_S}{q_s^2}\int\,d^2k_1\int\,d^2k_2e^{i\vec{k}_1\vec{r}_1+i\vec{k}_2\vec{r}_2}\int\,d^2k'(k')^2\phi(k',\,q)$$

$$N_0^{\omega}(k',\,k_2;\,q)\int\,d^2k''(k'')^2\phi(k'',\,q)N_0^{\omega}(k_1,\,k'';\,q)$$

"Effective" Pomeron at large b

- Soft kernel

$$K_{soft}(k_1, k_2\,, q) = \Delta_S \phi(k_1, q) \phi(k_2, q)$$

$$\phi(k, q) = e^{-\frac{k^2}{2\,q_s^2} - \frac{(\vec{q}-\vec{k})^2}{2\,q_s^2}}$$

- Soft Pomeron

$$\frac{A}{2}\,\pi\,q_s^2\,e^{-q^2\,/\,2\,q_s^2} \;=\; \Delta_S \,-\, \alpha'\,q^2$$

$$\frac{q^2}{q_s^2} \,<\, 1\,,\;\; A \,=\, \frac{2\,\Delta_S}{\pi\,q_s^2}\,,\;\; \alpha' \,=\, \frac{A\,\pi}{4} \,=\, \frac{\Delta_S}{2\,q_s^2}$$

- The "effective" pole equation

$$1 - \frac{2\,\Delta_S}{\pi\,q_s^2}\int\, d^2k_1\,k_1^2\int\, d^2k_2\,k_2^2\,\phi(k_1, q)\,\phi(k_2, q)$$

$$\int \frac{d^2\,r_{1,t}}{(2\pi)^2} e^{-i\vec{k}_1\vec{r}_1} \int \frac{d^2\,r_{2,t}}{(2\pi)^2} e^{-i\vec{k}_2\vec{r}_2} N_0^{\omega}(r_{1,t}, r_{2,t}; q) = 0$$

"Effective" Pomeron at large b

1. In the region where

$2\,\nu_0 \; < \; 1$ and $2\,\nu_0 \; \ln(\frac{q_s^2}{q^2}) \; < \; 1$ with

$$\nu_0 \; = \; \frac{4\,\Delta_S}{D}\;\ln(\frac{q_s^2}{q^2})$$

the solution is given by

$$\omega \; = \; \omega_{S-H} \; = \; \omega_0 \; + \; (\,\Delta_S\,)^2 \; \frac{\ln^2(\frac{q_s^2}{q^2})}{2^{\,16\,\nu_0\,-\,4}\;D}$$

2. In the region

$2\,\nu_0 \; < \; 1$ and $2\,\nu_0 \; \ln(\frac{q_s^2}{q^2}) \; > \; 1$, where

$$\nu_0 \; = \; \sqrt{\frac{2\,\Delta_S}{D}}$$

the solution for 'effective' pole reads:

$$\omega \; = \; \omega_{S-H} \; = \; \omega_0 \; + \; \frac{\Delta_S}{2^{\,8\,\nu_0\,-\,1}} \; - \; 2\,\Delta_S\;\left(\frac{q^2}{16\;q_s^2}\right)^{2\nu_0} \; .$$

"Effective" Pomeron at large b

Solution in q representation

$$N_{S-H}(y, r_{1,t}, r_{2,t}; q) \; =$$

$$= C\pi\Delta_S e^{\omega_{S-H} y} \frac{(r_{1,t} r_{2,t})}{16^{4\nu_0 - 1} D\nu_0} \left(\frac{q}{q_s}\right)^{2\nu_0} (r_{1,t} q_s)^{2\nu_0} K_{2\nu_0}(|q| \, |r_{2,t}|/2)$$

Solution at large b

$$N_{S-H}(y, r_{1,t}, r_{2,t}; b) \; = \int_0^{q_s} \, d^2 \, q \, e^{i \, \vec{q} \, \vec{b}} \, N_{S-H}(y, r_{1,t}, r_{2,t}; q)$$

“Effective” Pomeron at large b

1. In the region where

$$4\,b\,q_s\left(\frac{1}{2\,\Delta_S\,y}\right)^{1/4\nu_0} < 1\,,\;\; b < b_{max} = \frac{1}{4\,q_s}\,(\,2\,\Delta_S\,y\,)^{1/4\nu_0}\;\;,$$

the amplitude is

$$N_{S-H}(y,r_{1,t},r_{2,t};b) = e^{\omega_0 y + y\frac{\Delta_S}{2^{8\nu_0-1}}}\Gamma(1+\frac{1}{2\nu_0})$$

$$\frac{2^{4\nu_0}\pi^2}{16^{4\nu_0-1}}\sqrt{\frac{2\Delta_S}{D}}\left(\frac{8}{\Delta_S y}\right)^{1/2\nu_0}\left(r_{1,t}r_{2,t}q_s^2\right)\left(\frac{r_{1,t}}{r_{2,t}}\right)^{2\nu_0}$$

with

$$\nu_0 = \sqrt{\frac{2\,\Delta_S}{D}}\,.$$

2. For impact parameters

$$b > b_{max} = \frac{1}{4\,q_s}\,(\,2\,\Delta_S\,y\,)^{1/4\nu_0}\;\;,$$

the amplitude reads as

$$N_{S-H}(y,r_{1,t},r_{2,t};b) =$$

$$= \frac{2\pi^2}{16^{4\,\nu_0-1}}\sqrt{\frac{8\Delta_S}{D}}\,\frac{(\,r_{1,t}\,r_{2,t}\,)^{1+2\nu_0}}{\left(b^2 + r_{2,t}^2/4\right)^{1+2\nu_0}}\,e^{\omega_0\,y + y\frac{\Delta_S}{2^{8\,\nu_0-1}}}\,\Gamma(1+2\nu_0)$$

“Effective” Pomeron at large b

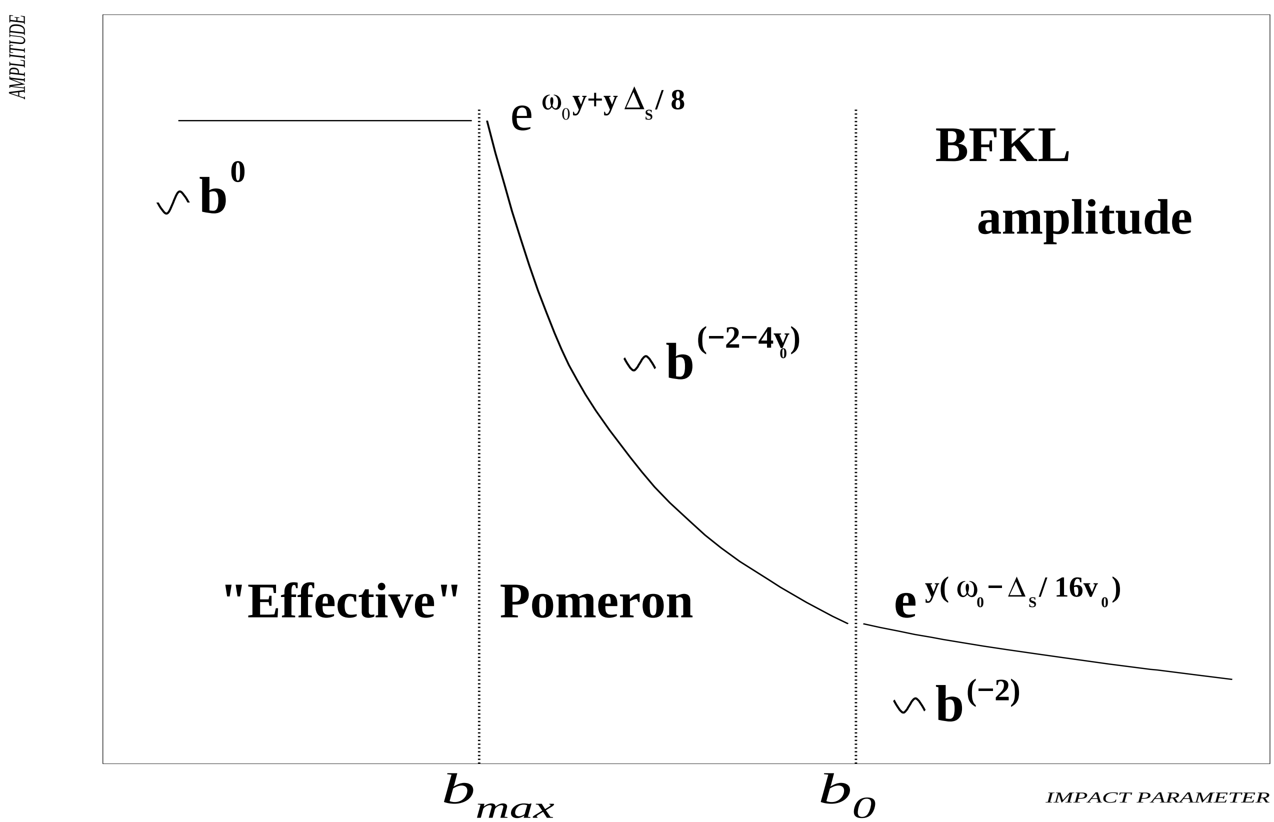
BFKL amplitude

- Amplitude at large b

$$N_{BFKL}(y, r_{1,t}, r_{2,t}; b) \propto \frac{1}{\sqrt{y}} \frac{r_{1,t}\, r_{2,t}}{b^2 + r_{2,t}^2/4}\, e^{\omega_0 y}$$

- Two amplitudes transition

$$b_0 = y^{1/8\nu_0} \sqrt{r_{1,t}\, r_{2,t}}\, e^{y \frac{\Delta_S}{2^8 \nu_0 + 1} \nu_0}$$



Conclusion

- The model does not lead to the unitarization of the cross section;
- The BFKL amplitude is the lower boundary for the “effective” Pomeron;