Summary of Working Group A Structure Functions and Low x Theory part

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Overview

- proton structure function: global fits R. Thorne, Wu-Ki Tung
- photon structure function: parametrisation A. Zembrzuski
- nuclear parton distributions S.Kumano
- lattice QCD results A.Shindler
- higher order calculations (NNLO splitting function) A.Vogt
- unintegrated parton distributions W.Broniowski, A.Szczurek, H.Jung, W.Broniowski
- low x
  - resummation at low x G.Salam
  - BFKL Pomeron at nonzero temperature L.Lipatov
  - perturbative-nonperturbative interface S.Bondarenko
  - studies of NLL BFKL A.Sabio-Vera, C.Royon
  - $\gamma^* g$ impact factor, $\gamma^*\gamma^*$ scattering A. Kyrieleis, M. Lublinsky
  - higher twists, renormalons A.Kotikov

Overview (continued)

- dipole picture and saturation at low x
  - nonlinear evolution (Balitsky-Kovchegov equation and beyond) L.Lipatov,R.Peschanski, K.Golec-Biernat, E.Iancu,M.Kozlov
  - saturation in impact parameter space E.Naftali
  - odderon in dipole model Y.Kovchegov
  - instantons (*WGB Summary*) A.Utermann
  - shadowing in diffraction and DVCS (*WGB summary*) S.Munier,T.Rogers, L.Favart
  - hadronic collisions C.Marquet
  - shadowing in nuclei W.Schafer

In total: 31 theory talks

Lattice

Structure functions on lattice

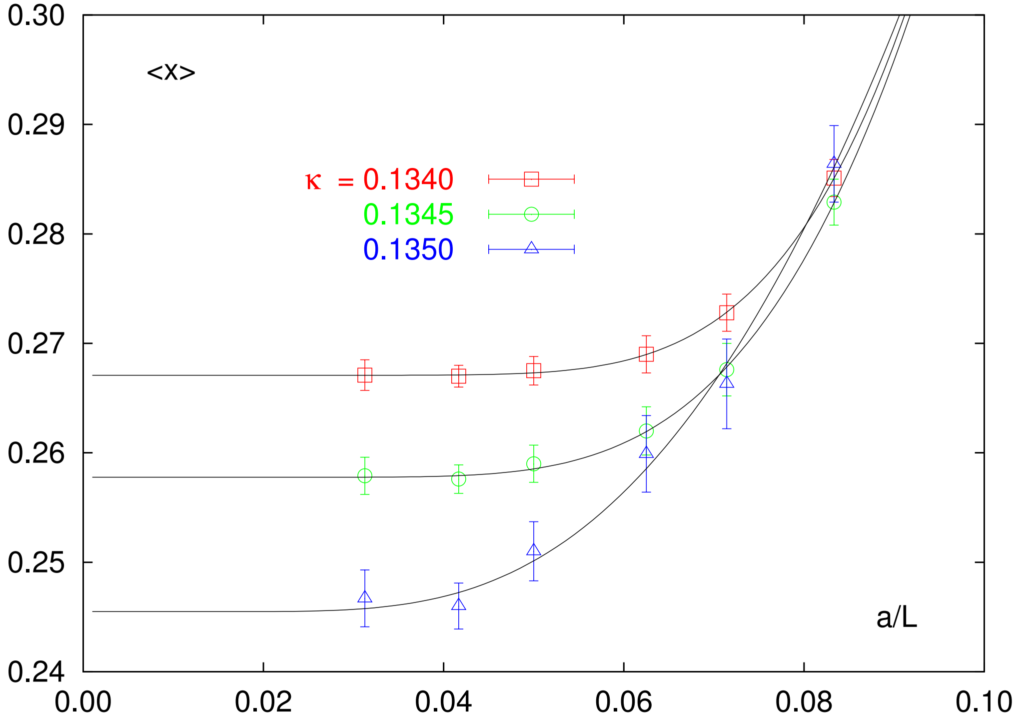
A.Shindler: Review of recent results on the moments of structure functions from lattice QCD.

Detailed study of systematic uncertainties:

- non-perturbative renormalisation
- continuum limit
- finite volume effects
- chiral limit (lattice formulation of the Dirac operator)
- quenching

Structure functions on lattice

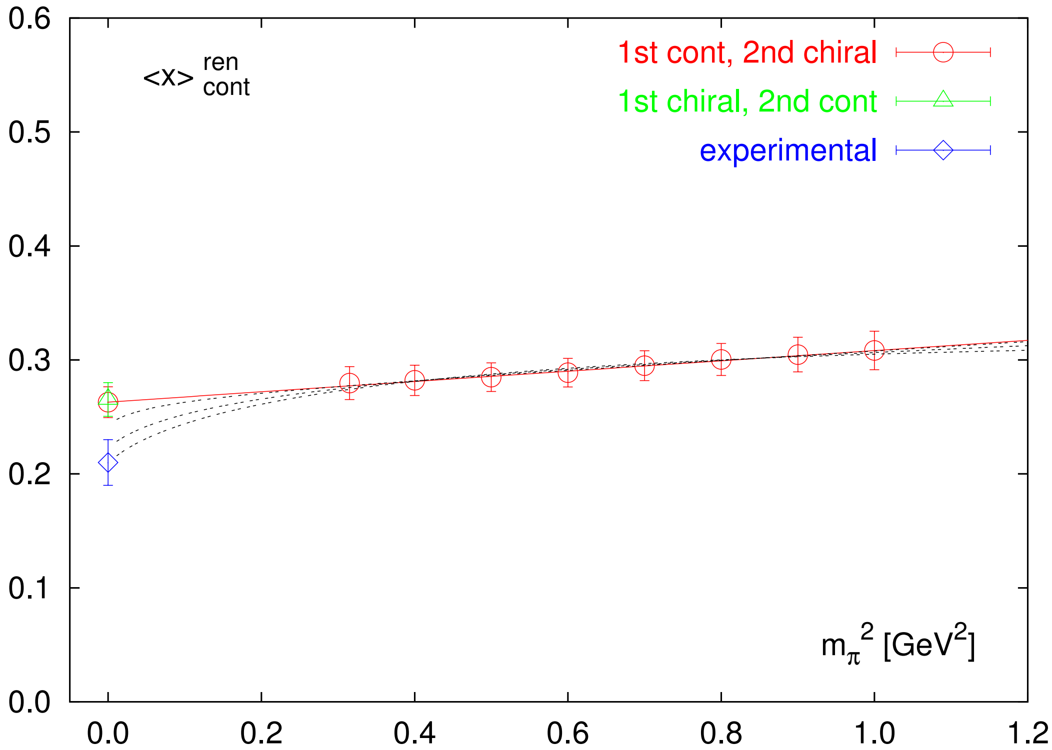
Pion matrix element of the twist 2 operator which corresponds to average momentum of the nonsinglet quark density: $\langle x \rangle_\pi$



Significant finite volume effects in this case. FVE are expected to be also very large in case of nucleon which might be a possible explanation for the current devation of lattice calculations from experiment in case of $\langle x \rangle_N$.

Structure functions on lattice

Calculation of $\langle x \rangle_\pi$ using non-perturbative renormalisation



Chiral extrapolation is an important issue. Non-linear fit gives value close to experiment, however more studies are needed, especially for lower quark masses.

Splitting functions

Splitting function at NNLO

A.Vogt: Complete results for next-to-next-to-leading order (NNLO) non-singlet and singlet splitting functions.

Small x limit of nonsinglet functions:

$$P^i_{(x\to 0)}(x) = D^i_0 \ln^4 x + D^i_1 \ln^3 x + D^i_2 \ln^2 x + D^i_3 \ln x$$

where

$$D^+_0 \simeq 1.58, \qquad D^+_1 \simeq 29.63 - 2.37 n_f,$$
$$D^+_2 \simeq 295.04 - 32.20 n_f + 0.59 n_f^2, \qquad D^+_3 \simeq 1261.11 - 152.60 n_f + 4.35 n_f^2$$

Strongly increasing coefficients of small x terms.

New, unpredicted small x term in P^s_{ns} :

$$\sim d^{abc} d_{abc} n_c \alpha_s^3 \ln^4 x$$

Splitting function at NNLO

Small x limit in singlet case:

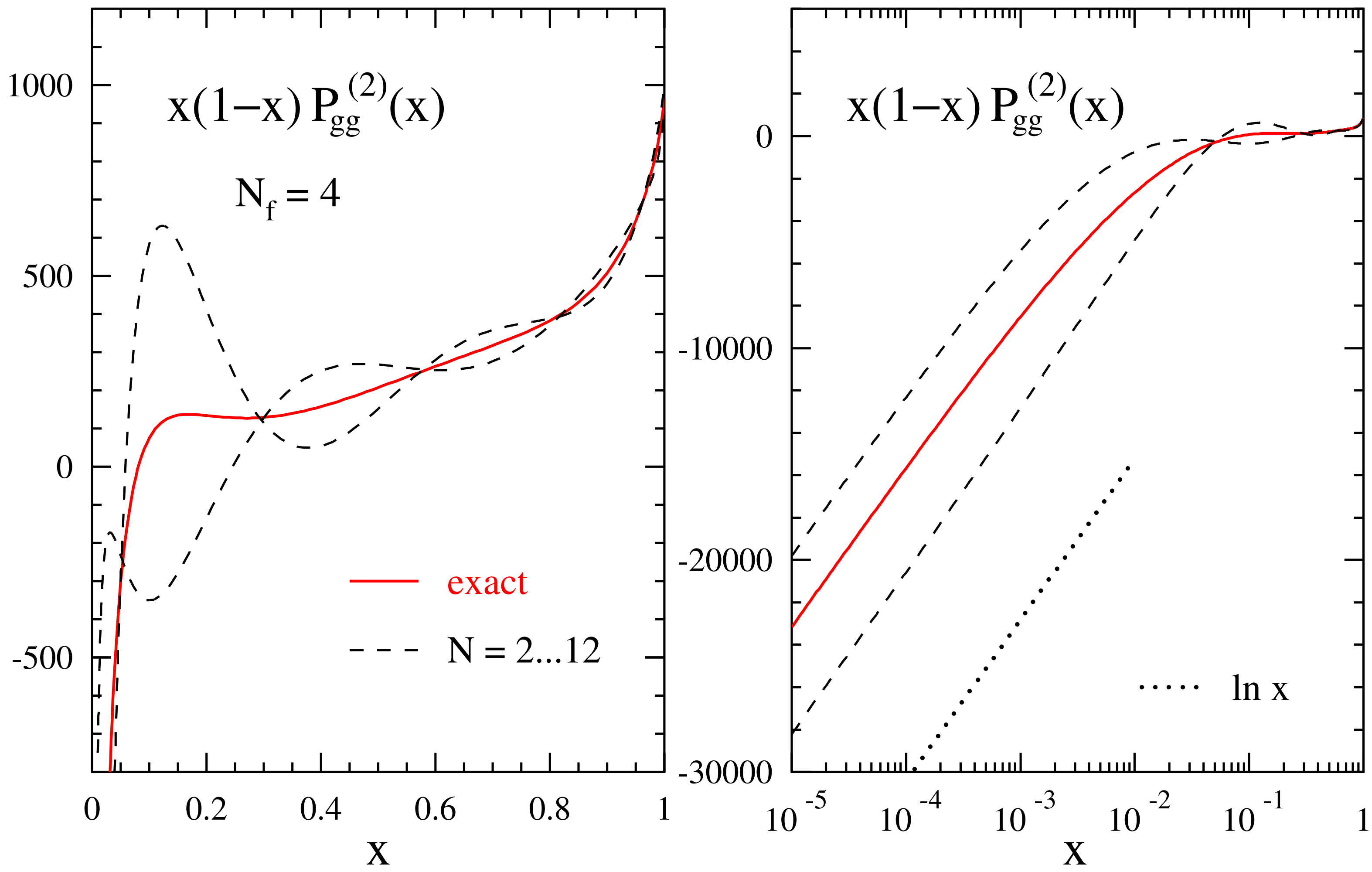
$$P_{ab}^{(2)} = E_1^{ab}\frac{\ln 1/x}{x} + E_2^{ab}\frac{1}{x} + \ldots$$

Small x terms E_1^{gg}, E_1^{qg} consistent with previous calculations: NLLx BFKL and Catani-Hautmann.

Exact values of P_{ab} within the error bands of the previous estimates.

One should not expect big differences from previous NNLO MRST estimates.

Splitting function at NNLO



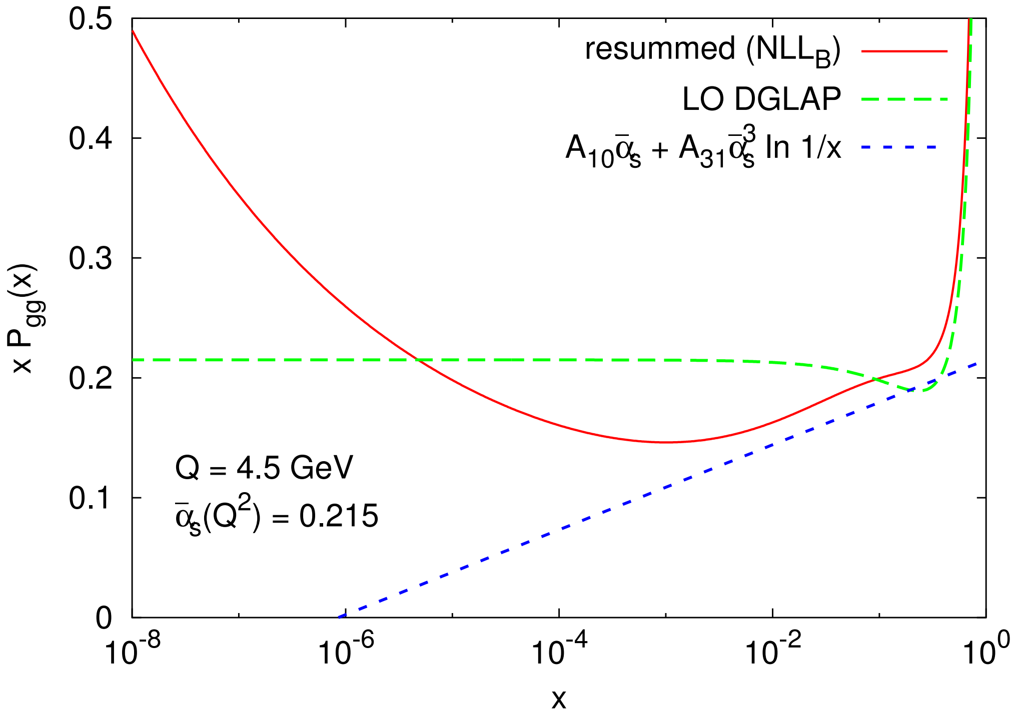
Note: P_{gg} is negative

Resummation at low x

G.Salam: Calculation of P_{gg} splitting function from the improved small x evolution equation with collinear logarithms.

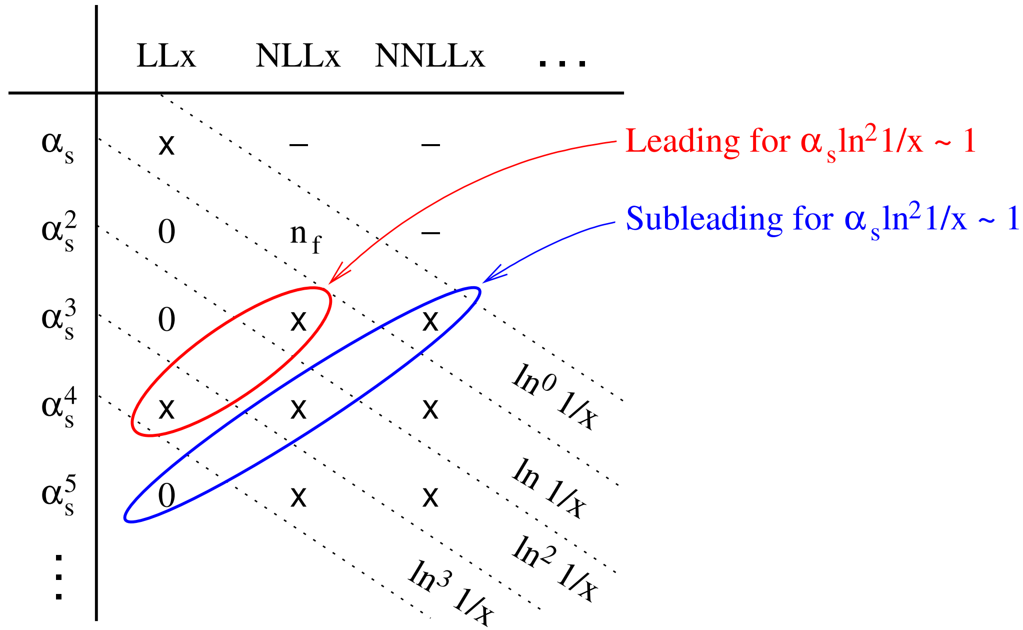
Interesting feature: *DIP* present at $x \sim 10^{-3}$, instead of immediate increase at low x.

Observed in all low x resummation methods (see Altarelli et.al, Thorne).



Dip in P_{gg}

New classification of small x terms: $\bar{\alpha}_s \log^2 1/x \sim 1$.



$$xP_{gg}(x) \simeq -|A_{31}|\bar{\alpha}_s^3 \log \frac{1}{x} + |A_{43}|\bar{\alpha}_s^4 \log^3 \frac{1}{x} + \ldots$$

Initial negative slope, starts at NNLO.
Poorly convergent expansion in $\sqrt{\bar{\alpha}_s}$, though NNLO quite stable, up to the position of the dip. Implications for HERA: NNLO vs Resummation.

Low x

BFKL Pomeron at $T > 0$

Collision of heavy nuclei $\rightarrow$ formation of quark-gluon plasma

Parton-parton scattering leads to thermalisation of plasma

The confining potential between $q\bar{q}$ disappears, which leads to J/Ψ suppression.

A similar mechanism could occur for glueballs.

L. Lipatov: Study of influence of $T > 0$ on a Pomeron: cyllinder topology

Results:

- resulting equation still has conformal symmetry, despite the presence of additional scale T
- energy dependence of the Pomeron the same as at $T = 0$
- extension to nonlinear equation
- different topologies $\rightarrow$ connection with string dynamics

Saturation

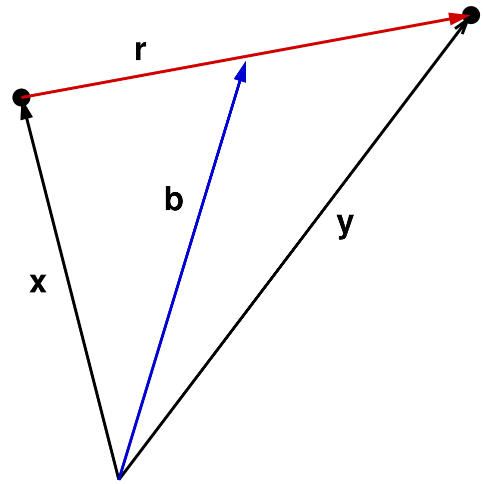
Balitsky-Kovchegov non-linear equation

$$\frac{\partial N_Y(\underline{x}_0,\underline{x}_1)}{\partial Y} = \bar{\alpha}_s \int \frac{d^2\underline{x}_2}{2\pi} \frac{(\underline{x}_0-\underline{x}_1)^2}{(\underline{x}_0-\underline{x}_2)^2(\underline{x}_1-\underline{x}_2)^2} [N_Y(\underline{x}_0,\underline{x}_2) + N_Y(\underline{x}_2,\underline{x}_1) - N_Y(\underline{x}_0,\underline{x}_1) - N_Y(\underline{x}_0,\underline{x}_2)N_Y(\underline{x}_2,\underline{x}_1)]$$

$N_Y(\underline{x}_0,\underline{x}_1)$ forward amplitude for scattering of the $q\bar{q}$ dipole on a nucleus target. Linear + rescattering term.

$$Y = \ln(\tfrac{1}{x}) \qquad \mathbf{r} = \underline{x}_0 - \underline{x}_1$$

$$\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi} \qquad \mathbf{b} = \frac{\underline{x}_0+\underline{x}_1}{2}$$



E.Naftali: study of solution with impact parameter + confinement

BFKL formalism and CDP

L.Lipatov: Using Möbius representation one can derive:

- Original BFKL equation (in LLx) is completely equivalent to the Mueller-BFKL equation in the Color Dipole Picture.
- The equation with the Triple Pomeron vertex is equivalent to Balitsky-Kovchegov equation (in the large N_c)
- New equation for finite N_c

$$\frac{dN_{xy}}{dY} = \bar{\alpha}_s \int \frac{d^2z}{2\pi} \frac{xy^2}{xz^2 yz^2} f_{xz,yz}$$

where

$$f_{xz,yz} = N_{xz} + N_{yz} - N_{xy} - G^{(4)}_{xz,yz} - \tfrac{1}{2(N_c^2-1)} (N_{xz} + N_{yz} - N_{xy})^2$$
$$G^{(4)}_{xz,yz} = N_{xz} N_{yz} + \Delta G^{(4)}$$

- powerful approach, possibly NLLx corrections can be implemented in the CDP

Travelling waves and Saturation scale

In the approximation of large nucleus $R \gg r$, impact parameter b in BK equation can be neglected leading to 1+1 dim. problem

$$\partial_Y N = \bar{\alpha}_s \chi(-\partial_L) N - \bar{\alpha}_s N^2$$

with χ BFKL eigenvalue.

R.Peschanski: BK equation falls into class of nonlinear equation with general properties:

- $N = 0$ is unstable fixed point w.r.t. linear evolution
- nonlinearity tames the growth when $N \sim \mathcal{O}(1)$
- the initial condition must be sufficiently steep at large k, ($N \sim 1/k^2$)

Relation with other fields in physics: Disordered phenomena, spin glass

phase transitions, polymer diffusion

Travelling wave fronts

N is a travelling wave

$$N(\ln k^2 - \ln Q_s^2(Y))$$

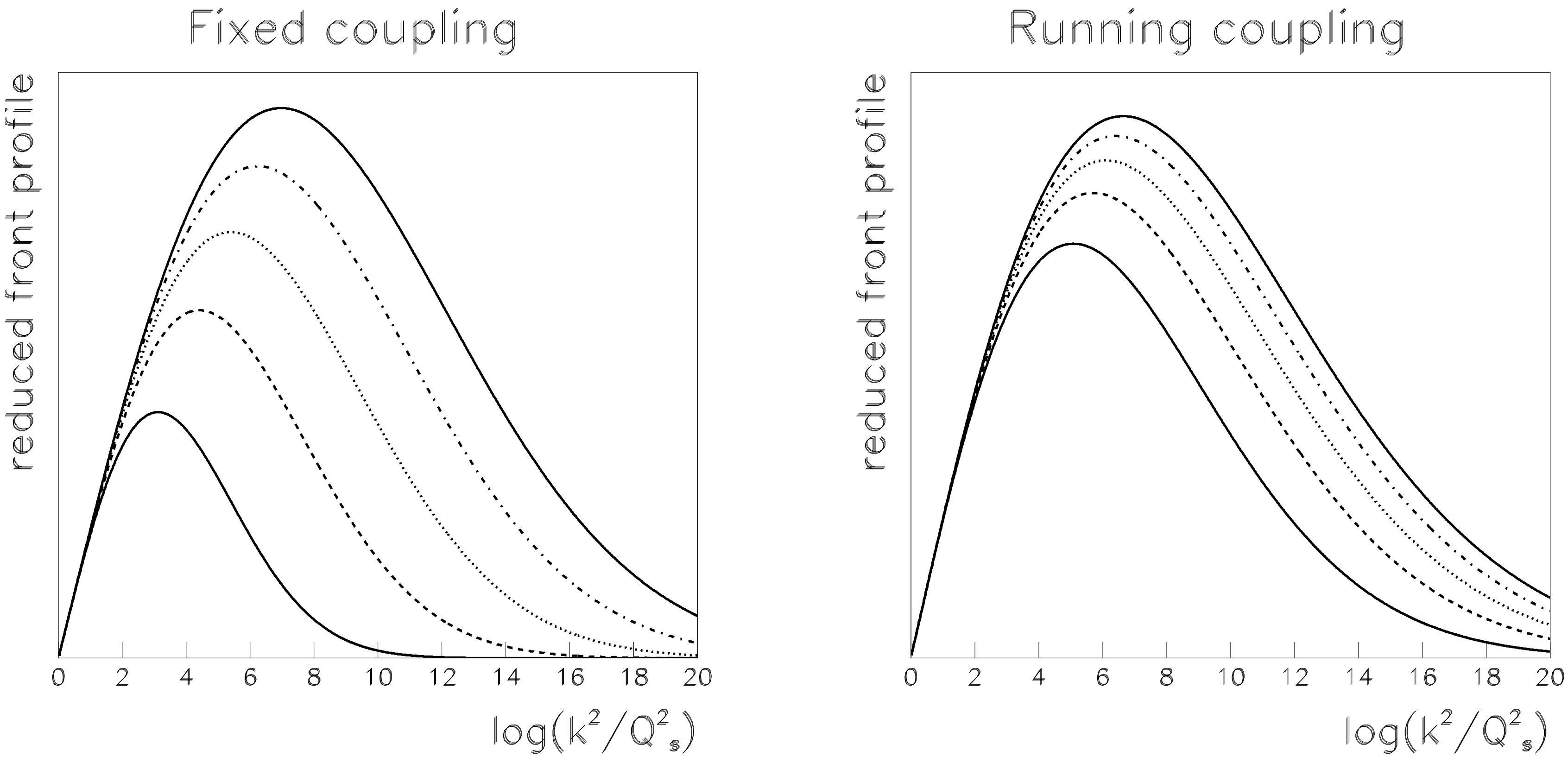
which corresponds to geometric scaling. A systematic expansion has been provided for $Q_s(Y)$

$$\ln Q_s^2(Y) \;=\; \bar{\alpha}_s \frac{\chi(\gamma_c)}{\gamma_c} Y - \frac{3}{2\gamma_c} \ln Y - \frac{3}{\gamma_c^2} \sqrt{\frac{2\pi}{\bar{\alpha}_s \chi''(\gamma_c)}} \frac{1}{\sqrt{Y}}$$

Solution N around the wave front

$$N(k,Y) \simeq \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c} e^{-\frac{\ln^2 k^2/Q_s^2(Y)}{2\bar{\alpha}_s \chi''(\gamma_c) Y}} \times \{\gamma_c \ln(k^2/Q_s^2(Y) + \dots\}$$

Travelling wave fronts

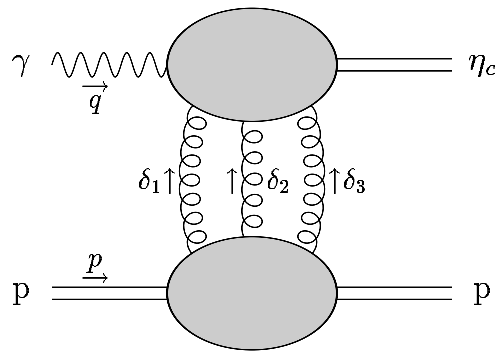


Rigorous results for $Q_s(Y)$ and N also in the running coupling case.

Numerical study: K. Golec-Biernat

Odderon: recap

- Odderon: partner of Pomeron but with $C = -1$
- It gives contribution to the difference between particle-particle and particle-antiparticle collisions cross sections.
- Test at HERA: an exclusive photo(electro)-production of η_c pseudoscalar meson.



- QCD evolution equation for the Odderon - BKP equation

Odderon in dipole picture

Y.Kovchegov: Odderon evolution equation in rapidity Y

$$\frac{\partial \mathcal{O}_Y(\underline{x}_0, \underline{x}_1)}{\partial Y} = \bar{\alpha}_s \int \frac{d^2 \underline{x}_2}{2\pi} \frac{(\underline{x}_0 - \underline{x}_1)^2}{(\underline{x}_0 - \underline{x}_1)^2 (\underline{x}_1 - \underline{x}_2)^2} \cdot$$
$$\cdot \left[\mathcal{O}_Y(\underline{x}_0, \underline{x}_1) + \mathcal{O}_Y(\underline{x}_2, \underline{x}_1) - \mathcal{O}_Y(\underline{x}_0, \underline{x}_1)\right]$$

with C-odd initial conditions

$$\mathcal{O}_{Y=0}(\underline{x}_0, \underline{x}_1) \;=\; -\mathcal{O}_{Y=0}(\underline{x}_1, \underline{x}_0) \;=\; c_0 \alpha_s^3 \ln^3 \frac{x_0}{x_1}$$

$\underline{x}_0, \underline{x}_1$ are transverse coordinates of $q\bar{q}$ dipole pair. The evolution equation for Odderon is the same as for Pomeron, the difference being in C-odd or C-even initial conditions.

Odderon in dipole picture

The leading high energy intercept is then

$$\omega_{\rm Odd} - 1 \;=\; \bar{\alpha}_s \chi(n=1, \nu=0) = 0$$

the same as Bartels-Lipatov and Vacca.
Including saturation: coming through the Pomeron amplitude N (which includes multiple scatterings)

$$\frac{\partial \mathcal{O}_Y(\underline{x}_0, \underline{x}_1)}{\partial Y} = K \otimes \mathcal{O}_Y(\underline{x}_0, \underline{x}_1) - \frac{\alpha_s N_c}{2\pi^2} \int d^2 \underline{x}_2 \frac{\underline{x}_{01}{}^2}{\underline{x}_{02}{}^2 \underline{x}_{12}{}^2} .$$
$$[\mathcal{O}_Y(\underline{x}_0, \underline{x}_2) N_Y(\underline{x}_2, \underline{x}_1) + N_Y(\underline{x}_0, \underline{x}_2) \mathcal{O}_Y(\underline{x}_2, \underline{x}_1)]$$

The solution not known, likely to be decreasing function on energy.

S - matrix in the high energy limit

Two different formalisms at high energy which include unitarity corrections:

Color Glass Condensate
(JIMWLK equation)

Color Dipole Picture

E.Iancu: Two approaches are in fact equivalent.

This means that the full solution to JIMWLK equation should match the Monte Carlo simulations for the onium-onium scattering done by G.Salam.

Role of fluctuations

Monte Carlo simulations done by G.Salam show the crucial role of fluctuations in the onium wave function $\rightarrow$
correlations between multiple scatterings.

Full Monte Carlo:

$$S_Y \sim \exp(-\kappa \alpha_s^2 Y^2)$$

Incoherent scattering:

$$S_Y \sim \exp(-\kappa_0 \alpha_s^2 e^{Y\lambda})$$

E.Iancu: S matrix is much larger when one takes the fluctuations into account.

Unitarity effects can be therefore significant and set in early, even when S is still quite large (or amplitude $N = 1 - S$ relatively small).

Saturation in hadronic collisions

C.Marquet: Extension of the GBW saturation model for study of Mueller-Navelet jets. Assuming k_T factorisation theorem:

$$\sigma = \int d^2 r_1 d^2 r_2 \phi^{(1)}(r_1, Q_1^2)\, \phi^{(2)}(r_1, Q_1^2)\, \sigma_{dd}(\Delta\eta, r_1, r_2)$$

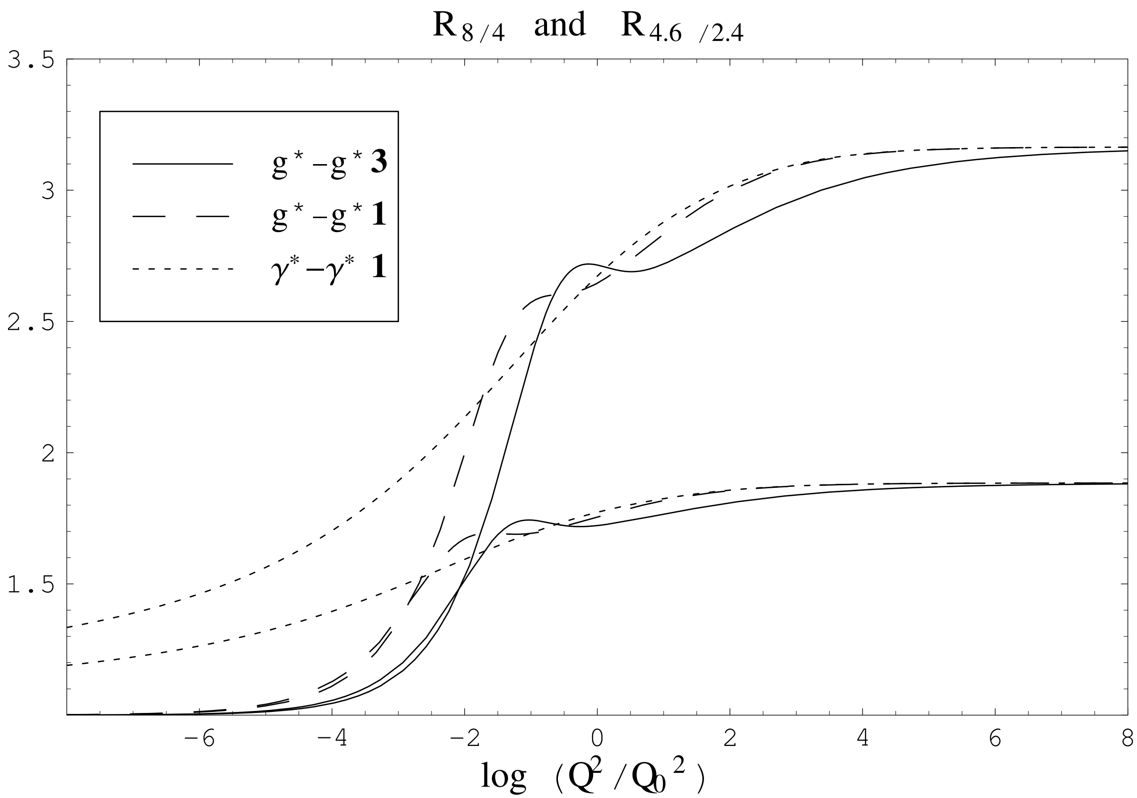
with

$$\sigma_{dd}(\Delta\eta, r_1, r_2) = \sigma_0 \left\{ 1 - \exp(-\frac{r_{eff}^2}{4R_0^2(\Delta\eta)}) \right\}$$

taken from GBW generalisation to $\gamma^*\gamma^*$ collisions (N.Timneau et.al). r_{eff}

is a combination of r_1 and r_2.

Saturation in hadronic collisions



Cross section ratios:

$$R_{i|j} = \frac{\sigma(Q_1, Q_2, \Delta\eta_i)}{\sigma(Q_1, Q_2, \Delta\eta_j)}$$

More abrupt transition to saturation than in the $\gamma^*\gamma^*$ case.

Some open questions: qqg state, validity of k_T factorisation

Saturation at HERA

BK equation $\longrightarrow$ F_2 from dipole picture $\longrightarrow$ HERA data

but

BK equation is not complete: running α_s?, $NLLx$?, impact parameter dependence?, applicability at small Q^2?

Dipole picture: is it valid beyond LLx?
Need to study NLLx photon-gluon impact factor A.Kyrieleis

Usually description of HERA data with some model containing numbers of parameters which are not well under control.

F_2 data too inclusive, perhaps better to study more exclusive processes:

diffraction, DVCS (S.Munier, L.Favart, T.Rogers)

Unintegrated parton distributions

Unintegrated parton distributions

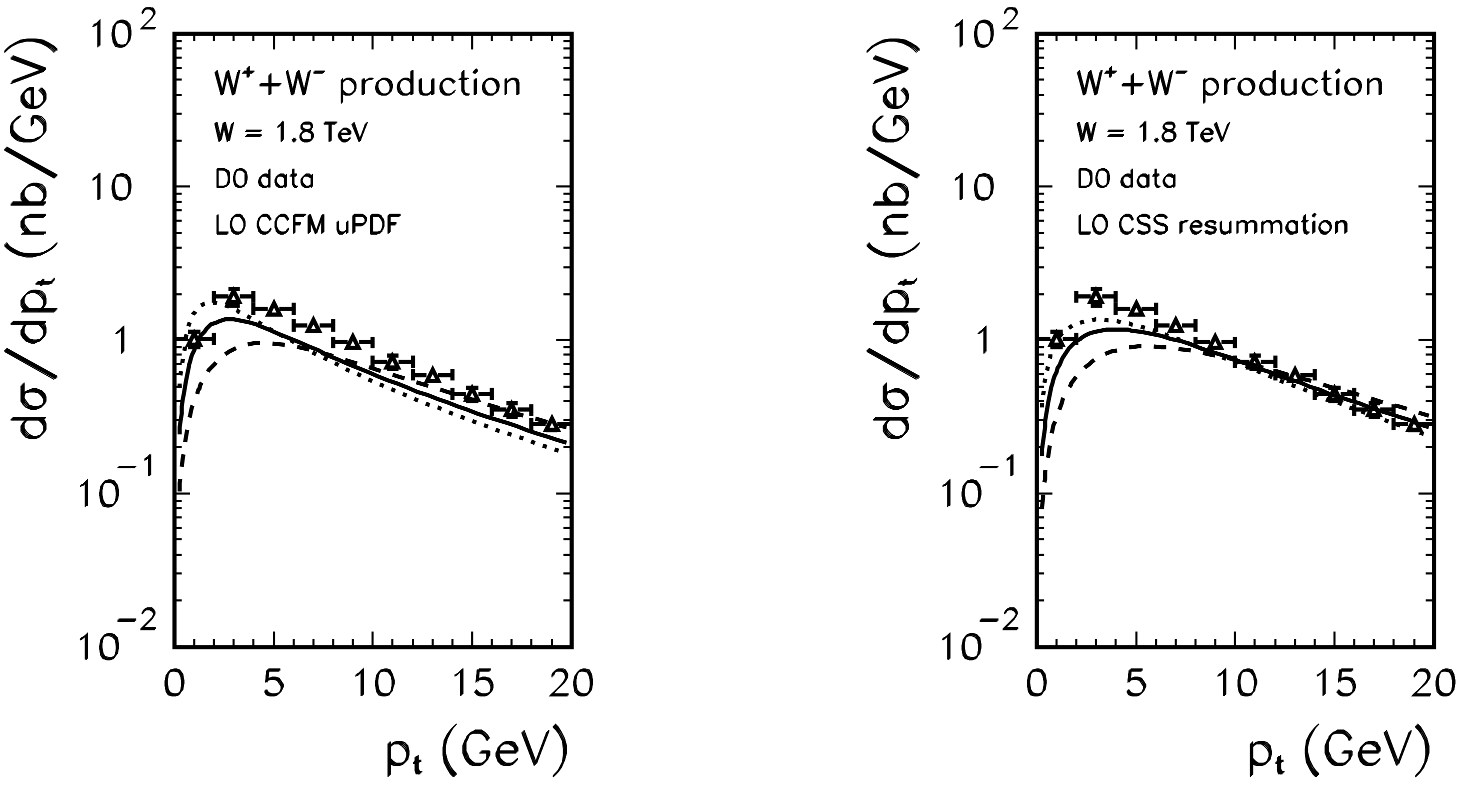
W. Broniowski, A. Szczurek, H. Jung, L. Lonnblad: unintegrated parton distributions from CCFM, one loop approximation to CCFM, Linked Dipole Chain model

Various applications at hadron colliders:

- heavy quark production
- p_T distribution of $W^{\pm}, Z^0$
- p_T distribution of the Higgs
- exclusive diffractive Higgs
- π^+, π^- spectra

p_T *distribution of* W *at Tevatron*

A.Szczurek



Soft resummation formulae (CSS) can be obtained from CCFM equation in the one-loop approximation (papers by J.Kwieciński et al)