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A SIMPLE MODEL FOR GENERALISED PARTON DISTRIBUTIONS OF THE PION

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We present here an extension of our model for the pion, which we used previously to calculate its diagonal structure function, to the off-forward case. The imaginary part of the off-forward $\gamma^\star\pi \to \gamma^\star\pi$ scattering amplitude is evaluated in the chiral limit ($m_\pi = 0$) and related to the twist-two and twist-three generalised parton distributions H, H^3, $\tilde{H}^3$. Non-perturbative effects, linked to the size of the pion and still preserving gauge invariance, are included. Remarkable new relations between H, H^3 and $\tilde{H}^3$ are obtained and discussed.

1 Introduction

Structure functions are useful tools to understand the structure of hadrons. At large Q^2, they are related to parton distributions. Although their Q^2-evolution is consistent with perturbative QCD, their bulk properties come from nonperturbative effects. The latter are often treated by low-energy models, such as NJL, which establish a connection with the low Q^2 physics. There has been extensive work on diagonal distributions along these lines (see Ref. [1] and references therein for the pion case).

The interest has now turned to the off-diagonal case [2,3,4]. For the latter, the off-forward structure functions are related to the off-forward γ^*-hadron amplitude and appear as convolutions of generalised parton distributions. These carry information about correlations between partons. In order to illustrate the properties of these quantities, we undertook to calculate them in the case of the pion. In Ref. [1], we first calculated the forward amplitude and the quark distribution in a simple model, in which the pion field is coupled to (constituent) quark fields through a γ^5 vertex. Furthermore, pion size effects are introduced through a gauge-invariant procedure by requiring that the squared relative momentum of the quarks inside the pion is smaller than a cut-off value. The most remarkable result of this investigation is that the momentum fraction carried by the quarks is smaller than one, although gluonic degrees of freedom are not included. Here, we report on the extension of our model to the off-diagonal case [2].

In the following, we calculate the imaginary part of the off-forward photon-pion scattering amplitude, and of the structure functions $F_1, \ldots, F_5$, related to the five independent tensor structures in the scattering amplitude, and we discuss their behaviour. We relate them to vector and axial vector form factors and to the twist-two and twist-three generalised parton distributions (GPD's) H, H^3 and $\tilde{H}^3$. We shall show that, within our model and in the high-Q^2 limit, the non-diagonal structure functions F_3 and F_4 are related to F_1, while F_5 happens to be a higher twist. These results lead to new relations for the GPD's in the neutral pion case.

2 TENSORIAL STRUCTURE OF THE $\gamma^\star\pi \to \gamma^\star\pi$ AMPLITUDE

We adopt the kinematics shown in Fig. 1. We use the Lorentz invariants $t = \Delta^2$, $Q^2 = -q^2$, $x = Q^2/2p\cdot q$ and $\xi = \Delta\cdot q/2p\cdot q$. The diagonal limit is characterised by $\xi = t = 0$, the elastic limit by $\xi = 0$, and the deeply virtual compton scattering (DVCS) limit by $\xi = -x$ for $t \ll Q^2$.

The hadronic tensor $T_{\mu\nu}(q,p,\Delta)$ can be written, for a scalar or pseudoscalar target, as [3]

$$T_{\mu\nu} = -\mathcal{P}_{\mu\sigma}g^{\sigma\tau}\mathcal{P}_{\tau\nu}F_1 + \frac{\mathcal{P}_{\mu\sigma}p^\sigma p^\tau\mathcal{P}_{\tau\nu}}{p\cdot q}F_2 + \frac{\mathcal{P}_{\mu\sigma}(p^\sigma(\Delta^\tau-2\xi p^\tau)+(\Delta^\sigma-2\xi p^\sigma)p^\tau)\mathcal{P}_{\tau\nu}}{2p\cdot q}F_3$$
$$+\frac{\mathcal{P}_{\mu\sigma}(p^\sigma(\Delta^\tau-2\xi p^\tau)-(\Delta^\sigma-2\xi p^\sigma)p^\tau)\mathcal{P}_{\tau\nu}}{2p\cdot q}F_4 + \mathcal{P}_{\mu\sigma}(\Delta^\sigma-2\xi p^\sigma)(\Delta^\tau-2\xi p^\tau)\mathcal{P}_{\tau\nu}F_5. \quad (1)$$

Current conservation is guarenteed by means of the projector $\mathcal{P}_{\mu\nu} = g_{\mu\nu} - \frac{q_{2\mu}q_{1\nu}}{q_1\cdot q_2}$, where q_1 and q_2 are the momenta of the incoming and outgoing photons, respectively. The structure functions F_i are functions of the invariant quantities x, ξ and t. They are all even functions of ξ, except for F_3, which is odd.

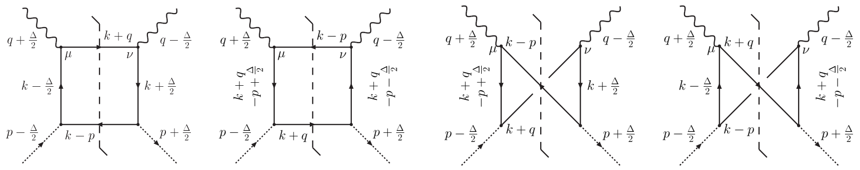

Figure 1. The simplest diagrams contributing to the imaginary part of the amplitude for the scattering $\gamma^\star\pi \to \gamma^\star\pi$. Dashed lines represent the discontinuity of the amplitudes, *i.e.* their imaginary parts.

3 THE MODEL

The model introduced in our previous work [1] includes massive pion and massive quark fields and a pion-quark coupling described by the Lagrangian interaction density $\mathcal{L}_{int} = ig(\overline{\psi}\ \vec{\tau}\gamma_5\psi).\ \vec{\pi}$, where ψ is the quark field, $\vec{\pi} = (\pi^+,\pi^0,\pi^-)$ is the pion field and $\vec{\tau}$ is the isospin operator.

At leading order in the loop expansion, four diagrams contribute, see in Fig. 1. We have evaluated their imaginary part, using the integration variables $\tau = k^2$, $k_\rho = |\vec{k}|$, ϕ and θ, the polar angles of $\vec{k}$ with respect to the direction of the incoming photon. Actually, due to the discontinuity of the diagrams, indicated in Fig. 1, we must integrate on τ and ϕ only. We do not give the expressions here. They can be found in Ref. [2] . However, we can sketch our procedure for imposing a finite size to the pion. The relative four-momentum squared of the quarks inside the pion is given by

$$O^\pm = \left(2k - p \pm \frac{\Delta}{2}\right)^2 = 2\tau + 2m_q^2 - m_\pi^2 + \frac{t}{2} \pm 2k\cdot\Delta, \quad (2)$$

for pion-quark vertices like the ones in the first diagram of Fig. 1. Note that this can be rewritten as

$$O^{\pm} = \left(k \pm \frac{\Delta}{2}\right)^2 + 2m_q^2 - m_\pi^2. \tag{3}$$

The first quantity in the r.h.s. being nothing but the squared momentum transfer for the $\gamma^\star \pi \to q\overline{q}$ process, $O^{\pm}$ can be written as a function of the external variables for this process and of the masses. Similar expressions hold for other vertices. Generalizing the procedure of Ref. [1], we require $|O^{\pm}| < \Lambda^2$ either for one or the other vertex of each diagram. Gauge invariance is therefore preserved by this cut-off, as it can be thought of as a constraint on the intermediate state cut lines. In practice, this is equivalent to requiring one of the two following conditions:

$$\tau < -\tfrac{\Lambda^2}{2} + \tfrac{m_\pi^2}{2} - m_q^2 - \tfrac{t}{4} + |k \cdot \Delta| \,, \; \tau > \tfrac{\Lambda^2}{2} - \tfrac{m_\pi^2}{2} + 3m_q^2 + \tfrac{t}{4} - \tfrac{Q^2}{x} - \left|\tfrac{\xi Q^2}{x} + k \cdot \Delta\right| . \tag{4}$$

As explained in Ref. [1], owing to these conditions and for small t, the crossed diagrams are suppressed by a power Λ^2/Q^2, compared to the box diagrams.

We keep the coupling constant g as in the diagonal case, where it was determined by imposing that there are only two constituent quarks in the pion, or equivalently that the following relation

$$\int_0^1 F_1 dx = \frac{5}{18} \tag{5}$$

holds, which makes g dependent upon Q^2. It turns out that, with the cut-off, g reaches an asymptotic value for Q^2 above 2 GeV^2.

4 Results for the structure functions

4.1 General features

From the imaginary part of the total amplitude, the imaginary part of the five structure functions F_i can be obtained by a projection on the corresponding tensors. For any fixed value of ξ not close to ± 1, we recovered for F_1 and F_2 the same behaviour as in the diagonal case. We checked indeed that the diagonal limit is recovered for $\xi = 0$ and $t = 0$. Furthermore the structure functions F_3, F_4, F_5 depend little on ξ except when this variable is close to ± 1. In the particular case of DVCS, in the presence of finite-size effects, the value of F_1 gets significantly reduced, especially for small x, as $|t|$ increases, whereas that effect is much less noticeable without cut-off. In the elastic case, the same suppression at small x is observed, especially when the cut-off is applied.

The effect of the variation of Q^2 were also studied. As in the diagonal case [1], we can conclude that the details of the non-perturbative effects cease to matter for Q^2 greater than 2 GeV^2, that is significantly larger than Λ^2.

4.2 High-Q^2 limit: new relations

Having determined the 5 functions F_i's in the context of our model, we shall now consider their behaviour at high Q^2. Expanding the ratios of $\frac{F_2}{F_1}, \frac{F_3}{F_1}, \frac{F_4}{F_1}, \frac{F_5}{F_1}$, we obtain the following asymptotic behaviour:

$$F_2 = 2xF_1 + \mathcal{O}(1/Q^2), \qquad F_3 = \frac{2x\xi}{\xi^2-1}F_1 + \mathcal{O}(1/Q^2), \tag{6}$$

$$F_4 = \frac{2x}{\xi^2-1}F_1 + \mathcal{O}(1/Q^2), \qquad F_5 = \mathcal{O}(1/Q^2). \tag{7}$$

The first relation is similar (at leading order in $1/Q^2$ and with the replacement of x by x_B) to the Callan-Gross relation between the diagonal structure functions F_1 and F_2, valid for spin one-half constituents in general. Except for F_5, which is small at large Q^2, these relations show that F_2, F_3 and F_4 are simply related to F_1 at leading order. They also clearly display and therefore confirm the symmetries of these functions. The fact of getting such simple relations between the F_i's (at leading order) constitutes a remarkable result of our model. Furthermore, we checked that the term $\mathcal{O}(1/Q^2)$ in the first relation is numerically quite small, even for moderate Q^2. One may wonder whether these results are typical of our model, or more general.

5 Linking the F_i's to H, H^3, and $\tilde{H}^3$

Having at hand the five functions F_i's that parametrise the amplitude for $\gamma^\star\pi \to \gamma^\star\pi$, we can link them to the generalised parton distributions. For this purpose, we make use of a tensorial expression coming from the twist-three analysis of the process, which singles out the twist-two $\mathcal{H}$ and the twist-three $\mathcal{H}^3, \tilde{\mathcal{H}}^3$ form factors. Following Ref. [3], we write[a]:

$$T_{\mu\nu} = -\mathcal{P}_{\sigma\mu} g^{\sigma\tau} \mathcal{P}_{\nu\tau} \tfrac{q\cdot V_1}{2p\cdot q} + (\mathcal{P}_{\sigma\mu} p^\sigma \mathcal{P}_{\nu\rho} + \mathcal{P}_{\rho\mu} p^\sigma \mathcal{P}_{\nu\sigma}) \tfrac{V_2^\rho}{p\cdot q} - \mathcal{P}_{\sigma\mu} i\epsilon^{\sigma\tau q\rho} \mathcal{P}_{\nu\tau} \tfrac{A_{1\,\rho}}{2p\cdot q}. \tag{8}$$

where the V_i's and A_1 read

$$V_{1\,\rho} = 2p_\rho \mathcal{H} + (\Delta_\rho - 2\xi p_\rho)\mathcal{H}^3 + \text{twist } 4, A_{1\,\rho} = \frac{i\epsilon_{\rho\Delta p q}}{p\cdot q}\,\tilde{\mathcal{H}}^3, \tag{9}$$

$$V_{2\,\rho} = xV_{1\,\rho} - \frac{x}{2}\frac{p_\rho}{p\cdot q} q\cdot V_1 + \frac{i}{4}\frac{\epsilon_{\rho\sigma\Delta q}}{p\cdot q} A_1^\sigma + \text{twist } 4. \tag{10}$$

In Ref. [3], gauge invariance of Eq. (8) beyond the twist-three accuracy was in fact restored by hand, contrarily to the present calculation for which the amplitude is explicitly gauge invariant.

To relate the F_i's to the $\mathcal{H}$'s, we project the amplitude (8) onto the five projectors contained in Eq. (1) and identify the results with the F_i's. Note that, in the neutral pion case, the imaginary part of the form factors $\mathcal{H}$, $\mathcal{H}^3$ and $\tilde{\mathcal{H}}^3$ directly gives the GPD's H, H^3 and $\tilde{H}^3$ up to a factor 2π. As we have kept the off-shellness of the photons arbitrary, we in fact can relate the imaginary parts of F_i to the GPD's for arbitrary x and ξ (up to $\mathcal{O}(1/Q^2)$ terms):

$$\frac{F_1}{2\pi} = H, \; \frac{F_2}{2\pi} = 2xH, \; \frac{F_3}{2\pi} = \frac{2x}{x^2-\xi^2}\left(H^3x^2 + \tilde{H}^3\xi x - H\xi\right), \tag{11}$$

$$\frac{F_4}{2\pi} = \frac{2x}{x^2-\xi^2}\left(H^3\xi x + \tilde{H}^3 x^2 - Hx\right), \; \frac{F_5}{2\pi} = \mathcal{O}(1/Q^2) \tag{12}$$

[a] Please note that Ref. [3] uses $\mathcal{P}_{\nu\mu}$ instead of $\mathcal{P}_{\mu\nu}$ as projector.

Replacing the F_i's by the expressions (6), we can write (up to $\mathcal{O}(1/Q^2)$ terms)

$$\tilde{H}^3 = \frac{(x-1)}{x(\xi^2-1)} H \text{ and } H^3 = \frac{(x-1)\xi}{x(\xi^2-1)} H = \xi \tilde{H}^3. \tag{13}$$

As F_1 to F_4 can be written in term of only one of them, *e.g.* F_1, it is not surprising that H^3 and $\tilde{H}^3$ are simply related to H. Note that polynomiality of the Mellin moments of H, H^3 and $\tilde{H}^3$, together with Eqs. (13), imply that H must be the product of ξ^2-1 with a polynomial in ξ, P_H. Morevover, the fact that $\tilde{H}_3$ is almost independent of ξ shows that P_H is very close to a constant.

To convince ourselves that relations (13) are new, we have compared them to the Wandzura-Wilczek approximation [5], given for the pion case in [3,6]. First of all, it is well-known that these relations are discontinuous at $\xi = \pm x$, which is not the case for (13). Furthermore, we compared the results of the Wandzura-Wilczek approximation with our results. We found that the two are numerically very different. Hence relations (13), derived in an explicitly gauge-invariant model, do not come from "kinematical" twist corrections, but emerge from the dynamics of the spectator quark propagator and from finite-size effects.

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