Supersymmetry

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- ⋆ The Lightest Higgs Boson in the Large $\tan\beta$ regime of MSSM
- ⋆ The Lightest Higgs Boson in the NMSSM

- Focus Points in the MSSM - The Lightest Higgs Boson mass
- Conclusions

The Standard Model and its Drawbacks

- The Standard Model (SM): theory of strong, weak, and electromagnetic interactions based on the gauge principle
  - ⋆ The group of the SM: $SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$
  - ⋆ Field content of the SM:
    - *Gauge sector*: Spin$= 1$

$$
\begin{array}{cclll}
\textit{gluons} & G^a_\mu : & (8,1,0) & SU_c(3) & g_s \\
\textit{intermediate weak bosons} & W^i_\mu : & (1,3,0) & SU_L(2) & g \\
\textit{abelian boson} & B_\mu : & (1,1,0) & U_Y(1) & g'
\end{array}
$$

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- *Fermion sector*: Spin$= 1/2$

quarks

$$
\begin{array}{lcccccc}
Q^i_{\alpha L} = & \left(\begin{array}{c} U^i_\alpha \\ D^i_\alpha \end{array}\right)_L & = & \left(\begin{array}{c} u^i \\ d^i \end{array}\right)_L & \left(\begin{array}{c} c^i \\ s^i \end{array}\right)_L & \left(\begin{array}{c} t^i \\ b^i \end{array}\right)_L & (3, 2, 1/3) \\
 & U^i_{\alpha R} & = & u_{iR} & c_{iR} & t_{iR} & (3^*, 1, 4/3) \\
 & D^i_{\alpha R} & = & d_{iR} & s_{iR} & b_{iR} & (3^*, 1, -2/3) \\
\textit{leptons} & L_{\alpha L} & = & \left(\begin{array}{c} \nu_e \\ e \end{array}\right)_L & \left(\begin{array}{c} \nu_\mu \\ \mu \end{array}\right)_L & \left(\begin{array}{c} \nu_\tau \\ \tau \end{array}\right)_L & (1, 2, -1) \\
 & E_{\alpha R} & = & e_R & \mu_R & \tau_R & (1, 1, -2)
\end{array}
$$

$i = 1, 2, 3$ - color, $\alpha = 1, 2, 3, \ldots$ - generation.

- *Higgs (scalar) sector*: Spin$= 0$

$$
H = \left(\begin{array}{c} H^0 \\ H^- \end{array}\right), \quad (1, 2, -1)
$$

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⋆ The Lagrangian of the SM:

$$L = L_{gauge} + L_{Yukawa} + L_{Higgs},$$

where

$$\begin{aligned} L_{gauge} \;=\; & -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} \\ & + i\bar{L}_\alpha\gamma^\mu D_\mu L_\alpha + i\bar{Q}_\alpha\gamma^\mu D_\mu Q_\alpha + i\bar{E}_\alpha\gamma^\mu D_\mu E_\alpha \\ & + i\bar{U}_\alpha\gamma^\mu D_\mu U_\alpha + i\bar{D}_\alpha\gamma^\mu D_\mu D_\alpha + (D_\mu H)^\dagger(D_\mu H), \end{aligned}$$

with

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu, \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned}$$

and

$$
\begin{aligned}
D_\mu L_\alpha &= \left(\partial_\mu - i\frac{g}{2}\tau^i W^i_\mu + i\frac{g'}{2}B_\mu\right) L_\alpha, \\
D_\mu E_\alpha &= \left(\partial_\mu + ig' B_\mu\right) E_\alpha, \\
D_\mu Q_\alpha &= \left(\partial_\mu - i\frac{g}{2}\tau^i W^i_\mu - i\frac{g'}{6}B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu\right) Q_\alpha, \\
D_\mu U_\alpha &= \left(\partial_\mu - i\frac{2g'}{3}B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu\right) U_\alpha, \\
D_\mu D_\alpha &= \left(\partial_\mu + i\frac{g'}{3}B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu\right) D_\alpha.
\end{aligned}
$$

$$L_{Yukawa} = y^l_{\alpha\beta}\bar{L}_\alpha E_\beta H + y^d_{\alpha\beta}\bar{Q}_\alpha D_\beta H + y^u_{\alpha\beta}\bar{Q}_\alpha U_\beta \tilde{H} + h.c.,$$

where $\tilde{H} = i\tau_2 H^\dagger$.
In the end

$$L_{Higgs} = m^2 H^\dagger H - \lambda (H^\dagger H)^2.$$

⋆ Particle masses - spontaneous symmetry breaking of $SU_L(2)$ as a result of the non-zero vacuum expectation value of the Higgs field:

$$\langle H \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}, \quad v = m/\sqrt{2\lambda}.$$

As a result the gauge group is spontaneously broken

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \Rightarrow SU_c(3) \otimes U_{EM}(1).$$

The weak bosons are

$$W^{\pm}_{\mu} = \frac{W^1_{\mu} \mp i W^2_{\mu}}{\sqrt{2}}, \quad Z_{\mu} = -\sin\theta_W B_{\mu} + \cos\theta_W W^3_{\mu},$$

with masses

$$m_W = \frac{gv}{\sqrt{2}}, \quad m_Z = \frac{m_W}{\cos\theta_W}, \quad \tan\theta_W = \frac{g'}{g}.$$

The photon field remains massless

$$\gamma_{\mu} = \cos\theta_W B_{\mu} + \sin\theta_W W^3_{\mu}.$$

The matter fields acquire masses

$$M^u_{\alpha\beta} = y^u_{\alpha\beta} v, \quad M^d_{\alpha\beta} = y^d_{\alpha\beta} v, \quad M^l_{\alpha\beta} = y^l_{\alpha\beta} v, \quad m_H = \sqrt{2} m.$$

- Drawbacks of the SM
  - ⋆ large number of parameters.
  - ⋆ formal unification of strong and electroweak interactions.
  - ⋆ Higgs boson, its nature. It is not clear whether it is fundamental or composite.
  - ⋆ number of generations is arbitrary
  - ⋆ origin of the mass spectrum

  The answers lie beyond the SM

- Three possible ways:
  - ⋆ the same fundamental fields with new interactions
    - supersymmetry
    - Grand Unification
    - String Theory
  - ⋆ new fundamental fields with new interactions
    - compositeness
    - Technicolour
    - extended Technicolor
    - preons
  - ⋆ Exotic ways
    - gravity at TeV energies
    - large extra dimensions
    - brane worlds

Supersymmetry

- Motivation of SUSY
  - ⋆ Idea of unification toward smaller distances up to $l_{Planck} \sim 10^{-33}$. But:
    - – graviton has spin 2, gauge bosons have spin 1: they correspond to different representations of Poincare algebra.
    - – no-go theorems: their unification within unique algebra is forbidden
    - – the only possibility: supersymmetry algebra (possibility to combine fields with different spins into the same supermultiplet)
    - – Mathematical statment: the *uniqueness* of SUSY
    - – How does it work?
      If Q is generator of SUSY algebra, then

$$Q|boson>=|fermion> \quad and \quad Q|fermion>=|boson>$$

Thus, SUSY algebra links together various representations with different spins by following chain of states

$$s=2 \longrightarrow s=3/2 \longrightarrow s=1 \longrightarrow s=1/2 \longrightarrow s=0$$

As a result: we have possibility to unify all interactions as well as "matter" with "forces".

– The key relation in SUSY algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha,\dot{\alpha}} P_\mu.$$

Now using of the infinitesimal transformations $\delta_\epsilon = \epsilon^\alpha Q_\alpha$, $\bar{\delta}_{\bar{\epsilon}} = \bar{Q}_{\dot{\alpha}} \bar{\epsilon}^{\dot{\alpha}}$ one gets

$$\{\delta_\epsilon, \bar{\delta}_{\bar{\epsilon}}\} = 2(\epsilon \sigma^\mu \bar{\epsilon}) P_\mu.$$

Now choosing ϵ to be local, i.e., $\epsilon = \epsilon(x)$, then it is a local coordinate transformation - General Relativity.

Local SUSY $\Rightarrow$ *supergravity*

⋆ Unification of gauge couplings

– Philosophy of Grand Unification: gauge symmetry increases with energy

– at high energy: one $G_{GUT} = SU(5), SO(10), \ldots$ with one coupling constant g_{GUT}

– crucial point: *running coupling constants* given by the renormalization group equations

$$\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \frac{\alpha_i}{4\pi},$$

where

$$\begin{aligned} \alpha_1 &= (5/3)g'^2/(4\pi) = 5\alpha/(3\cos^2\theta_W), \\ \alpha_2 &= g^2/(4\pi) = \alpha/\sin^2\theta_W, \\ \alpha_3 &= g_s^2/(4\pi), \end{aligned}$$

where α - fine structure constant, b_i depend on the model and

$$\alpha_1(M_Z) = 0.017, \quad \alpha_2(M_Z) = 0.034, \quad \alpha_3(M_Z) = 0.118 \pm 0.003,$$

– The SM: unification is impossible !!!
– Minimal supersymmetric extensions lead to unification with

$$M_{SUSY} = 10^3 - 10^4 GeV, \; M_{GUT} \simeq 10^{16} GeV, \; \alpha_{GUT}^{-1} \simeq 25,$$

⋆ Solution of the hierarchy problem

– existence of two different scales $V > v$ (M_W and M_{GUT}) in GUT theories - *hierarchy problem*

– two different aspects of the problem

1. the very existence of the hierarchy: for desired spontaneous symmetry breaking one needs

$$\begin{aligned} m_H &\sim v \sim 10^2 Gev, \\ m_\Sigma &\sim V \sim 10^{16} GeV, \end{aligned}$$

where H, Σ - Higgs fields responsible for the SB of $SU_L(2)$ and GUT groups.

Question: how to get so small number $10^2/10^{16}$ in natural way.

2. preservation of the hierarchy: radiative corrections can destroy and will destroy it.

Supersymmetry - the only way how to cancel quadratic divergences.

Nice example: radiative corrections to the Higgs boson mass:

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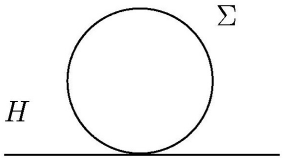
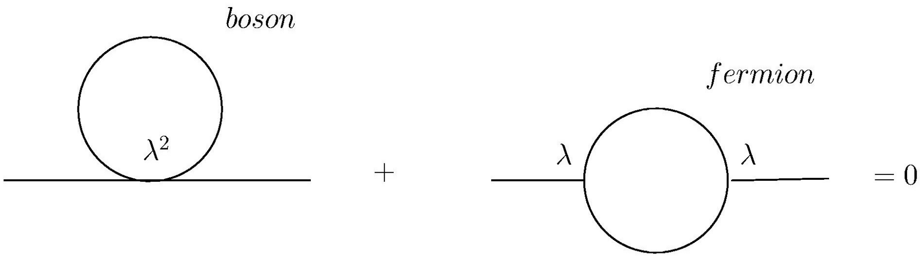


$$\delta m^2 \sim \lambda^2 M^2$$



When supersymmetry is broken with the SUSY breaking scale M_{SUSY}, then

$$\delta m_h^2 \sim \lambda^2 M_{SUSY}^2 \sim m_h^2$$

and $M_{SUSY} \sim 1TeV$.

- $\star$ Beyond GUTs - superstrings

- Basic facts about SUSY
  - $\star$ SUSY - symmetry between bosons and fermions

⋆ Algebra of SUSY - Super-Poincare Lie algebra:

$$
\begin{aligned}
&[P_\mu, P_\nu] = 0,\\
&[P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),\\
&[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),\\
&[B_r, B_s] = iC^t_{rs}B_t,\\
&[B_r, P_\mu] = [B_r, M_{\mu\sigma}] = 0,\\
&[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0,\\
&[Q^i_\alpha, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^i_\beta, \quad [\bar{Q}^i_{\dot\alpha}, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}^i_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha},\\
&[Q^i_\alpha, B_r] = (b_r)^i_j Q^j_\alpha, \quad [\bar{Q}^i_{\dot\alpha}, B_r] = -\bar{Q}^j_{\dot\alpha}(b_r)^i_j,\\
&\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu,\\
&\{Q^i_\alpha, Q^j_\beta\} = 2\epsilon_{\alpha\beta}Z^{ij}, \quad Z_{ij} = a^r_{ij}b_r, \quad Z^{ij} = Z^+_{ij},\\
&\{\bar{Q}^i_{\dot\alpha}, \bar{Q}^j_{\dot\beta}\} = -2\epsilon_{\dot\alpha\dot\beta}Z^{ij}, \quad [Z_{ij}, anything] = 0,
\end{aligned}
$$

where $i, j = 1, 2, ..., N$.

⋆ how many SUSY generators are possible: the restriction is $N \leq 4S$, where S is the maximal spin of the particles. Thus, $N \leq 8$ We shall consider only $N = 1$ supersymmetry.

⋆ Superspace and superfields

– *Superspace*: $x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}$

$$\{\theta_\alpha, \theta_\beta\} = 0, \quad \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0, \quad \theta_\alpha^2 = 0, \quad \bar{\theta}_{\dot{\alpha}}^2 = 0$$

– SUSY group element in superspace:

$$G(x, \theta, \bar{\theta}) = e^{i(-x^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q})},$$

then supertranslation:

$$\begin{aligned} x_\mu &\rightarrow x_\mu + i\theta\sigma_\mu\bar{\varepsilon} - i\varepsilon\sigma_\mu\bar{\theta}, \\ \theta &\rightarrow \theta + \varepsilon, \\ \bar{\theta} &\rightarrow \bar{\theta} + \bar{\varepsilon}. \end{aligned}$$

– Superfields:
1. Scalar superfield $F(x,\theta,\bar\theta)$ - SUSY invariant.
It is reducible: a) *chiral superfield*; b) *antichiral superfield*
Chiral superfield:

$$\bar D F = 0, \quad \bar D = -\frac{\partial}{\partial\bar\theta} - i\theta\sigma^\mu\partial_\mu \tag{1}$$

In components:

$$\begin{aligned}\Phi(x,\theta,\bar\theta) \;=\; & A(x) i\theta\sigma^\mu\bar\theta\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar\theta\bar\theta\Box A(x) \\ & +\sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar\theta + \theta\theta F(x),\end{aligned}$$

where $A(x)$ is complex scalar, $\psi(x)$ is Weyl spinor - *super-partners*
$F(x)$ - is an auxiliary field.
Antichiral superfield: Φ^+

$$DF = 0, \quad D = \frac{\partial}{\partial\theta} + i\sigma^\mu\bar\theta\partial_\mu.$$

2. real vector superfield: $V = V^+$:
In the so-called Wess-Zumino gauge:

$$V = -\theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x),$$

where v_μ - gauge vector field, λ - Majorana spinor.
Analog of the $F_{\mu\nu}$ in gauge theories - field strength tensor

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^V D_\alpha e^{-V}, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2 e^V \bar{D}_{\dot{\alpha}} e^{-V} \qquad (2)$$

- SUSY Lagrangians

  ⋆ Lagrangian without local gauge invariance

$$L = \int d^2\theta d^2\bar{\theta}\Phi_i^+\Phi_i + \int d^2\theta[\lambda_i\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k] + h.c.$$

First integral - kinetic term.
Second integral - superpotential W.

Explicit form:

$$
\begin{aligned}
L &= i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i - \frac{1}{2} m_{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \bar{\psi}_i \bar{\psi}_j \\
& \quad y_{ijk} \psi_i \psi_j A_k - y_{ijk}^* \bar{\psi}_i \bar{\psi}_j A_k^* - V(A_i, A_j).
\end{aligned}
$$

⋆ Gauge invariant SUSY Lagrangian

$$
\begin{aligned}
L &= \frac{1}{4} \int d^2\theta W^\alpha W_\alpha + \frac{1}{4} \int d^2\bar{\theta} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} \\
&= \frac{1}{2} D^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i\lambda \sigma^\mu D_\mu \bar{\lambda}.
\end{aligned}
$$

⋆ The scalar potential is completely defined by superpotential!!

⋆ The Lagrangian is practically fixed by symmetry requirement The only freedom: field content, gauge coupling, Yukawa couplings, and masses.

- Spontaneous breaking of SUSY
  Physics - supersymmetry have to be broken!
  Two possibilities: explicit or spontaneous
  Spontaneous breaking preserves needed properties of SUSY.

MSSM and NMSSM

- Particle content

  SUSY - number of bosonic degree of freedom = number of fermionic

  Thus, the SM is non-supersymmetric:

  1. no fermions with the quantum numbers of gauge bosons

  2. Higgs fields and his nonzero v.e.v.

  3. One needs at least two complex chiral Higgs multiplets to give masses to up and down quarks

  Conclusion: for each known boson - new fermion, and for each known fermion - new boson.

$\hat{L}$	leptons	$L_L = \binom{\nu_e}{e}_L$	sleptons	$\tilde{L}_L = \binom{\tilde{\nu}}{\tilde{e}}_L$	(1,2,-1)
$\hat{E}$		$l_R = e_R$		$\tilde{l}_R = \tilde{e}_R$	(1,1,-2)
$\hat{Q}$	quarks	$Q_L = \binom{u}{d}_L$	squarks	$\tilde{Q}_L = \binom{\tilde{u}}{\tilde{d}}_L$	(3,2,1/3)
$\hat{U}$		$U_R = u_R$		$\tilde{U}_R = \tilde{u}_R$	(3,1,4/3)
$\hat{D}$		$D_R = d_R$		$\tilde{D}_R = \tilde{d}_R$	(3,1,-2/3)
$\hat{H}_1$	higgsinos	$(\tilde{H}_1^0, \tilde{H}_1^-)_L$	Higgses	$\binom{H_1^0}{H_1^-}_L$	(1,2,-1)
$\hat{H}_2$		$(\tilde{H}_2^+, \tilde{H}_2^0)_L$		$\binom{H_2^+}{H_2^0}_L$	(1,2,1)

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$\hat{G}$	gluons	G^a_μ	gluinos	$\tilde{G}^a_\mu$	(8,1,0)
$\hat{V}$	$SU(2)$-gauge bosons	A^i_μ	gauginos	$\tilde{A}^i_\mu$	(1,3,0)
$\hat{V}'$	$U(1)$-gauge boson	B_μ	gauginos	$\tilde{B}_\mu$	(1,1,0)

- Higgs bosons

  Two Higgs doublets: 5 Higgs boson (physical states) - two CP-even neutral (h, H), one CP- odd neutral (A) and two charged $H^{\pm}$.

- Lagrangian of the MSSM

$$L = L_{gauge} + L_{Yukawa} + L_{breaking}, \tag{3}$$

where L_{gauge} is defined as

$$\begin{aligned} L_{gauge} &= \sum_{SU(3),SU(2),U(1)} \frac{1}{4}\left[\mathrm{Tr}W^\alpha W_\alpha + \mathrm{Tr}\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right] \\ &+ \sum_{Matter} \Phi_i^\dagger e^{g_3\hat{G}+g_2\hat{V}+g_1\hat{V}'}\Phi_i, \end{aligned} \tag{4}$$

and L_{Yukawa} part which is responsible for the generation of masses of the particles has the following form:

$$L_{Yukawa} = h^u_{\alpha\beta}\widehat{Q}_\alpha \widehat{U}^c_\beta \widehat{H}_2 + h^d_{\alpha\beta}\widehat{Q}_\alpha \widehat{D}^c_\beta \widehat{H}_1 + h^l_{\alpha\beta}\widehat{L}_\alpha \widehat{l}^c_\beta \widehat{H}_1 + \mu \widehat{H}_1 \widehat{H}_2. \quad (5)$$

- Lagrangian of the NMSSM

  superpotential:

$$W = \lambda H_1 H_2 S + \frac{\lambda'}{3} S^3 + ...,$$

  where S is new gauge singlet superfield.

  Then: 3 - CP-even Higgs bosons (one of them is the lightest), 2 - CP-odd, and 2 - charged.

- Breaking of SUSY in the MSSM:

  - gravity mediated SUSY breaking: two sector, SUSY is broken in hidden sector and it is mediated into the visible sector via gravity

$$
\begin{aligned}
-L_{Breaking} &= m_0^2 \sum_i |\varphi_i|^2 + \Big(\frac{1}{2} m_{1/2} \sum_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha \\
&+ A[h^U_{ab} \tilde{q}_a \tilde{u}^c_b h_2 + h^D_{ab} \tilde{q}_a \tilde{d}^c_b h_1 + h^L_{ab} \tilde{l}_a \tilde{e}^c_b h_1] \\
&+ B[\mu h_1 h_2] + h.c.\Big). \qquad (6)
\end{aligned}
$$

Here φ_i are all scalar fields, $\tilde{\lambda}_\alpha$ are the gaugino fields, h_1 and h_2 are the $SU(2)$ doublet Higgs fields. We assume so-called universality of the soft terms.

- Gaugino-higgsino mass terms

  nondiagonality of the matrices - mixing

$$
L_{G-H} = -\frac{1}{2} M_3 \bar{\lambda}_a \lambda_a - \frac{1}{2} \bar{\chi} M^{(0)} \chi - (\bar{\psi} M^{(c)} \psi + h.c.),
$$

where λ_a with $a = 1, 2, ..., 8$ are the Majorana gluino fields,

$$\chi \;=\; \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}$$

are the Majorana neutralino fields and

$$\psi = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix}$$

are Dirac chargino fields. The neutralino mass matrix is the following

$$M^{(0)} \;=\; \begin{pmatrix} M_1 & 0 & a & b \\ 0 & M_2 & c & d \\ a & c & 0 & -\mu \\ b & d & -\mu & 0 \end{pmatrix},$$

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where

$$\begin{aligned} a &= -M_Z \cos\beta \sin\theta_W, \\ b &= M_Z \sin\beta \sin\theta_W, \\ c &= M_Z \cos\beta \cos\theta_W, \\ d &= -M_Z \sin\beta \cos\theta_W \end{aligned}$$

where $\tan\beta = v_2/v_1$ and $\sin\theta_W$ is the usual sinus of the weak mixing angle.

For charginos we have the following mixing matrix

$$M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}.$$

The masses of its eigenstates $\tilde{\chi}^{\pm}_{1,2}$ are given by diagonalization of the above matrix

$$\begin{aligned} m^2_{\chi_{1,2}} &= \frac{1}{2}\big[M_2^2 + \mu^2 + 2M_W^2 \\ &\mp \Big((M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2(2\beta) \\ &+ 4M_W^2(M_2^2 + \mu^2 + 2M_2\mu \sin(2\beta)\Big)^{1/2}\big]. \end{aligned}$$

- Squarks and sleptons masses

$$\begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & \tilde{m}_{tR}^2 \end{pmatrix},$$
$$\begin{pmatrix} \tilde{m}_{bL}^2 & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & \tilde{m}_{bR}^2 \end{pmatrix},$$
$$\begin{pmatrix} \tilde{m}_{\tau L}^2 & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & \tilde{m}_{\tau R}^2 \end{pmatrix},$$

where

$$
\begin{aligned}
\tilde{m}_{tL}^2 &= \tilde{m}_Q^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2)\cos(2\beta), \\
\tilde{m}_{tR}^2 &= \tilde{m}_U^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2)\cos(2\beta), \\
\tilde{m}_{bL}^2 &= \tilde{m}_Q^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2)\cos(2\beta), \\
\tilde{m}_{bR}^2 &= \tilde{m}_D^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2)\cos(2\beta), \\
\tilde{m}_{\tau L}^2 &= \tilde{m}_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2)\cos(2\beta), \\
\tilde{m}_{\tau R}^2 &= \tilde{m}_E^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos(2\beta).
\end{aligned}
$$

Tree-level masses:

$$
\begin{aligned}
\tilde{m}^2_{t_{1,2}} &= \frac{1}{2}\Big[\tilde{m}^2_{t_L} + \tilde{m}^2_{t_R} \\
&\mp \sqrt{(\tilde{m}^2_{t_L} - \tilde{m}^2_{t_R})^2 + 4m_t^2(A_t - \mu\,\cot\beta)^2}\Big], \\
\tilde{m}^2_{b_{1,2}} &= \frac{1}{2}\Big[\tilde{m}^2_{b_L} + \tilde{m}^2_{b_R} \\
&\mp \sqrt{(\tilde{m}^2_{b_L} - \tilde{m}^2_{b_R})^2 + 4m_b^2(A_b - \mu\,\tan\beta)^2}\Big], \\
\tilde{m}^2_{\tau_{1,2}} &= \frac{1}{2}\Big[\tilde{m}^2_{\tau_L} + \tilde{m}^2_{\tau_R} \\
&\mp \sqrt{(\tilde{m}^2_{\tau_L} - \tilde{m}^2_{\tau_R})^2 + 4m_\tau^2(A_\tau - \mu\,\tan\beta)^2}\Big].
\end{aligned}
$$

- Masses of the lightest Higgs boson

  In MSSM – <u>two</u> Higgs doublet:

$$
H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}
$$

$\Rightarrow$ 5 physical Higgs bosons:

– neutral CP- even: h, H
– neutral CP- odd: A
– charged: $H^{\pm}$

one-loop effective potential:

$$\begin{aligned} V &= V_{tree} + \Delta V, \\ V_{tree} &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + h.c.) \\ &+ \frac{g^2+g'^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^+H_2|^2, \end{aligned}$$

where $m_i^2 = m_{H_i}^2 + \mu^2, i = 1, 2$ and $m_3^2 = -B\mu$.
– Minimization of the potential:

$$\begin{aligned} v^2 &= \frac{4((m_1^2+\Sigma_1) - (m_2^2+\Sigma_2)\tan^2\beta)}{(g^2+g'^2)(\tan^2\beta - 1)}, \\ \sin(2\beta) &= \frac{2m_3^2}{m_1^2+m_2^2+\Sigma_1+\Sigma_2}, \end{aligned}$$

where $v^2 = v_1^2 + v_2^2$, $v_1 =< H_1 >= v\cos\beta$, $v_2 =< H_2 >= v\sin\beta$ and $\tan\beta = v_2/v_1$. Futher: $m_1^2 = m_2^2 = m_0^2 + \mu_0^2$ (GUT scale)

– tree level masses:

$$
\begin{aligned}
m_A^2 &= m_1^2 + m_2^2, \\
m_{H^\pm}^2 &= m_A^2 + M_W^2, \\
m_{h,H}^2 &= \frac{1}{2}(M_Z^2 + m_A^2 \\
&\mp \sqrt{(M_Z^2 + m_A^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta})
\end{aligned}
$$

- In the NMSSM: $\mu = \lambda y$, where $y =< S >$

- Parameter freedom in the MSSM
  - ⋆ gauge couplings α_i, $i = 1, 2, 3$
  - ⋆ Yukawa couplings h^i_{ab}
  - ⋆ Higgs mixing parameter μ
  - ⋆ soft breaking parameters (universal case)
    - m_0 - common scalar mass
    - $m_{1/2}$ - common gaugino mass
    - A - common trilinear scalar coupling
    - B - bilinear Higgs coupling
    - $\tan\beta = v_2/v_1$ - ratio of two v.e.v

- The 3rd generation 1-loop RGEs in the MSSM:

$$\frac{dY_t}{dt} = Y_t\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{15}\tilde{\alpha}_1 - 6Y_t - Y_b\right),$$

$$\frac{dY_b}{dt} = Y_b\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{7}{15}\tilde{\alpha}_1 - Y_t - 6Y_b - Y_\tau\right),$$

$$\frac{dY_\tau}{dt} = Y_\tau\left(3\tilde{\alpha}_2 + \frac{9}{5}\tilde{\alpha}_1 - 3Y_b - 4Y_\tau\right),$$

$$\frac{dA_t}{dt} = -\left(\frac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \frac{13}{15}\tilde{\alpha}_1 M_1\right) - 6Y_t A_t - Y_b A_b,$$

$$\frac{dA_b}{dt} = -\left(\frac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \frac{7}{15}\tilde{\alpha}_1 M_1\right) - 6Y_b A_b - Y_t A_t - Y_\tau A_\tau,$$

$$\frac{dA_\tau}{dt} = -\left(3\tilde{\alpha}_2 M_2 + \frac{9}{5}\tilde{\alpha}_1 M_1\right) - 3Y_b A_b - 4Y_\tau A_\tau,$$

$$
\begin{aligned}
\frac{dB}{dt} &= -3\left(\tilde{\alpha}_2 M_2 + \frac{1}{5}\tilde{\alpha}_1 M_1\right) \\
&\quad -3Y_t A_t - 3Y_b A_b - Y_\tau A_\tau, \\
\frac{dm_Q^2}{dt} &= \left(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \frac{1}{15}\tilde{\alpha}_1 M_1^2\right) \\
&\quad -Y_t\left(m_Q^2 + m_U^2 + m_{H_2}^2 + A_t^2\right) \\
&\quad -Y_b\left(m_Q^2 + m_D^2 + m_{H_1}^2 + A_b^2\right), \\
\frac{dm_U^2}{dt} &= \left(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{16}{15}\tilde{\alpha}_1 M_1^2\right) \\
&\quad -2Y_t\left(m_Q^2 + m_U^2 + m_{H_2}^2 + A_t^2\right), \\
\frac{dm_D^2}{dt} &= \left(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{4}{15}\tilde{\alpha}_1 M_1^2\right) \\
&\quad -2Y_b\left(m_Q^2 + m_D^2 + m_{H_1}^2 + A_b^2\right),
\end{aligned}
$$

$$
\begin{aligned}
\frac{dm_{H_1}^2}{dt} &= 3\left(\tilde{\alpha}_2 M_2^2 + \frac{1}{5}\tilde{\alpha}_1 M_1^2\right) \\
&\quad -3Y_b\left(m_Q^2 + m_D^2 + m_{H_1}^2 + A_b^2\right) \\
&\quad -Y_\tau\left(m_L^2 + m_E^2 + m_{H_1}^2 + A_\tau^2\right), \\
\frac{dm_{H_2}^2}{dt} &= 3\left(\tilde{\alpha}_2 M_2^2 + \frac{1}{5}\tilde{\alpha}_1 M_1^2\right) \\
&\quad -3Y_t\left(m_Q^2 + m_U^2 + m_{H_2}^2 + A_t^2\right), \\
\frac{dm_L^2}{dt} &= 3\left(\tilde{\alpha}_2 M_2^2 + \frac{1}{5}\tilde{\alpha}_1 M_1^2\right) \\
&\quad -Y_\tau\left(m_L^2 + m_E^2 + m_{H_1}^2 + A_\tau^2\right), \\
\frac{dm_E^2}{dt} &= \frac{12}{5}\tilde{\alpha}_1 M_1^2 - 2Y_\tau\left(m_L^2 + m_E^2 + m_{H_1}^2 + A_\tau^2\right), \\
\frac{dM_i}{dt} &= -b_i\tilde{\alpha}_i^2 M_i, \\
\frac{d\tilde{\alpha}_i}{dt} &= -b_i\tilde{\alpha}_i^2,
\end{aligned}
$$

where

$$
\begin{aligned}
Y_i &= h_i^2/(4\pi)^2, \\
\tilde{\alpha}_i &= \alpha_i/(4\pi) = g_i^2/(4\pi)^2, \\
(b_1, b_2, b_3) &= (33/5, 1, -3), \\
t &= \log(M_{GUT}^2/Q^2)
\end{aligned}
$$

- In the NMSSM we have in addition

$$
\begin{aligned}
\frac{dY_\lambda}{dt} &= -Y_\lambda(3Y_t + 4Y_\lambda + 2Y_{\lambda'} - 3\tilde{\alpha}_2 - \frac{3}{5}\tilde{\alpha}_1)\,, \\
\frac{dY_{\lambda'}}{dt} &= -6Y_{\lambda'}(Y_\lambda + Y_{\lambda'})\,,
\end{aligned}
$$

where $Y_\lambda = \lambda^2/(4\pi)^2$ and $Y_{\lambda'} = \lambda'^2/(4\pi)^2$.

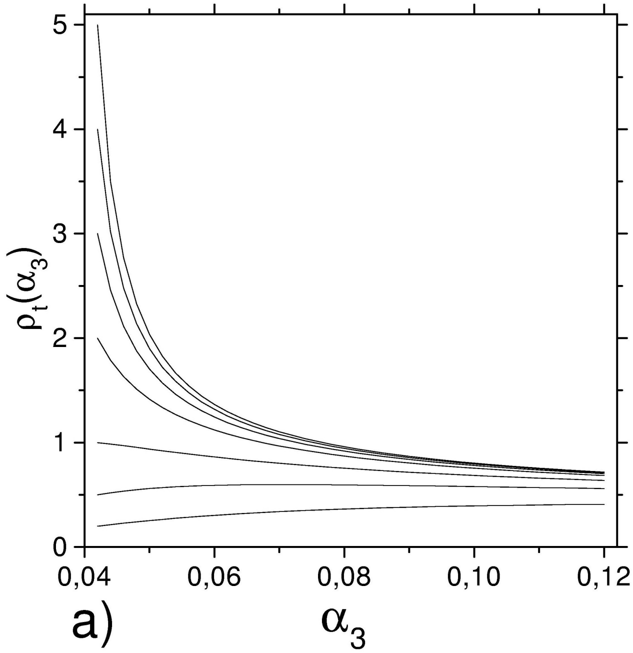
Infrared Quasi Fixed Points

- infrared behavior of RGEs (small $\tan\beta$):
  example:

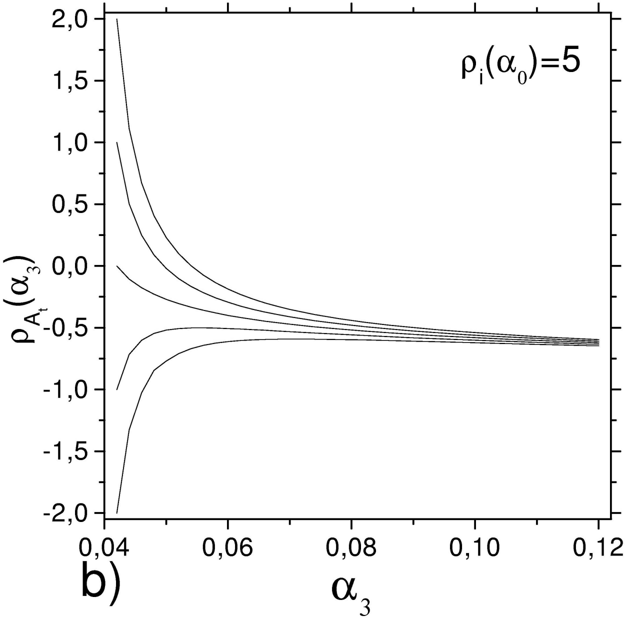
$$
\begin{aligned}
Y_t(t) &= \frac{Y_0 E(t)}{1+6Y_0 F(t)}; \quad Y_0 = Y_t(0), \\
E(t) &= (1+\beta_3 t)^{16/(3b_3)}(1+\beta_2 t)^{3/b_2}(1+\beta_1 t)^{13/(15b_1)}, \\
F(t) &= \int_0^t E(t')dt'.
\end{aligned}
$$

if $Y_0 = Y_t(0) \to \infty$:

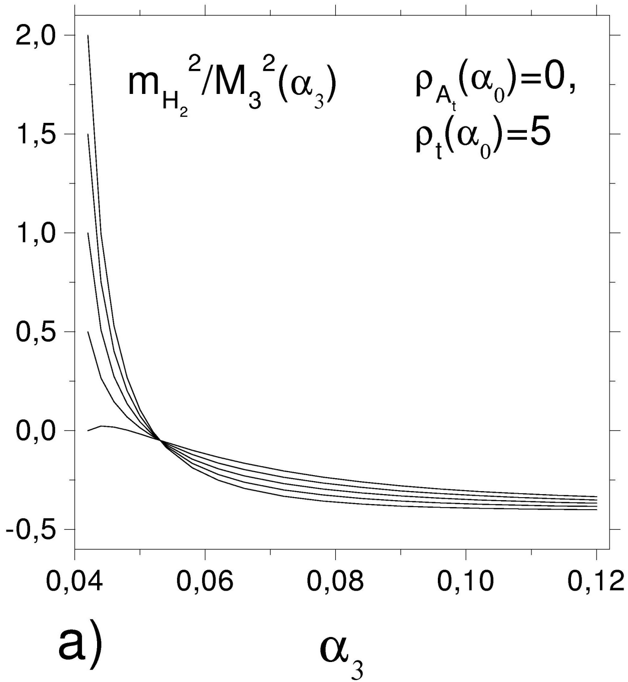
$$
Y(t) \Rightarrow Y_{FP} = \frac{E(t)}{6F(t)}
$$

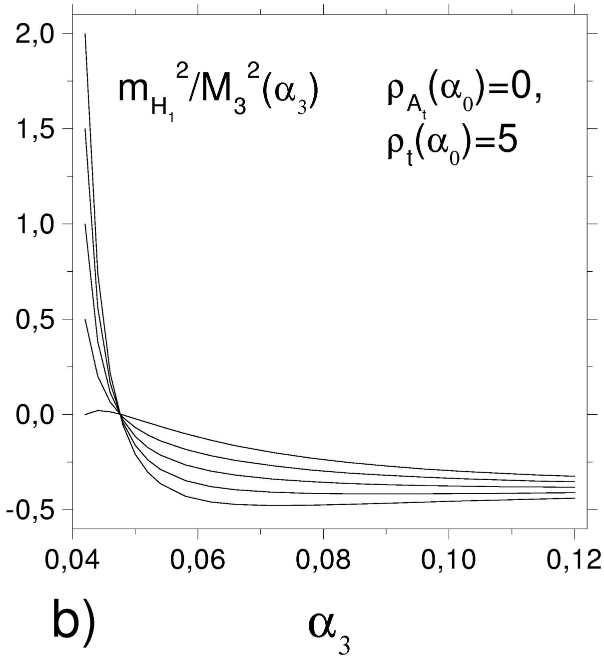


where $\rho_t = Y_t/\tilde{\alpha}_3$.

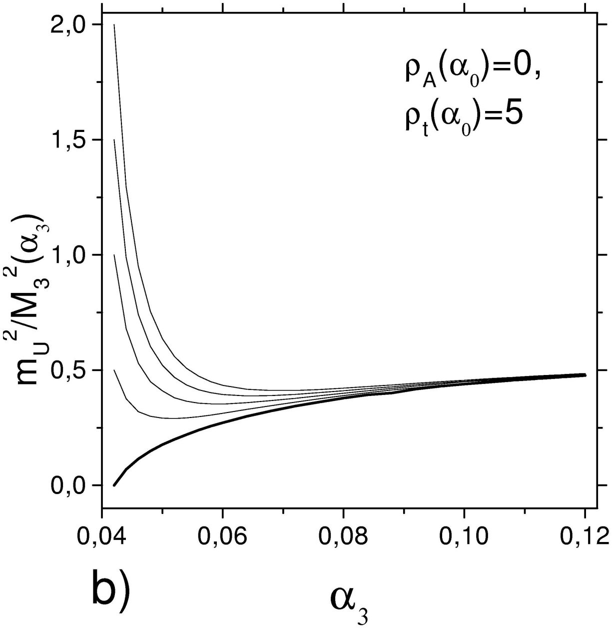


where $\rho_{A_t} = A_t/M_3$.


2,0
1,5
1,0
0,5
0,0
-0,5
$m_{H_2}{}^2/M_3{}^2(\alpha_3)$
$\rho_{A_t}(\alpha_0)=0,$
$\rho_t(\alpha_0)=5$
0,04
0,06
0,08
0,10
0,12
a)
α_3



$m_{H_1}{}^2/M_3{}^2(\alpha_3)$
$\rho_{A_t}(\alpha_0)=0,$
$\rho_t(\alpha_0)=5$
2,0
1,5
1,0
0,5
0,0
-0,5
0,04
0,06
0,08
0,10
0,12
b)
α_3

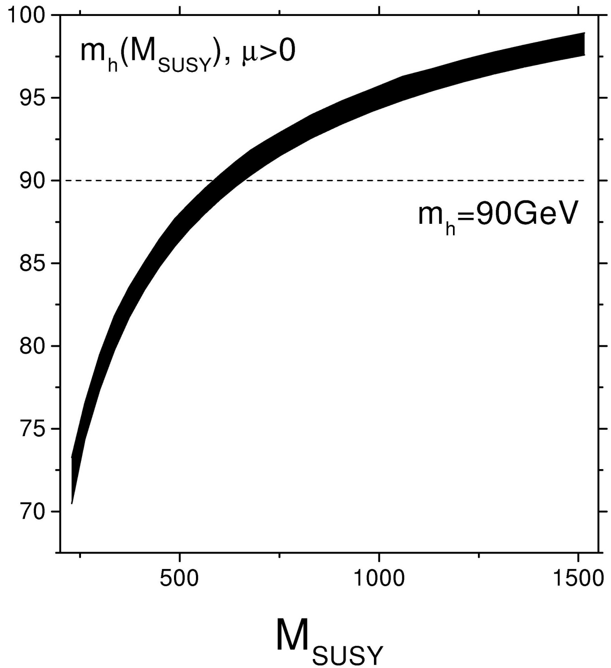

2,0
1,5
1,0
0,5
0,0
0,04
0,06
0,08
0,10
0,12
$m_U^2/M_3^2(\alpha_3)$
$\rho_A(\alpha_0)=0,$
$\rho_t(\alpha_0)=5$
b)
α_3

- Possible deviations from IRQFPs must be taken into account
- infrared behavior of RGEs in the large $\tan\beta$ case:
  - $\star$ all equations are relevant
  - $\star$ no exact solutions exist
  - $\star$ two possible ways:

    a) numerical solutions

    b) iterative solutions

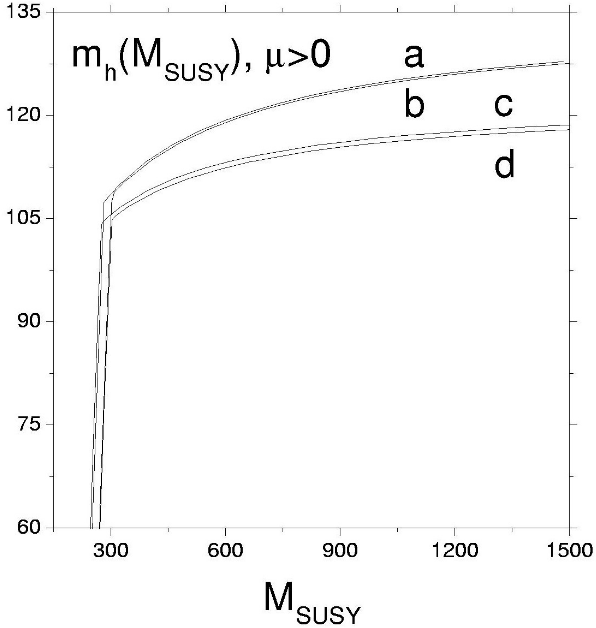
• The lightest Higgs boson in the MSSM: $\tan\beta \sim 1$

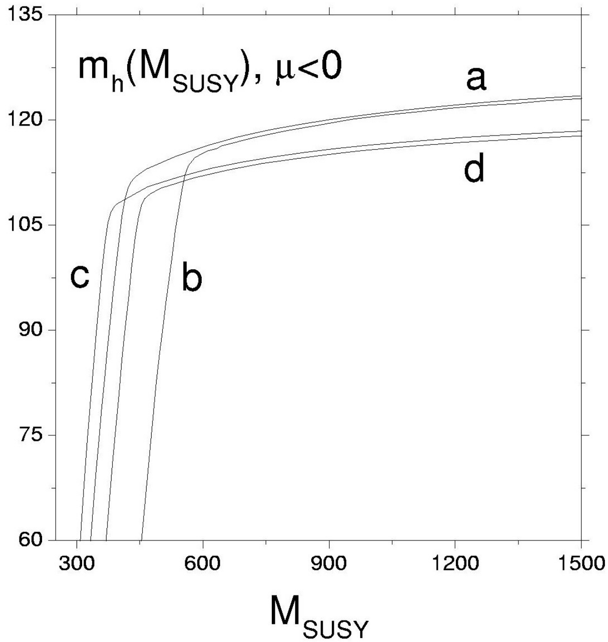


at $M_{SUSY} = (\tilde{m}_{t_1}\tilde{m}_{t_2})^{1/2} = 1TeV$:

$$m_h \ = \ (94.3 + 1.6 + 0.6 \pm 5 \pm 0.4) \text{ GeV}$$

- The lightest Higgs boson in the MSSM: $\tan\beta \sim 50$

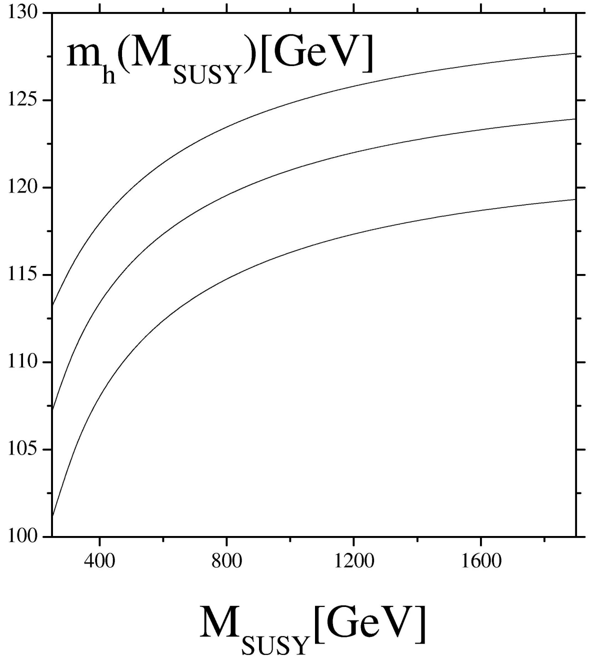




at $M_{SUSY} = 1$ TeV $(M_3 \approx 1.3$TeV$)$:

$$m_h \;=\; 123.7 - 0.9 - 6.5 \pm 5 \text{ GeV, for } \mu > 0\,,$$
$$m_h \;=\; 119.1 - 0.4 - 2.8 \pm 5 \text{ GeV, for } \mu < 0\,.$$

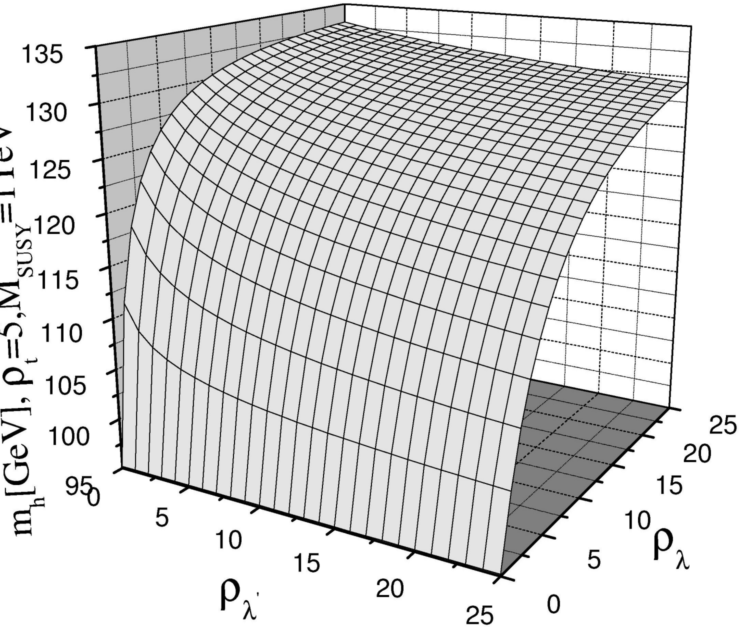
- The lightest Higgs boson in the NMSSM: $\tan\beta \sim 1$



at $M_{SUSY} = 1$TeV

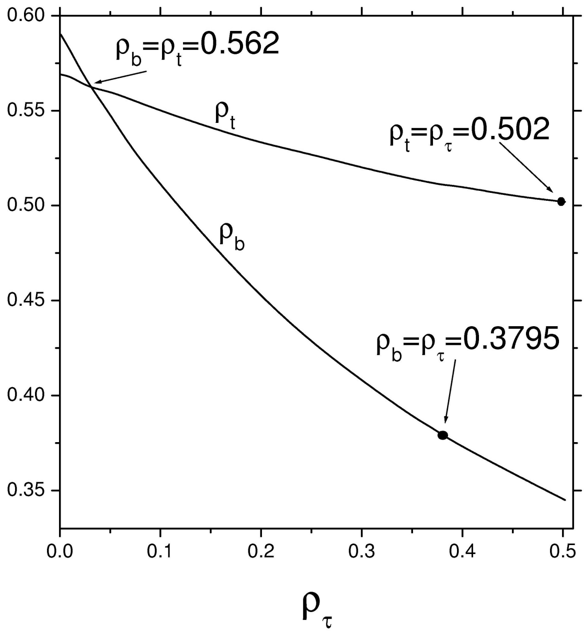
$$m_h = 121^{+1.8+1.3}_{-3.0-1.3} \pm 5GeV. \quad (7)$$

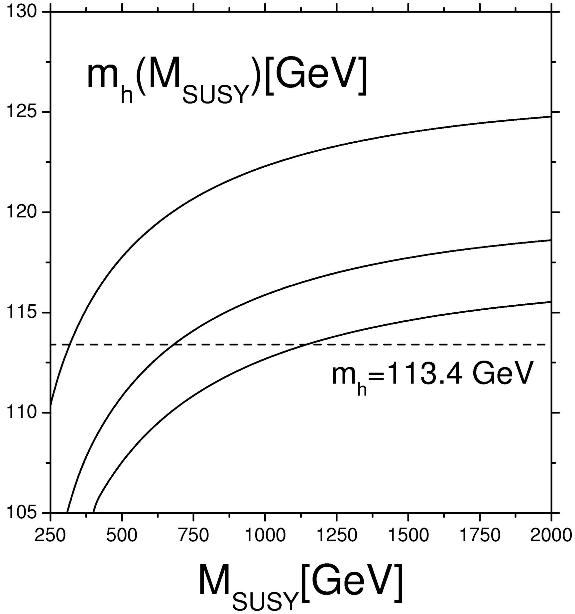
Back
Close


m_h[GeV], ρ_t=5,M_{SUSY}=1TeV
135
130
125
120
115
110
105
100
95
0
5
10
15
20
25
$\rho_{\lambda'}$
0
5
10
15
20
25
ρ_λ

Focus Points in the MSSM - mass of the lightest Higgs boson

- to have both focus points at the M_Z scale





with $m_t^{pole} = 174.2$ GeV and $\tan\beta = 58$ and

$$m_h = 115.9\, {}^{+6.4}_{-3.2} \pm 0.4 \text{ GeV}, \qquad \text{for } M_{SUSY} = 1 \text{ TeV}.$$

Conclusions

- There is still space for SUSY Higgs
- The results was published:

  - G. K. Yeghiyan, M. Jurcisin, D. I. Kazakov, Mod. Phys. Lett. A 14 (1999) 601-619;

  - M. Jurcisin, D. I. Kazakov, Mod. Phys. Lett. A 14 (1999) 671-687;

  - S. Codoban, M. Jurcisin, D.I. Kazakov, Phys. Lett. B 477 (2000) 223-232;

  - D.I. Kazakov, A.V. Gladyshev, M. Jurcisin, Phys.Atom.Nucl.62 (1999) 2059-2063;

  - S. Codoban and M. Jurcisin, Acta Phys. Slov. 52 (2002) 253.

  - E. Jurcisinova, M. Jurcisin, Focus points and the Lightest Higgs Boson Mass in the Minimal Supersymmetric Standard Model, hep-ph/0512