Relativistic hydrodynamics[1] with strangeness production

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[1] based on H.T. Elze,J. Rafelski, LT: Phys.Lett. **B506**, 123 2001

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There is a need for

Hydro model with dynamical chemical reactions and multi-fluid included

Dissipative hydro primer

The energy-momentum tensor for an imperfect fluid

$$T^{\mu\nu} \equiv (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \delta T^{\mu\nu} = T^{\nu\mu} \quad ,$$

The evolution of the system is described by:

$$\partial_\mu T^{\mu\nu} = f^\nu \quad ,$$

Continuity equations

$$\partial_\mu \rho_i^\mu \equiv \partial_\mu \rho_i u^\mu = 0 \quad ,$$

Dissipative terms to vanish in the local rest frame

$$T^{00}_{(0)} = \epsilon \quad \longrightarrow \quad \delta T^{00}_{(0)} = 0 \quad \longrightarrow \quad u_\mu u_\nu \delta T^{\mu\nu} = 0 \quad ,$$

Diffusion, particle production, chemical reactions

In order to incorporate diffusion and particle production or chemical reactions, we now generalize

$$\partial_\mu \tilde{\rho}_i^\mu \equiv \partial_\mu(\rho_i^\mu + \triangle^{\mu\nu}\partial_\nu \mathcal{R}_i) = \mathcal{J}_i \quad ,$$

$$\triangle_{\mu\nu} \equiv g_{\mu\nu} - u_\mu u_\nu$$

In the local rest frame:

$$\mathcal{J}_i = \sum_j [G_{i\leftarrow j}\rho_j - L_{j\leftarrow i}\rho_i] \quad ,$$

The diffusion term $\propto \partial \mathcal{R}_i$ will be determined consistently with the Second Law of Thermodynamics

Since the entropy of the QGP phase at hadronization determines the observable particle multiplicity, it is crucial to understand its production.

The entropy production in a dissipative relativistic fluid is given by:

$$T\partial_\mu s^\mu = -\frac{\kappa}{T}\mathcal{Q}_\mu \triangle^{\mu\nu}\mathcal{Q}_\nu + \frac{\eta}{2}\mathcal{W}_\alpha^\beta \triangle_\beta^\gamma \mathcal{W}_\gamma^\delta \triangle_\delta^\alpha + \zeta(\partial_\mu u^\mu)^2 \ ,$$

$$u^2 = 1(\hbar = c = k_B = 1)$$

$$\mathcal{Q}_\mu \equiv \partial_\mu T - Tu^\nu \partial_\nu u_\mu \ \text{, heat-flow four-vector}$$

$$\mathcal{W}_{\mu\nu} \equiv \partial_\mu u_\nu + \partial_\nu u_\mu - \frac{2}{3} g_{\mu\nu}\partial_\gamma u^\gamma \ \text{, shear tensor}$$

Entropy Production

The (local) equilibrium relations,

$$-PV = \Omega(T, V, \mu_i) = U - TS - \sum_i \mu_i N_i \quad ,$$

Duhem-Gibbs relation for the densities

$$Ts = \epsilon + P - \sum_i \mu_i \rho_i \quad .$$

The First Law of Thermodynamics imply:

$$0 = T\partial_\mu \tilde{s}^\mu + \sum_i \mu_i (\mathcal{J}_i - \partial_\mu \triangle^{\mu\nu} \partial_\nu \mathcal{R}_i) + u_\nu \partial_\mu \delta T^{\mu\nu} \quad ,$$

with an auxiliary entropy four-current, $\tilde{s}^\mu \equiv s u^\mu$, and $\rho_i^\mu \equiv \rho_i u^\mu$

Entropy four-current

The proper entropy four-current:

$$s^\mu \equiv \tilde{s}^\mu + T^{-1} u_\nu \delta T^{\mu\nu} + \sum_i \mathcal{L}_i \triangle^{\mu\nu} \partial_\nu \mathcal{R}_i \quad .$$

An imperfect fluid evolving under the influence of conservative external forces

$$\begin{aligned} T\partial_\mu s^\mu \;\; = \;\; & -\sum_i \mu_i \mathcal{J}_i - T^{-1}(\partial_\mu T) u_\nu \delta T^{\mu\nu} + (\partial_\mu u_\nu)\delta T^{\mu\nu} \\ & + \sum_i (\mu_i + T\mathcal{L}_i)\partial_\mu \triangle^{\mu\nu} \partial_\nu \mathcal{R}_i + T \sum_i (\partial_\mu \mathcal{L}_i) \triangle^{\mu\nu} \partial_\nu \mathcal{R}_i \quad , \end{aligned}$$

Second Law of Thermodynamics

$$\sum_i \mu_i \mathcal{J}_i \leq 0 \ .$$

$$\delta T^{\mu\nu} = \kappa(\triangle^{\mu\gamma} u^\nu + \triangle^{\nu\gamma} u^\mu)\mathcal{Q}_\gamma + \eta \triangle^{\mu\gamma} \triangle^{\nu\delta} \mathcal{W}_{\gamma\delta} + \zeta \triangle^{\mu\nu} \partial_\gamma u^\gamma \ ,$$

$$\begin{aligned} \mathcal{Q}_\mu &\equiv \partial_\mu T - T u^\nu \partial_\nu u_\mu \ , \\ \mathcal{W}_{\mu\nu} &\equiv \partial_\mu u_\nu + \partial_\nu u_\mu - \frac{2}{3} g_{\mu\nu} \partial_\gamma u^\gamma \ . \end{aligned}$$

$$\mathcal{L}_i \equiv -\frac{\mu_i}{T} \ , \quad \mathcal{R}_i \equiv \sum_j \sigma_{ij} \frac{\mu_j}{T} \ ,$$

Chemo- and Thermo-diffusion

The diffusion current

$$\vec{j}_i \equiv -\nabla \mathcal{R}_i = -\sum_j \frac{\sigma_{ij}}{T}(\nabla \mu_j - \frac{\mu_j}{T}\nabla T) \quad .$$

involves chemo- and thermo-diffusion contributions

Entropy Production

All sources of entropy production

$$T\partial_\mu s^\mu = -\sum_i \mu_i \mathcal{J}_i - T\sum_{i,j}\sigma_{ij}(\partial_\mu \frac{\mu_i}{T})\triangle^{\mu\nu}(\partial_\nu \frac{\mu_j}{T})$$
$$-\frac{\kappa}{T}\mathcal{Q}_\mu \triangle^{\mu\nu}\mathcal{Q}_\nu + \frac{\eta}{2}\mathcal{W}_\alpha^\beta \triangle_\beta^\gamma \mathcal{W}_\gamma^\delta \triangle_\delta^\alpha + \zeta(\partial_\mu u^\mu)^2 \ .$$

Strangeness Production

Strangeness production in plasma is dominated by gluons

$$g + g \longrightarrow s + \bar{s}$$

with small contributions of quarks annihilation

$$u + \bar{u} \longrightarrow s + \bar{s}\,; \qquad d + \bar{d} \longrightarrow s + \bar{s}$$

In the local rest frame, we obtain:

$$\partial_t \rho_i \approx \mathcal{J}_i \equiv \sum_j [G_{i\leftarrow j}\rho_j - L_{j\leftarrow i}\rho_i]$$

Population equation

Gain term

$$G_s \equiv \langle \sigma_s^{gg} v_{gg} \rangle (\bar{\rho}_g)^2 + \sum_{q=u,d} \langle \sigma_s^{q\bar{q}} v_{q\bar{q}} \rangle_{th} \bar{\rho}_q \bar{\rho}_{\bar{q}} \equiv \mathcal{G}$$

Loss term

$$L_s \equiv [\langle \sigma_{\mathrm{g}}^{s\bar{s}} v_{s\bar{s}} \rangle + \langle \sigma_q^{s\bar{s}} v_{s\bar{s}} \rangle] \rho_s^2 \equiv \mathcal{L} \rho_s^2$$

Source term for strangeness

$$\mathcal{J}_s = \partial_t \rho_s = \mathcal{G} - \mathcal{L} \rho_s^2$$

Continuity equation is replaced by

$$\partial_t \rho_s = \mathcal{G} - \mathcal{L} \rho_s^2$$

Event by event

Taking event by event averages $\langle\langle\cdot\rangle\rangle$

$$\partial_t \langle\langle \rho_s \rangle\rangle = \mathcal{G} - \mathcal{L} \langle\langle \rho_s^2 \rangle\rangle$$

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If $\langle\langle \rho_s \rangle\rangle^2 \gg \delta\rho_s^2$

$$\rho_s(\tau) = \sqrt{\frac{\mathcal{G}}{\mathcal{L}}}\tanh(\tau\sqrt{\mathcal{G}\mathcal{L}})$$

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In the local rest frame + x-dependence!

Conclusions

- the entropy producing mechanisms of chemo- and thermo-diffusion together with the contribution of particle production is introduced
- we have an imperfect relativistic fluid with composition changing processes
- chemical equilibrium

$$J_i = 0$$

- thermal equilibrium

$$\partial_\mu s^\mu = 0$$

Problems

- Potential instabilities due to the first-order approach
- Numerical implementation
- Additional dissipative processes: surface radiation