Renormalization in hot QCD and jet quenching

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– Strange Quark Matter, Levoča, June 2007 –

hadronic matter at large density/temperature is Quark-Gluon plasma

$\Uparrow$

jet quenching as most instructive probe

$\Downarrow$

required: quantitative understanding of underlying physics

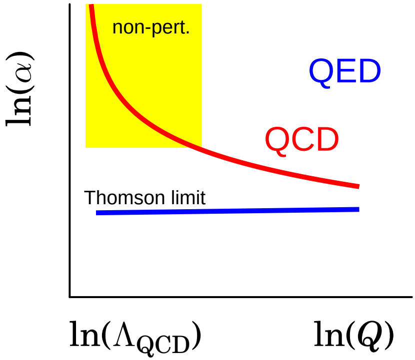
$$\text{analytical result} = \alpha^n \times \mathrm{dimension}\,[1 + \mathrm{corrections}]$$

without quantifying 'the' coupling α:

NO predictive power

Coupling: QCD vs. QED

$$\alpha(Q^2) = \frac{4\pi/\beta_0}{\ln(Q^2/\Lambda^2 + \ldots)}$$



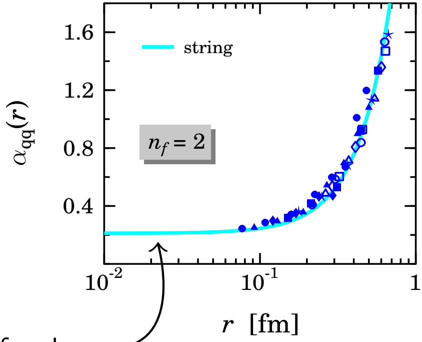
	β_0	Λ
QED	$-\frac{4}{3}$	10^{xx} GeV
QCD	$11 - \frac{2}{3} n_f$	~ 0.2 GeV

- **NO** QCD coupling *'per se'*
- in particular for heavy ion phenomenology, knowledge of relevant scale is crucial

1. **nonperturbative coupling from lattice QCD**

$$\text{potential } V_{qq}(r) \longrightarrow \text{coupling } \alpha_{qq} = \frac{3}{4}\, r^2 \frac{\partial V_{qq}}{\partial r}$$

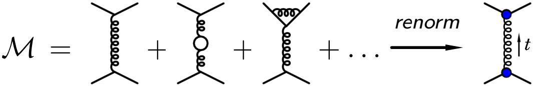
$$\text{parameterization: } V_{qq}^{\sigma} = -\frac{4}{3}\frac{a}{r} + \sigma r \quad \rightarrow \quad \alpha^{\sigma} = a + \frac{3}{4}\sigma r^2$$



[Kaczmarek et al.]

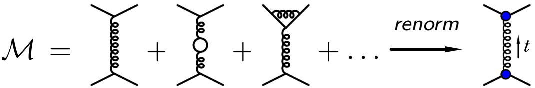
NO asympt. freedom

2. **pQCD: loop corrections to tree-level amplitude**



running coupling
$\alpha(t = 1/r^2)$

❷ pQCD: loop corrections to tree-level amplitude



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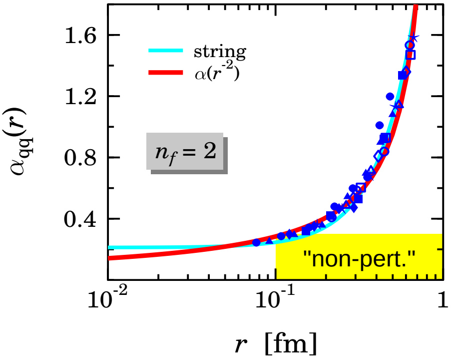


provocative observation

pQCD works *quantitatively*

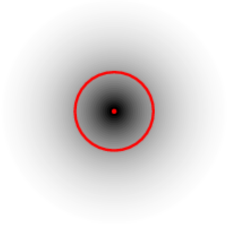
even at $\alpha \sim \mathbf{1}$, NB with

$\Lambda_{[n_f=2]} = 0.2\text{GeV}$

[AP, hep-ph/0601119]

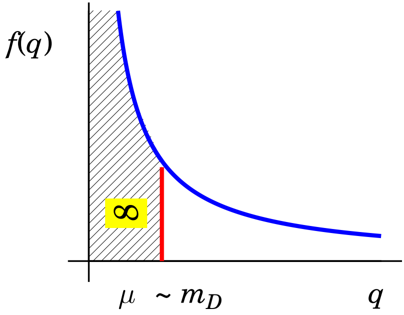
screening of

(1) charges in plasmas



$$V(r) \sim \frac{\exp(-r/l_D)}{r}$$

(2) divergences in integrals

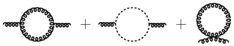


Debye mass $m_D = l_D^{-1}$ is important *IR regulator* in thermal field theory

QCD Debye mass

m_D^2 from longitudinal gluon polarization

$$\Pi =$$



$$= \Pi^{vac} + \Pi^{T}$$

standard textbook(!) prescription

1. **drop** vacuum part

$$\Pi^{vac} \sim \alpha P^2 \left(\epsilon^{-1} + \ln(-P^2/\mu^2)\right)$$

2. **guess** coupling in $\Pi^{T} \sim \alpha T^2$

$$\alpha \to \alpha(Q_T^2)\,, \quad Q_T = 2\pi T$$

$$\tilde{m}_D^2 = \left(1 + \tfrac{1}{6} n_f\right) 4\pi\alpha(Q_T^2)\, T^2$$

QCD Debye mass

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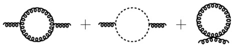
$\Pi =$  $= \Pi^{vac} + \Pi^T$

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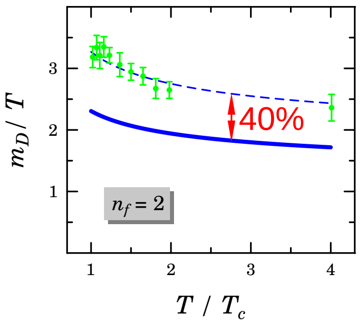
compare to lattice QCD



[Kaczmarek et al.]

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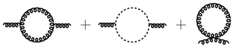
$\Pi =$  $= \Pi^{vac} + \Pi^{T}$

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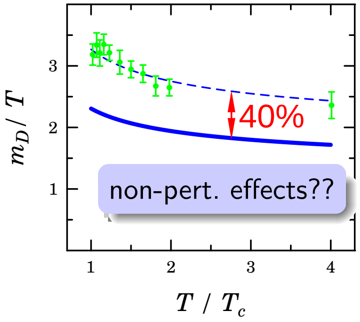
$$\tilde{m}_D^2 = \left(1 + \tfrac{1}{6} n_f\right) 4\pi\alpha(Q_T^2)\, T^2$$

WRONG!

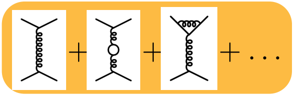
compare to lattice QCD



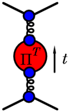
[Kaczmarek et al.]

loop correction to amplitude in heat bath

$\mathcal{M} =$



$= \mathcal{M}^{vac} + \mathcal{M}^{T} \xrightarrow{\text{renorm}}$



$\alpha(t)$

$\Pi^{T}[\,\alpha(t)\,]$

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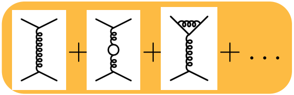
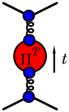
$\mathcal{M} =$  $= \mathcal{M}^{vac} + \mathcal{M}^{T} \xrightarrow{\textit{renorm}}$ 

$\alpha(t)$

$\Pi^T[\,\alpha(t)\,]$

applicable 'leading order' result

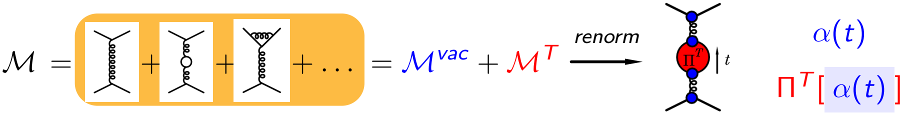
$$m_D^2 = \left(1 + \tfrac{1}{6} n_f\right) 4\pi\alpha(m_D^2)\, T^2$$

- predictive (Λ is fixed)
- RG and gauge invariant

[AP, hep-ph/0601119]

QCD Debye mass

loop correction to amplitude in heat bath



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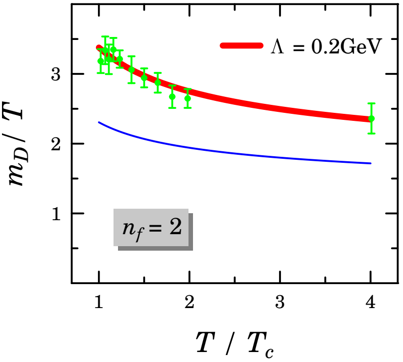
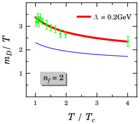
works *quantitatively* for $T \sim T_c$



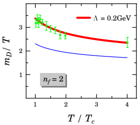
Once more: Why this is more than academic

	uncertainty, '*k*-factor'
$\alpha^{1/2}$	~ 1.4

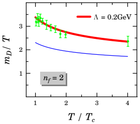


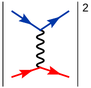
Once more: Why this is more than academic

	uncertainty, 'k-factor'
$\alpha^{1/2}$	~ 1.4
α^{1}	~ 2



	uncertainty, 'k-factor'
$\alpha^{1/2}$	~ 1.4
α^{1}	~ 2
α^{2}	~ 5





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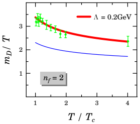
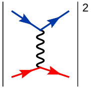
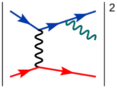
α^3	$\sim \mathcal{O}(10)$







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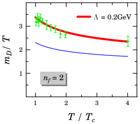

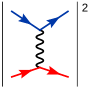

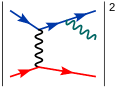

recall: jet quenching interpreted as radiative energy loss

$\Downarrow$

huge transport coefficient,

mabye factor 10 larger than expected

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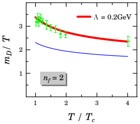

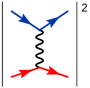

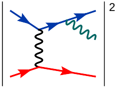

recall: jet quenching interpreted as radiative energy loss

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Do we really understand the *BASICS??*

Parton energy loss

- *paradigm:* radiative dominates collisional energy loss

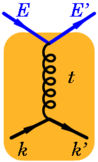
$$\frac{dE^{rad}}{dx} \sim E^{1/2} \quad \text{vs.} \quad \frac{dE^{coll}_{Bjorken}}{dx} \sim \alpha^2 \ln E$$

- *observation:* heavy quarks are also quenched, although they radiate less than light flavors

$\Rightarrow$ Is there a sizable collisional contribution?

QCD collisional energy loss

Bjorken '82: $$\frac{dE^{coll}}{dx} \sim \int_{k^3} \frac{n(k)}{2k} \int^{Ek} dt\, t\, \frac{d\sigma}{dt}$$



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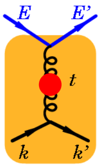

$$\frac{d\sigma}{dt} \sim \frac{\alpha^2}{t^2} \quad : \quad \mathcal{I}_{Bjorken} \sim \int_{\mu^2}^{Ek} dt\, t\, \frac{\alpha^2}{t^2} = \alpha^2 \ln \frac{Ek}{\mu^2}$$

divergence,
IR-cutoff **required**

Gyulassy, Braaten, Thoma calculate $\mu \sim m_D$ with *hard thermal loop* theory

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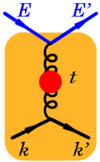

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divergence, IR-cutoff **required**

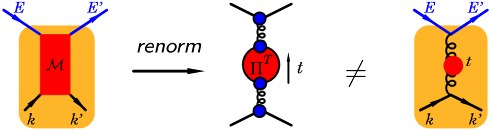
Gyulassy, Braaten, Thoma calculate $\mu \sim m_D$ with *hard thermal loop* theory

conceptual inconsistency

thermal fluctuation needed, **quantum contributions 'forgotten'**

$\Rightarrow$ value of α unspecified in $dE^{coll}_{Bjorken}/dx \sim \alpha^2 T^2 \ln(ET/m_D^2)$

revise Bjorken's formula



$$\frac{d\sigma}{dt} \sim \frac{\alpha^2(t)}{t^2} \quad : \quad \mathcal{I} \sim \int_{m_D^2}^{Ek} dt\, \frac{t}{t^2 \ln^2(t/\Lambda^2)} \sim \alpha(m_D^2) - \alpha(Ek)$$

structural modification of $\mathcal{I}_{Bjorken} \sim \alpha^2 \ln \frac{Ek}{\mu^2}$

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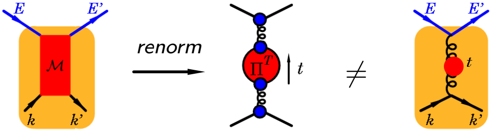

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structural modification of $\mathcal{I}_{Bjorken} \sim \alpha^2 \ln \frac{Ek}{\mu^2}$

applicable 'leading order' result

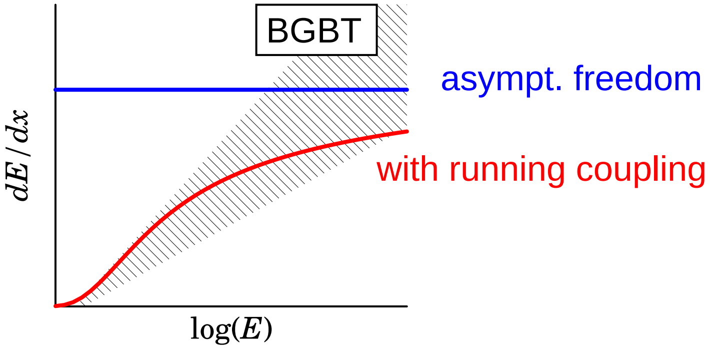
$$\frac{dE^{coll}}{dx} \overset{E\to\infty}{\longrightarrow} \alpha(m_D^2)\, T^2 \quad \text{vs.} \quad \frac{dE^{coll}_{Bjorken}}{dx} \sim \alpha^2 T^2 \ln \frac{ET}{m_D^2}$$

- predictive! (Λ has been fixed)

[AP, PRL (2006)]

saturation due to asymptotic freedom: $\frac{dE^{coll}}{dx} \to \alpha(m_D^2)\,T^2$

vs. $\frac{dE^{coll}_{Bjorken}}{dx} \sim \alpha^2 T^2 \ln(ET/m_D^2)$



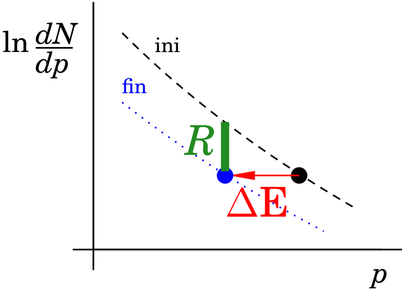
[BDMS]: **mean energy loss inappropriate to describe jet quenching**

'shift' formula misleading!

$$R_{shift} = \frac{dN(p+\Delta E)/dp}{dN/dp}$$

with $\Delta E = L\, dE/dx$

fails for 'steep' spectrum



energy loss ΔE is stochastic

need probability distribution function (*'quenching weights'*)

Occam's razor

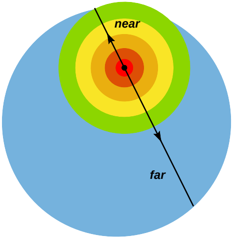
MC simulation

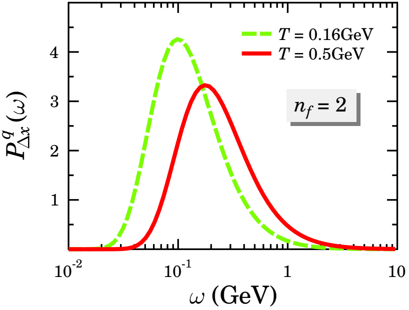
- *only* collisional energy loss
- central collisions and mid rapidity
- Bjorken geometry (R) and dynamics

$$T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3}$$

- local energy-loss probability (eikonal approx., $\mu^2 = \frac{1}{2} m_D^2$)

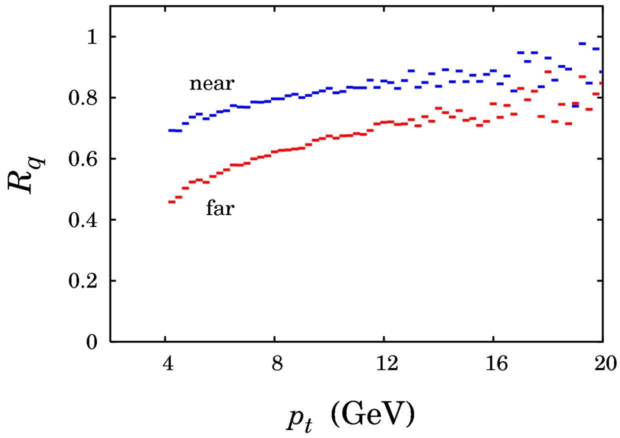
[AP, PRC 2006]



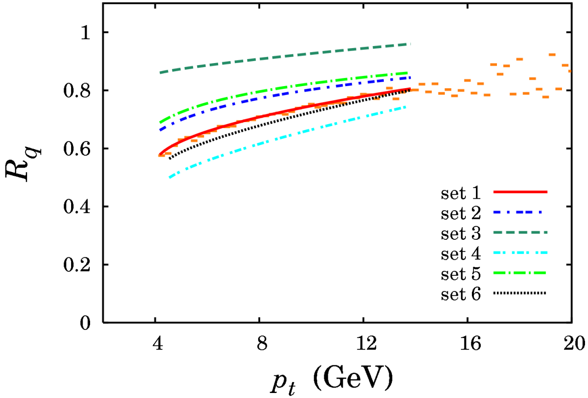


Jet quenching

partonic suppression ratio $R_q = \frac{dN/dp|_{fin}}{dN/dp|_{ini}}$



$$R = 5\,\text{fm}$$
$$T_0 = 0.5\,\text{GeV}$$
$$\tau_0 = 0.2\,\text{fm}$$



collisions do contribute to jet quenching

[AP, PRC 2006]

	set 1	set 2	set 3	set 4	set 5	set 6
R (fm)	5	3	5	5	5	7
T_0 (GeV)	0.5	0.5	0.3	0.7	0.5	0.5
τ_0 (fm)	0.2	0.2	0.2	0.2	0.1	0.2
τ_c (fm)	6.1	6.1	1.3	16.7	3.1	6.1

Resumé

principle of cognition

describe one phenomenon $\longrightarrow$ predict others

QCD:

- fix Λ once and for all: heavy quark potential at $T = 0$
- *(re-)calculate*: Debye mass

  collisional energy loss