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The Newtonian Gravitation Constant: Modern Status of measurement and New CODATA Value

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Henry Cavendish (1731-1810)

Earth's density:

$\sigma=(5.448\pm0.033)\,\mathrm{gcm}^{-3}$

First G value:

$G=(6.67\pm0.07)\times10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$

CODATA 2010 value

$G = (6.67384 \pm 0.00080)\times10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$

The best world experiments on the measurement of G and CODATA values

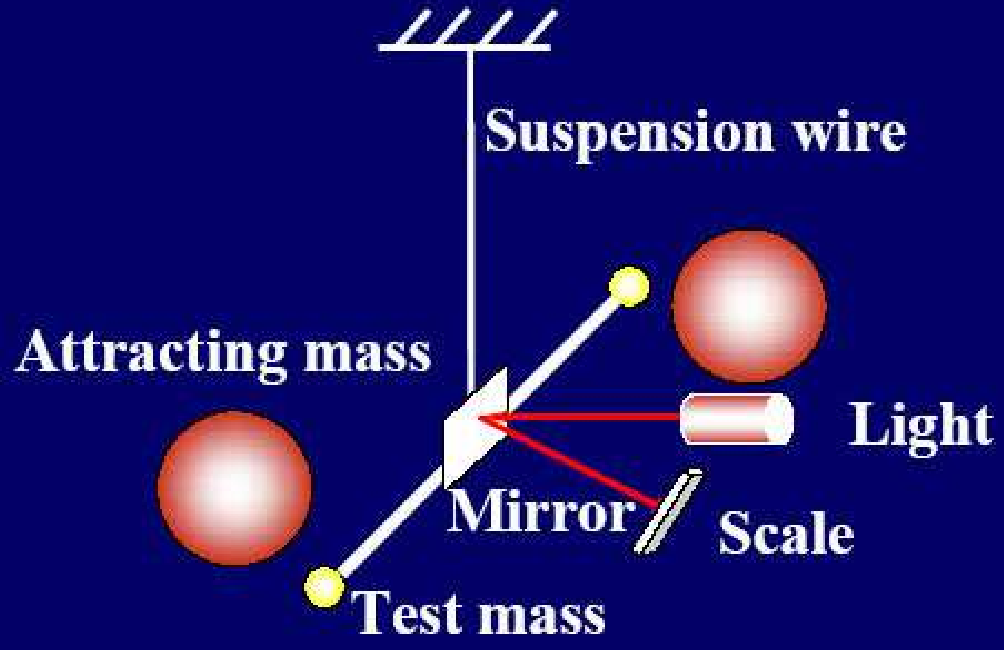
The Task Group on Fundamental Constants of the Committee on Data for Science and Technology (CODATA) was established in 1969 to periodically provide the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry based on all the relevant data available at a given point in time.

The first set of recommended values of the constants provided by CODATA was published in 1973

Authors, year of publication		**Value of $G\times10^{-11}$ м$^{3}kg^{-1}s^{-2}$**	**STD $\times10^{-11}$ $m^{3}kg^{-1}s^{-}$**	***ppm***
[1] Heil and Chrzanowski, Nat. Bur. of Stand., Washington	1942	6.6720	0.0041	615
CODATA	**1973**	**6.6720**	**0.0041**	**615**
[2] Facy and Ponticis, Metrol. Nationale, Yvelines, France	1972	6.6714	0.0006	90
[3] Sagitov, Milyukov, et al. Moscow University, USSR	1979	6.6745	0.0008	120
[4] Luther and Towler, Nat. Bur. of Stand., Washington	1982	6.6726	0.0005	75
CODATA	**1986**	**6.67259**	**0.00085**	**128**
[5] Michaelis, et al.,Phys. Techn. Bundesanstalt, Germany	1995	6.7154	0.0006	90
[6] Karagioz, Izmailov, Committee of Standards, Moscow	1996	6.6729	0.0005	75
[7] Bagley and Luther, Los Alamos National Lab, USA	1997	6.6740	0.0007	105
CODATA	**1998**	**6.673**	**0.010**	**1500**
[8] Jun Luo, et al., HUST, China	1999	6.6699	0.0007	105
[9] Fitzgerald and Armstrong, Meas.St.Lab, New Zealnd	1999	6.6742	0.0007	105
[10] Gundlach and Merkowich, University of Washington,	2000	6.674215	0.000092	14
[11] Quinn, Speake et all. BIPM, France	2001	6.67559	0.00027	41
[12] Schlamminger et all. University of Zurich	2002	6.67407	0.00022	33
[13] Kleinevoß, University of Wuppertal, Germany	2002	6.67422	0.00098	150
[14] Armstrong and Fitzgerald, Meas.St.Lab, New Zealnd	2003	6.67387	0.00027	40
CODATA	**2002**	**6.6742**	**0.0010**	**150**
[15] Hu, Guo, and Luo, HUST, China	2005	6.6723	0.0009	130
[16] Schlamminger et all. University of Zurich	2006	6.674252	0.000109	16
CODATA	**2006**	**6.67428**	**0.00067**	**100**
[17] Jun Luo, et al. HUST, China	2009	6.67349	0.00018	26
[18] Parks and Faller, University of Colorado	2010	6.67234	0.00014	21
CODATA	**2010**	**6.67384**	**0.00080**	**120**

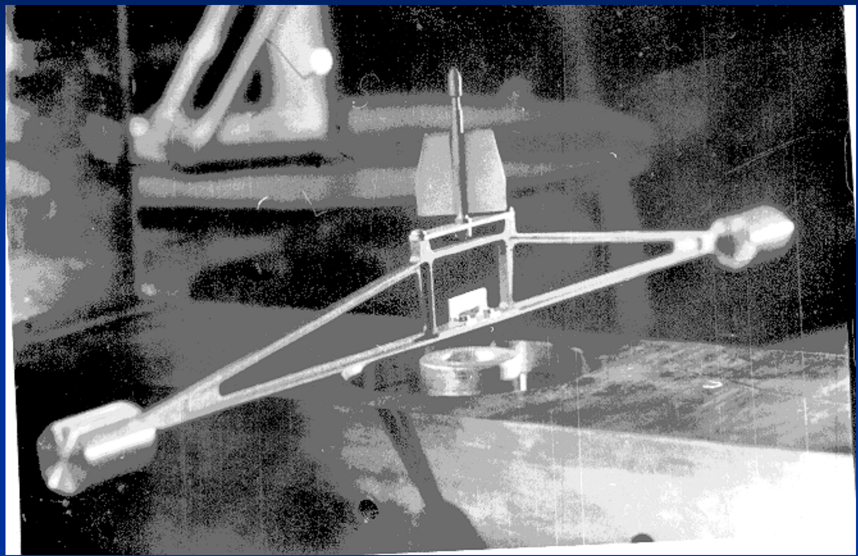
Measurement of G with the torsion balance:

1. Cavendish type experiment: measurement of torsion angle;
2. Time – of - swing method: measurement of frequency of torsion oscillation;



The torsion balances and time of swing method

No 1



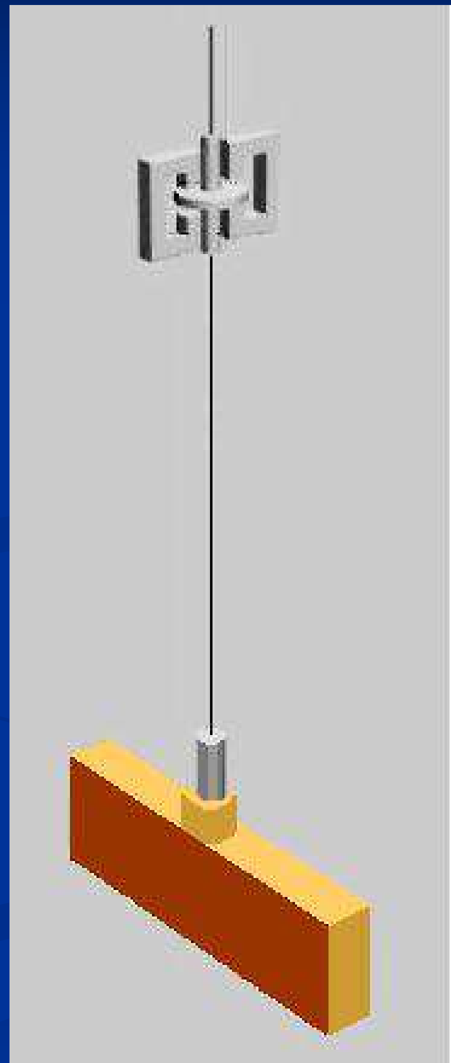
$$J\varphi + [K + G(\partial\Gamma/\partial\varphi)]\varphi = 0$$

$$\omega^2 = \frac{K + G(\partial\Gamma/\partial\varphi)}{J}$$

$$if \quad K = const$$

$$G = \frac{J\left[(\omega^2)_n - (\omega^2)_f\right]}{(\partial\Gamma/\partial\varphi)_n - (\partial\Gamma/\partial\varphi)_f}$$

No 2



Effect of anelasticty

$$if\ K \neq const$$

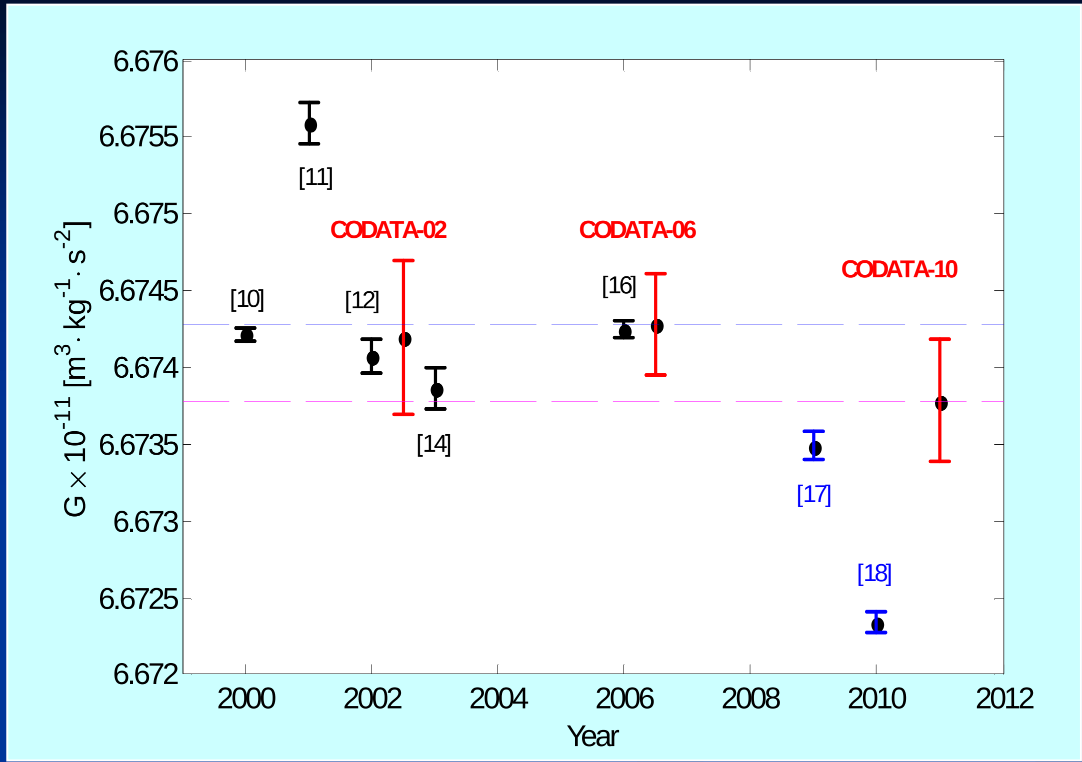
$$G = \frac{J(\omega_n^2 - \omega_f^2) - (K_n(\omega) - K_f(\omega))}{(\partial\Gamma/\partial\varphi)_n - (\partial\Gamma/\partial\varphi)_f}$$

$$G = \frac{J\Delta(\omega^2) - \Delta K(\Delta\omega)}{\Delta(\partial\Gamma/\partial\phi)} = \frac{J\Delta(\omega^2)}{\Delta C_g}\left[1 - \frac{\Delta K}{J\Delta(\omega^2)}\right]$$

$$\frac{\Delta G}{G} = \frac{\Delta K(\omega)}{J\Delta(\omega^2)} \approx \frac{1}{\pi Q}$$

An systematic error of big *G* due to anelasticty of the torsion wire (Kuroda effect)

Q≈1500 – 2000 **Δ G/G ≈ 160 – 200 ppm**



Authors, year of publication		**Value of $G \times 10^{-11}$ $м^3 kg^{-1} s^{-2}$**	**STD $\times 10^{-11}$ $m^3 kg^{-1} s^{-}$**	***ppm***
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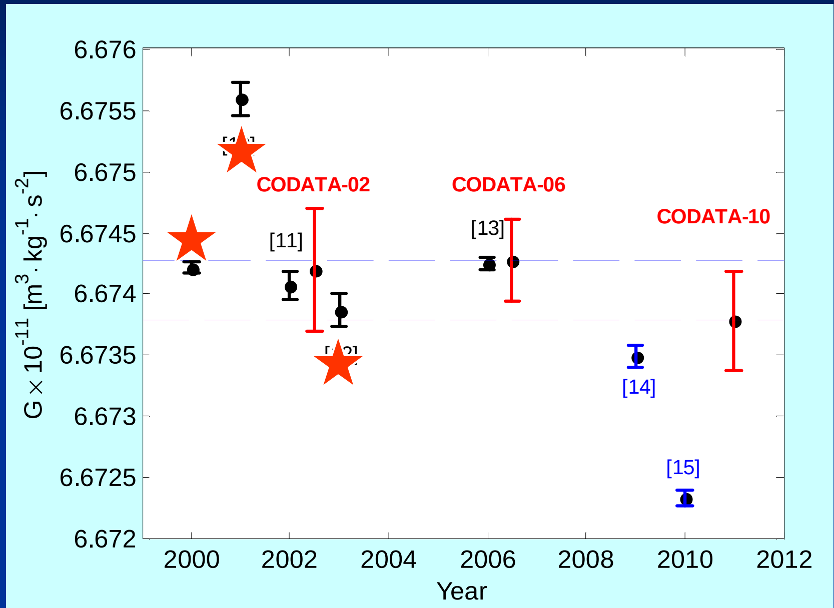
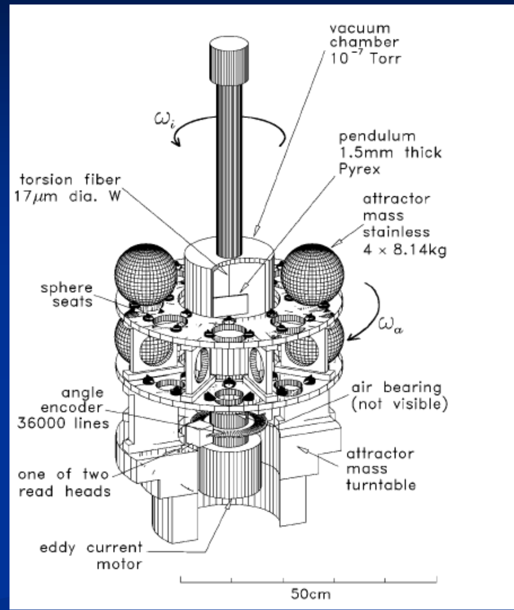
The ways to overcome the problem of anelasticity:

- Minimizing of the effect of the anelasticity by compensating the twist of the torsion fiber;
- Replacing the torsion balance with a beam balance;
- Replacing the torsion balance with a simple pendulum;
- Direct measurement of the effect of anelasticity.
- Increasing mechanical Q of the system (quartz torsion wire).

1. Compensation of the twist of the torsion fiber





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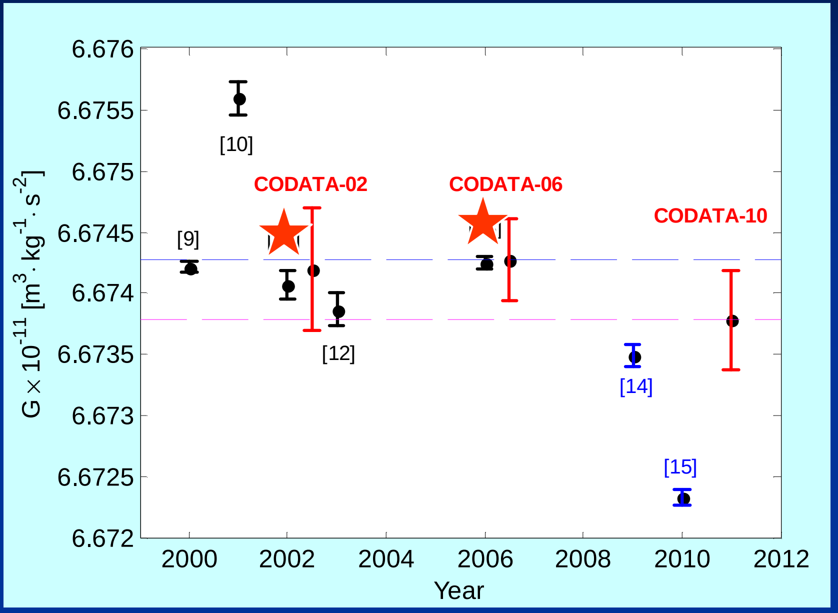
2. Replacing the torsion balance with a beam balance



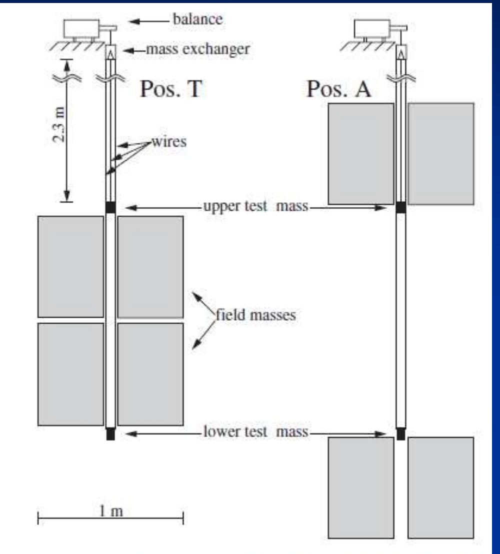


FIG. 1. Principle of the measurement. The FM's are shown in the position together (Pos. T) and the position apart (Pos. A).

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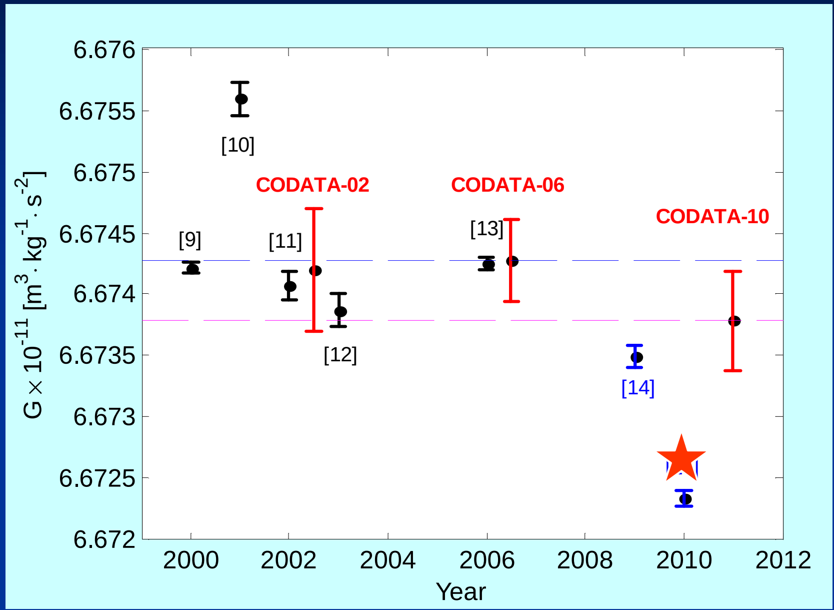
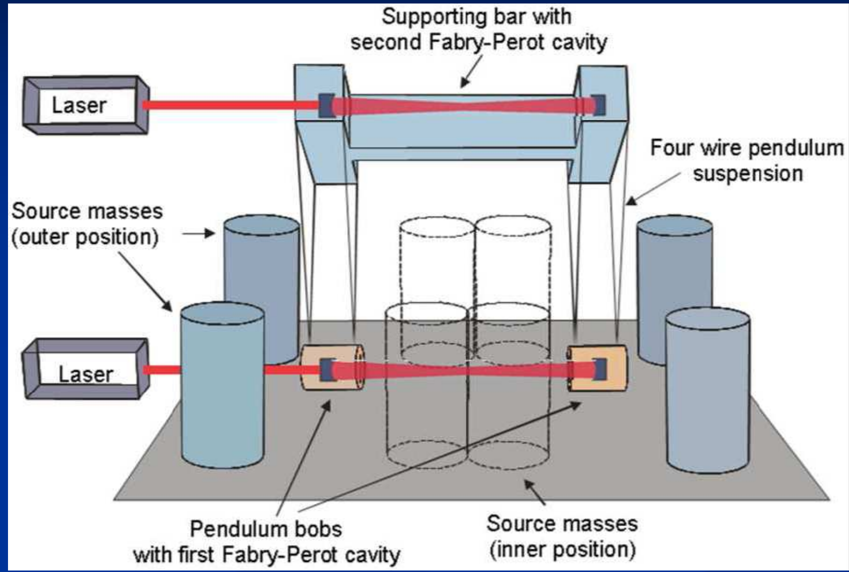
3. Replacing the torsion balance with a simple pendulum





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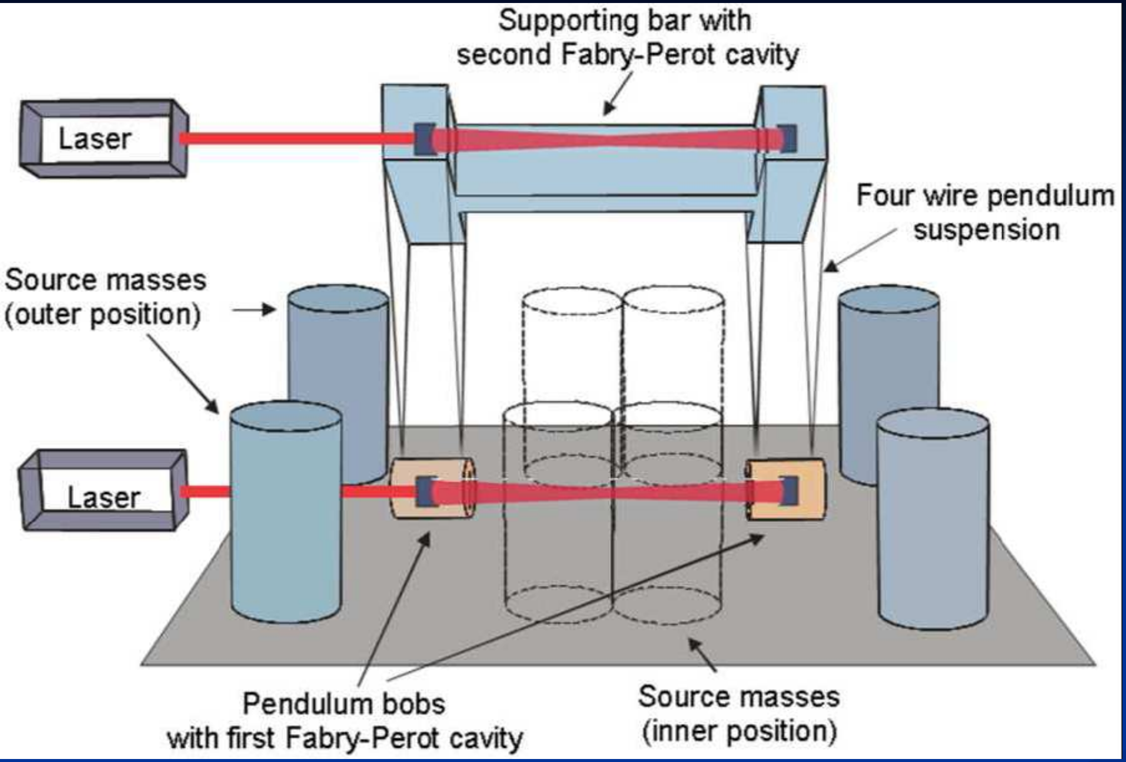
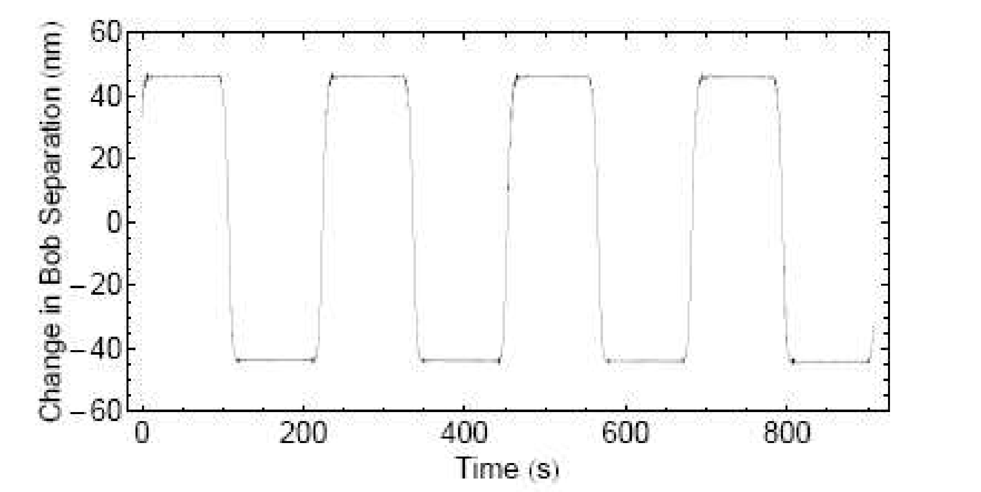




Principle of experiment:

A Fabry-Perot interferometer measures the spacing between the two pendulum bobs with respect to a suspension-point- located reference cavity.

Technical parameters:

The bobs: copper, 780 g.

The pendulum length : 72 см

The spacing between the bob centers :

34 см

Tungsten source masses : 120 kg.,

The masses are floated on air bearings

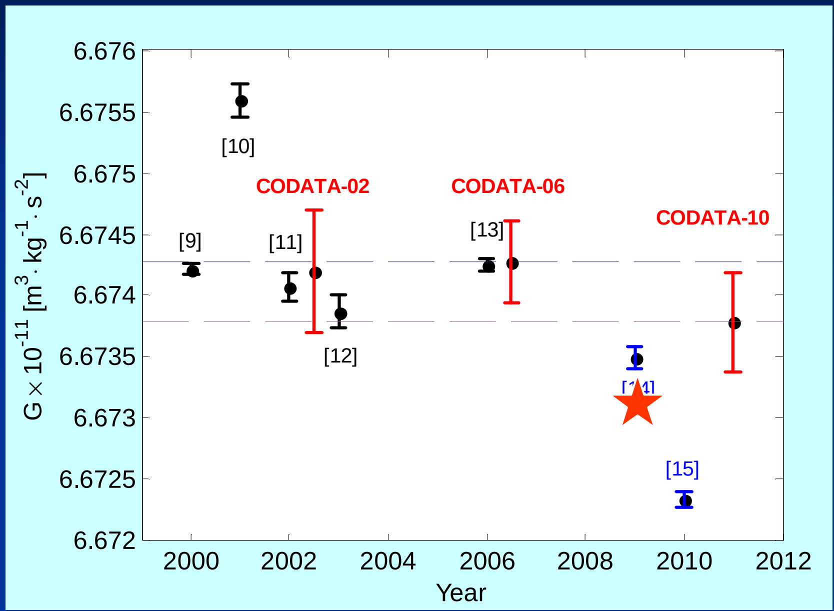
The pendulums are enclosed in vacuum chamber

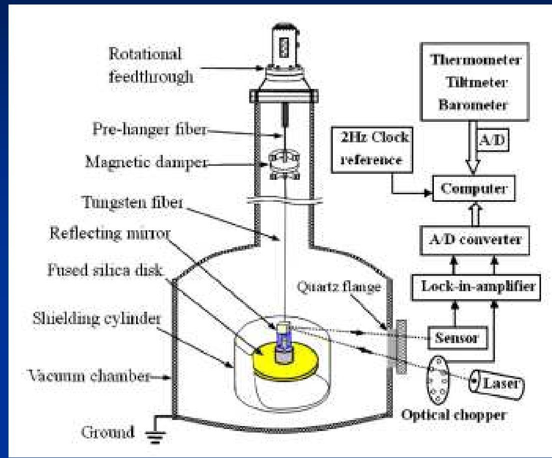
Magnets (not shown) outside of the vacuum system and below the pendulum bobs damp the swinging motion of the pendulums

He-Ne Laser, 1 μW, finesse 400

General view of the experimental setup

4. Direct measurement of the effect of anelasticity





$$\Delta K / \Delta(\omega^2) = (0.954 \pm 0.084) \times 10^{-8}\ kgm^2$$

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CODATA	**2010**	**6.67384**	**0.00080**	**120**

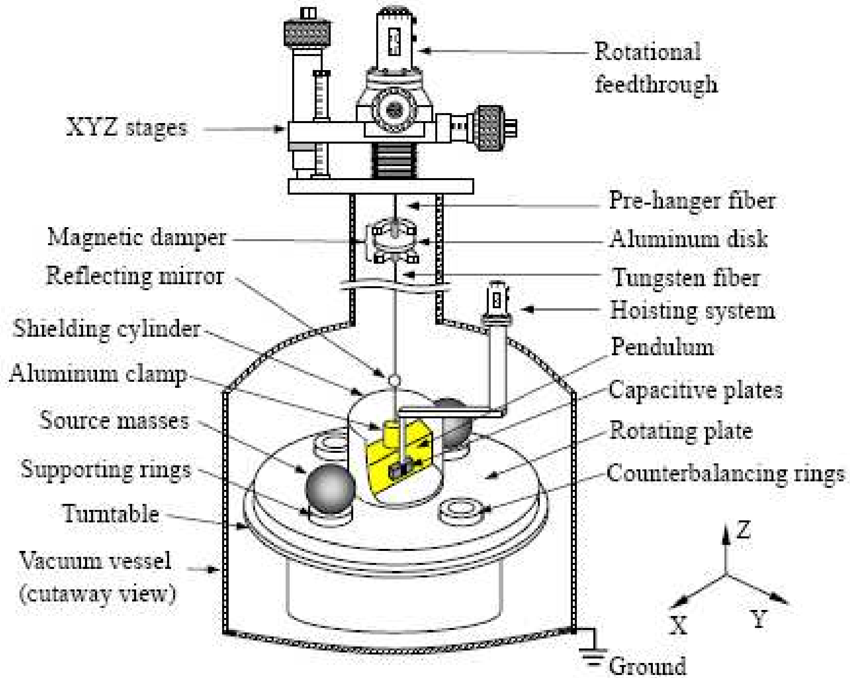
ΔG/G = (211.80±18.69)

Principle of measurement

$$\frac{\Delta G}{G} = \frac{\Delta K(\omega)}{J_G \Delta(\omega^2)} = \frac{J_2\omega_2^2 - J_1\omega_1^2}{J_G(\omega_2^2 - \omega_1^2)}$$

$$\frac{\Delta G}{G} = \frac{J_1}{J_G[(\omega_2/\omega_1)^2 - 1]}\left[\left(\frac{J_2}{J_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2 - 1\right]$$

General and Schematic view of the HUST apparatus for measurement of G



890 mm long, 25 μm diameter tungsten fiber

The torsion balance and source sphere masses

Stainless steel spheres

M=778 g; D=5.71 mm

Rectangular glass block coated with gold

m=75.59 g

91.52 x 12.01 x 27.58 mm

➢ Support rings and the spheres

Thermal expanding coefficient of the plate and the rings is

$(0 \pm 1) \times 10^{-7}$ ℃$^{-1}$

Vacuum chamber

The daily temperature change is less than 0.01 ℃ during the experiments

Remote controlled turntable Huber model:410

Counterbalancing rings Coated with Au

Supporting rings Coated with Au

Supporting plate Coated with Al

Error budget (1)

Error Sources	Corrections	ΔG/G, ppm	
Pendulum			5.07
Dimensions		1.95	
Attitude		0.13	
Nonaligment with fiber		0.45	
Flatness		0.34	
Clamp		1.65	
Density inhomogenity		≤0.21	
Coating layer	-24.28	4.33	
Edge flaw	-0.12	0.17	
Source masses			10.68
Masses		0.82	
Distance of GC		9.64	
Density inhomogenity		4.50	
XYZ positions		0.48	

Error budget (2)

Error Sources	Corrections	ΔG/G, ppm	
Fiber			18.76
Nonlinearity		<0.70	
Thermoelasticity	-39.83	1.52	
Anelasticity	-211.80	18.69	
Aging		<0.01	
Gravitation Nonlinearity			0.30
Magnetic damper			0.31
Magnetic field			0.40
Electrostatic field			0.10
Statistical $\Delta(\omega^2)$			14.18
Total			26.33

New value of Gravitational Constant

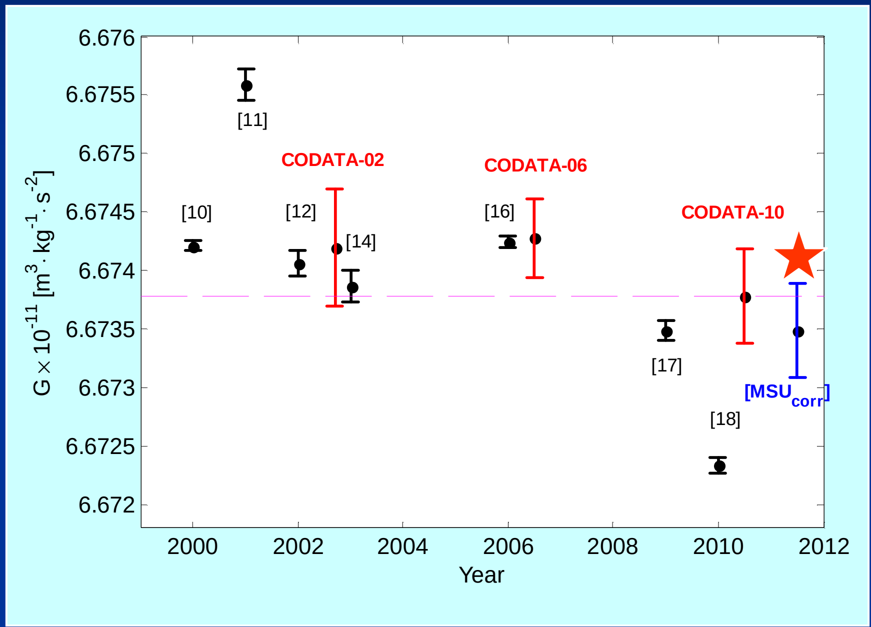
- $G=(6.67349 \pm 0.00018)\times 10\text{-}11\ m^3 kg^{-1} s^{-2}$

with a standard uncertainty 26 ppm

Jun Luo, et al //Phys. Rev. Lett., **102**, 240801 (2009)

Liang-Cheng Tu, et al // Phys. Rev. D 82, 022001 (2010)

Correction of the *G* value, measured in MSU, due to the effect of anelasticity



$$Q=(2.091 \pm 0.040)\times 10^3$$

$$\Delta G/G = -152(3)\ \text{ppm}$$

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Milyukov, Moscow University, USSR	**2011**	**6.67349**	**0.00081**	**120**

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Conclusion

Henry Gavendish : “The apparatus is very simple” (*Philos. Trans. R. Soc. London, **88**, 469, 1798)*

2010

James Faller:

1. “The measurement is very hard” (*Phys. Rev. Lett., **105**, 2010)*
2. “Big G is the Mt. Everest of precision measurement science, and it should be climbed.”