Electronic refrigeration and thermometry in nanostructures at low temperatures

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Nanostructures
Temperature
Energy relaxation
Thermometry
Refrigeration
Present topics of interest
What new can micro- and nanostructures provide at low temperatures?

1. New physical phenomena and versatile fabrication techniques lead to new electronic device concepts.
   - quantum mechanical effects
   - single-electron effects
   - superconductivity
   - non-equilibrium effects

2. Thermal properties are very different from those of macroscopic systems, especially at low temperature. Local refrigeration and thermometry become possible.
Technological basis: micro- and nanofabrication

1. Lithography and thin film deposition (structures in 2D)
   A. Kemppinen et al.,
   Single-electron transistor

2. Etching (structures in 3D)
   A. Luukanen et al.,
   Antenna Coupled Microbolometer
Temperature in an electronic device

\[ f(E) = \frac{1}{1 + e^{(E - \mu)/k_B T}} \]

- **f(E)**
- **E**
- **E = \mu**
- **hot**
- **intermediate**
- **cold**
Generic thermal model for an electronic "thermometer"
The energy distribution of electrons in a small metal conductor

The distribution is determined by energy relaxation:

**Equilibrium** – Thermometer measures the temperature of the "bath"

**Quasi-equilibrium** – Thermometer measures the temperature of the electron system which can be different from that of the "bath"

**Non-equilibrium** – There is no well defined temperature measured by the "thermometer"

Illustration: diffusive normal metal wire
H. Pothier et al. 1997
Electron-phonon relaxation in metals at low $T$

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Hot-electron effects in metals

F. C. Wellstood,* C. Urbina, † and John Clarke
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(Received 21 July 1993)

\[ \dot{Q}_{ep} = \Sigma \Omega (T_e^5 - T_p^5) \]

**FIG. 2.** Emission and absorption of phonons of wave vector $q$ by an electron of wave vector $k$. 

Tunnelling and thermometry by tunnel junctions

Symbols:

Normal junction

Superconducting junction
Tunnel barrier

Examples of aluminium-oxide tunnel barriers
Basics of tunnel junctions

Tunneling from occupied states to empty states
Energy is conserved

\[ I_{1 \rightarrow 2}(V) = \int \mathcal{T}^2 eN_1(E - eV)f_1(E - eV)N_2(E)[1 - f_2(E)]dE \]

\[ I_{2 \rightarrow 1}(V) = \int \mathcal{T}^2 eN_2(E)f_2(E)N_1(E - eV)[1 - f_1(E - eV)]dE \]

\[ I(V) = \mathcal{T}^2 \int eN_1(E - eV)N_2(E)[f_1(E - eV) - f_2(E)]dE \]
Metal – Insulator – Metal (MIM or NIN) tunnel junction

Now, density of states (DOS) is almost constant over the small energy interval:

$$I(V) = T^2 \int eN_1(E - eV)N_2(E)[f_1(E - eV) - f_2(E)]dE$$

If $f_1 = f_2 = f$, we may use

$$I(V) = eT^2 N_1(0)N_2(0) \int [f_1(E - eV) - f_2(E)]dE$$

$$\int [f(E - eV) - f(E)]dE = eV$$

$$I = \frac{V}{R_T}$$

$$R_T = [e^2T^2 N_1(0)N_2(0)]^{-1}$$

Ohmic, no temperature dependence

NOT A THERMOMETER!
Noise?

Average current through a basic tunnel junction is not sensing $T$. How about fluctuations around the mean current?

Equilibrium noise: $I_{\text{ave}} = 0$

Tunnel junction behaves like a resistor

$$S_I = 4k_B T/R_T \quad S_V = 4k_B T R_T$$

Noise thermometry is possible, but it is seldom used as such in nanostructures.
Non-equilibrium: Shot noise thermometry (SNT)

\[ I = \frac{V}{R} \]

\[ S_I \approx 2eI \]

\[ I = 0 \]

\[ S_I \approx \frac{4k_B T}{R} \]

SNT – quantitative analysis

Idea: measure the crossover voltage between thermal noise and shot noise; not necessary to know the absolute calibration for noise measurement.

\[ S_I(V) \approx \frac{2eV}{R} \coth\left(\frac{eV}{2k_BT}\right) \]

\[ P(V) \propto S_I(V) \Delta f \]
Single-electron tunneling and Coulomb blockade thermometry
Single-electron transistor (SET)

\[ R_{T1}, C_1 \quad ne \quad R_{T2}, C_2 \]

Unit of charging energy:

\[ E_C = \frac{e^2}{2C_\Sigma} \]
Coulomb Blockade Thermometry – principal idea

With increasing temperature the IV curve of a SET transistor gets increasingly smeared.
Basic properties of a Coulomb blockade thermometer (CBT)

An array of $N$ tunnel junctions in weak Coulomb blockade, $E_C << k_B T$:

$$V_{1/2} \approx 5.439 N k_B T / e$$

**primary thermometer**

$$\frac{\Delta G}{G_T} = \frac{1}{6} \frac{E_C}{k_B T}$$

**secondary thermometer**

$$E_C \equiv [(N-1)/N] e^2/C$$

J.P. et al., PRL 73, 2903 (1994)
CBT performance at low $T$

Experiments against MCT at CNRS Grenoble and at PTB

Solution: Larger islands and make a (known) correction.

CBBT in magnetic field

Magnetic field has no observable influence on CBT

Single junction thermometer (SJT)

In both cases:

\[ V_{1/2} = 5.439 \frac{Nk_B T}{e} \]

PRL 101, 206801 (2008)
Single Junction Thermometer for $T > 77$ K

FABRICATED BY Yu. Pashkin, NEC
Single Junction Thermometer

\[ \frac{\Delta G}{G_T} \]

- Error of ~1% mainly due to background
NIS-tunnelling and thermometry

\[ I = \frac{1}{eR_T} \int N_S(E)[f_N(E - eV) - f_S(E)]dE = \frac{1}{2eR_T} \int N_S(E)[f_N(E - eV) - f_N(E + eV)]dE \]

\[ N_S(E) = \frac{|E|}{\sqrt{E^2 - \Delta^2}} \]

\[ I = \frac{1}{eR_T} \int_0^{eV} \frac{E}{\sqrt{E^2 - \Delta^2}}dE = \frac{1}{R_T} \sqrt{V^2 - (\Delta/e)^2}, \quad T = 0 \]

\[ I = \frac{V}{R_T}, \quad T \geq T_C \]
NIS-thermometry

\[ I(V) = \frac{1}{2eR_T} \int_{-\infty}^{\infty} N_S(E)[f_N(E - eV) - f_N(E + eV)]dE \]

Probes electron temperature of N island (and not of S!)
Outlook

Electronic primary thermometers, SNT, CBT and SJT presented as prime examples, show great promise

1. as practical calibration-free on-chip thermometers over a wide temperature range
2. in realising the temperature scale, based on a fixed value of the Boltzmann constant, in the sub - 10 K regime

Problems:

Measurement of SNTs is not straightforward
CBTs suffer from self-heating at the lowest temperatures
NANO-REFRIGERATION
Refrigeration on-chip

Thermoelectric refrigeration

Peltier refrigerators, Peltier 1834

Thermionic refrigeration, Mahan, 1994
Korotkov and Likharev, 1999

Quantum-dot refrigerator, Edwards et. al., 1993
Experiment: Prance et al., PRL 102, 146602 (2009).

FIG. 1. (a) Schematic and (b) energy-level diagram of a theoretical design for a quantum-dot refrigerator which could be made using a ZnSe in GaAs/AlGaAs. The reservoir $R$ is cooled to $T_1$, so its Fermi-Dirac distribution is sharpened by resonant tunneling through quantum dots $D_1$ and $D_2$, to the electrodes $F_R$ and $F_L$. A heat load $L$ can be coupled electronically to $R$ via tunneling.
Refrigeration by tunnelling
Energy current in a tunnel junction

In order to find energy current (from conductor 1) one replaces \( e \) by \( E - eV \) in the corresponding expression:

\[
P(V) = T^2 \int (E - eV)N_1(E - eV)N_2(E)[f_1(E - eV) - f_2(E)]dE
\]

Compare to:

\[
I(V) = T^2 \int eN_1(E - eV)N_2(E)[f_1(E - eV) - f_2(E)]dE
\]

For a NIN junction, we obtain

\[
P(V) = \frac{1}{eR_T} \int (E - eV)[f(E - eV) - f(E)]dE = -\frac{V^2}{2R_T}
\]

This means that the Joule power is divided evenly between 1 and 2.
NIS junction as a refrigerator

Cooling power of a NIS junction:

\[ P(V) = \frac{1}{eR_T} \int (E - eV) N_S(E) [f_N(E - eV) - f_S(E)] \, dE \]

Optimum cooling power is reached at \( V \approx \Delta/e \):

\[ P_{\text{NIS, max}} \approx 0.59 \frac{\Delta^2}{e^2 R_T} \left( \frac{k_B T_N}{\Delta} \right)^{3/2} - \frac{\Delta^2}{e^2 R_T} \sqrt{\frac{2\pi k_B T_S}{\Delta}} \exp\left( - \frac{\Delta}{k_B T_S} \right) \]

Optimum cooling power of a NIS junction at \( T_S, T_N << T_C \)

Efficiency (coefficient of performance) of a NIS junction refrigerator:

\[ \eta \approx \frac{k_B T}{\Delta} \]
**Basic idea:**
- bias voltage $V_c$ modifies electron distribution $f(E)$ on N island
- probe junctions measure modified $f(E)$
Early experiments

M. Leivo et al., 1996
Cooling of a superconductor (SIS’IS cooler)

\[ \dot{Q} = \frac{1}{e^2 R T} \int_{-\infty}^{\infty} \left[ f(\varepsilon, T_{e2}) - f(\varepsilon - eV, T_{e1}) \right] \times N_2(\varepsilon) N_1(\varepsilon - eV) \varepsilon \, d\varepsilon \]

Ti – Al sample
\[ [T_C(Ti) = 0.5 \text{ K}, \quad T_C(Al) = 1.3 \text{ K}] \]

Experimental status

Nahum, Eiles, Martinis 1994 *Demonstration of NIS cooling*
Leivo, Pekola, Averin 1996, Kuzmin 2003, Rajauria et al. 2007 *Cooling electrons 300 mK -> 100 mK by SINIS*
Manninen et al. 1999 *Cooling by SIS’IS* see also Chi and Clarke 1979 and Blamire et al. 1991, Tirelli et al. 2008
Manninen et al. 1997, Luukanen et al. 2000 *Lattice refrigeration by SINIS*
Savin et al. 2001 *S – Schottky – Semic – Schottky – S cooling*
Clark et al. 2005, Miller et al. 2008 *x-ray detector refrigerated by SINIS*
Prance et al. 2009 *Electronic refrigeration of a 2DEG*
Kafanov et al. 2009 *RF-refrigeration*

For a review, see Rev. Mod. Phys. 78, 217 (2006).
RF-refrigerator

Question: can one cool the island of a single-electron box by gate?
Typical cooling cycle

\[
\langle Q^\pm \rangle \sim \mp k_B T
\]

Influence of photon assisted tunneling: N. Kopnin et al., PRB 77, 104517 (2008)

RF-refrigerator - experiment

S. Kafanov et al., PRL 2009
Hybrid single-electron turnstile (SINIS)
DC properties of the cooler

Source-drain voltage cools the island, measured current yields temperature
DC current provides thermometry for RF cooling

\[ \Gamma^- / \Gamma^+ = e^{-eV/kT} \]

Results of the RF experiments
Curiosity experiments and (till now) Gedanken experiments

Quantum of thermal conductance
Brownian refrigerator
Temperature fluctuations
Quantized conductance

Electrons:

Electrical conductance in a ballistic contact:

\[ \sigma_Q = \frac{2e^2}{h} \]

Thermal conductance:

\[ G_Q = \frac{\pi k_B^2}{6h} T \]

\( G_Q \) and \( \sigma_Q \) related by Wiedemann-Franz law

More generally:

Example of quantized thermal conductance: phonons in a nanobridge


\[ G = 4 \times 4 \times G_Q \]
Electromagnetic transfer of heat (photons)

Electron system

Electrical environment

Lattice

Schmidt et al., PRL 93, 045901 (2004)
Ojanen et al., PRB 76, 073414 (2007), PRL 100, 155902 (2008)
D. Segal, PRL 100, 105901 (2008)
Heat transported between two resistors

Radiative contribution to net heat flow between electrons of 1 and 2:

\[
P_{\nu} = \int_{0}^{\infty} \frac{d\omega}{2\pi} [S_{P12}(\omega) - S_{P21}(\omega)] = r \frac{\pi k_B^2}{12\hbar} (T_{e1}^2 - T_{e2}^2)
\]

\[
S_{P12}(\omega) - S_{P21}(\omega) = \frac{4R_1R_2\hbar\omega}{|Z_t(\omega)|^2} \left( \frac{1}{e^{\hbar\omega/k_B T_1} - 1} - \frac{1}{e^{\hbar\omega/k_B T_2} - 1} \right)
\]

Impedance matching:

\[
r \equiv \frac{4R_1R_2}{(R_1 + R_2)^2}
\]

Linearized expression for small temperature difference \(\Delta T = T_{e1} - T_{e2}\):

\[
P_{\nu} = r G_Q \Delta T
\]

\[
G_Q = \frac{\pi k_B^2}{6\hbar} T
\]

\[
G_{\nu} = r G_Q
\]
Classical or quantum heat transport?

\[ P_\nu = \int_0^\infty \frac{d\omega}{2\pi} \frac{4R_1 R_2 \hbar \omega}{|Z_t(\omega)|^2} \left( \frac{1}{e^{\hbar \omega / k_B T_1} - 1} - \frac{1}{e^{\hbar \omega / k_B T_2} - 1} \right) \]

\[
\frac{4R_1 R_2}{|Z_t(\omega)|^2}
\]

\[ \omega_c \quad k_B T / \hbar \]

"Classical"

\[ G_\nu \sim r k_B \omega_C \]

"Quantum"

\[ G_\nu = r G_Q \]
Classical or quantum heat transport?

Classical:
\[ \frac{\hbar}{k_B T} \frac{1}{RC} \ll 1 \quad \frac{\hbar}{k_B T} \frac{R}{L} \ll 1 \]

Quantum limited:
\[ \frac{\hbar}{k_B T} \frac{1}{RC} \gg 1 \quad \frac{\hbar}{k_B T} \frac{R}{L} \gg 1 \]

Johnson, Nyquist 1928

\[ T = 300 \text{ K}, \ell = 1 \text{ cm}: \quad \frac{\hbar}{k_B T} \frac{1}{RC} \sim \frac{\hbar}{k_B T} \frac{R}{L} < 10^{-3} \ll 1 \]

\[ T = 100 \text{ mK}, \ell = 100 \mu\text{m}: \quad \frac{\hbar}{k_B T} \frac{1}{RC} \sim \frac{\hbar}{k_B T} \frac{R}{L} \sim 10^2 \gg 1 \]
Demonstration of photonic heat conduction

Tunable impedance matching using DC-SQUIDs

\[ L_J = \frac{\hbar}{2eI_{C,0}|\cos(\pi \Phi/\Phi_0)|} \]

\[ T_0 = 167 \text{mK} \]

157mK

118mK

105mK

75mK

60mK

2nd experiment

SAMPLE A in a loop ("matched")
[SAMPLE B without loop ("not matched")]

\[ V_{th} \] \[ I_{th} \]

10 \( \mu \text{m} \)

3 \( \mu \text{m} \)
Heat transport in different set-ups

Loop geometry (Sample A)  Linear geometry (Sample B)

\[ P^A_\nu = G_Q \Delta T \]

for small temperature difference

\[ \frac{P^B_\nu}{P^A_\nu} = \frac{2}{5} \left( \frac{k_B T R C}{\hbar} \right)^2 \]

\[ \simeq 10^{-3} \]

in the present experiment
Results in the two sample geometries

\[
\frac{\Delta T_2}{\Delta T_1} = \frac{G_V + G_S}{G_V + G_S + G_{ep,2}}
\]

Heat transported by residual quasiparticles at \( T > 0.3 \) K and by photons (in the loop sample) at \( T < 0.3 \) K

A. Timofeev et al., PRL 2009
Brownian refrigerator
Heat flows from hot to cold by photon radiation

This happens between two resistors

\[ R_1, T_1 \]

\[ R_2, T_2 \]

The situation is identical if we replace one resistor by an ordinary tunnel junction

\[ R, T_R \]

\[ R_T, T_N \]
Harmonic vs stochastic drive in refrigeration

Sinusoidal bias – Refrigerates $N$ if frequency and amplitude are not too high

Stochastic drive – Refrigerates $N$ if spectrum is "suitable"

Brownian refrigerator?
Brownian refrigerator

\[ \dot{Q}_N = \frac{2}{e^2 R_T} \int dE' \int dE \ n_S(E) f_N(E') [1 - f_S(E)] P(E' - E) \]

Preliminary experiment on the Brownian refrigerator

A. Timofeev et al., unpublished
Temperature fluctuations
Temperature fluctuations

Consider a body and a small ”sub-body” in it. The probability $w$ of a fluctuation is

$$w \propto \exp(- R_{\text{min}}/k_B T),$$

where $R_{\text{min}}$ is the minimum work needed to carry the small sub-body reversibly from the equilibrium to a state with new values of thermodynamical quantities, and $T$ is the common (average for the sub-system) temperature of the system and the sub-system. We consider the basic case (constant volume) where

$$R_{\text{min}} = \delta E - T \delta S.$$

Here $E$ is the energy and $S$ the entropy of the sub-system. Expanding

$$\delta E = \frac{dE}{dS} \delta S + \frac{1}{2} \frac{d^2 E}{dS^2} (\delta S)^2,$$

and noting that $\frac{dE}{dS} = T$, we have

$$R_{\text{min}} = \frac{1}{2} \frac{d^2 E}{dS^2} (\delta S)^2 = \frac{1}{2} \frac{dT}{dS} (\delta S)^2 = \frac{1}{2} \frac{dS}{dT} (\delta T)^2.$$

Since $\frac{dS}{dT} = C_v/T$, where $C_v$ is the heat capacity of the sub-system, we may write

$$w \propto \exp\left(- \frac{C_v}{2k_B T^2} (\delta T)^2\right).$$

The mean-square of these Gaussian temperature fluctuations is given by

$$\langle \Delta T^2 \rangle = \frac{k_B T^2}{C_v}.$$
Classical temperature fluctuations

Assume that all $T_i$ are equal, and equal to the average of $T$ (equilibrium fluctuations)

$$S \dot{Q}_i = 2k_BT^2G_{th,i}$$

(fluctuation-dissipation theorem)

$$\sum_i \dot{Q}_i = C \dot{T} + \sum_i G_{th,i}(T - T_i)$$

(balance equation)

$$\sum_i S \dot{Q}_i = \omega^2 C^2 S_T + \left(\sum_i G_{th,i}\right)^2 S_T$$

(Fourier transform into noise spectra)

$$S_T(\omega) = \frac{2k_BT^2}{\sum_i G_{th,i}} \frac{1}{1 + \omega^2 C^2 / \left(\sum_i G_{th,i}\right)^2}$$
Classical temperature fluctuations

\[ S_T(\omega) = \frac{2k_B T^2}{\sum_i G_{th,i}} \left( 1 + \frac{\omega^2 C^2}{(\sum_i G_{th,i})^2} \right) \]

\[ \omega_c = \sum_i G_{th,i} / C \]

\[ \langle \delta T^2 \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_T(\omega) = k_B T^2 / C \]
Example system: electrons in the phonon bath

\[ \langle \Delta T^2 \rangle \propto \frac{T}{\mathcal{V}} \]

In this grain at \( T = 100 \text{ mK} \), \( \langle \Delta T^2 \rangle = (10 \text{ mK})^2 \).

Cut-off frequency \( f_c \) determined by electron-phonon relaxation rate, it varies in the range 10 kHz – 10 MHz: suitable for a measurement.
Preliminary measurements

Experimental setup

Magnitude response:

Phase response:

Resonance: $f_0 = 639 \text{ MHz}$
Quality factor: $Q = 280$
Optimal bias point: $I_p = 0$
CBT results for an array

Uniform array, $E_C \ll k_B T$:

$$G(V)/G_T = 1 - \frac{E_C}{k_B T} g \left( \frac{eV}{N k_B T} \right)$$

$$E_C \equiv \left[ \frac{(N-1)}{N} \right] e^2/C$$

$$g(x) = \frac{x \sinh(x) - 4 \sinh^2(x/2)}{8 \sinh^4(x/2)}$$
Long arrays suppress efficiently the errors due to co-tunnelling and finite impedance of the electromagnetic environment.

Parallel arrays lower the sensor impedance to a convenient value.

Another configuration is a 2D array, employed by T. Bergsten et al., APL 78, 1264 (2001).
This method is self-calibrating and can be considered (almost) primary.
On-Chip Cooling/small junctions

Cooling of electrons from 0.3 K to 0.1 K (M.M. Leivo, J.P. Pekola, and D.V. Averin, Appl. Phys. Lett. 68, 1996 (1996).)

Cooling of lattice from 0.2 K to 0.1 K (A. Luukanen, M.M. Leivo, J.K. Suoknuuti, A.J. Manninen, and J.P. Pekola, JLTP, 120, 281 (2000).)
Schottky barrier coolers with superconductor and heavily doped silicon

Schottky barrier coolers – results

Thermometry by S-Sm-S Schottky barrier

Cooling results
2nd experiment

SAMPLE B ("mismatched")
Results in the two sample geometries

**sample A**

**sample B**

**BLUE LINE:** Directly cooled resistor

**RED LINE:** Remote resistor